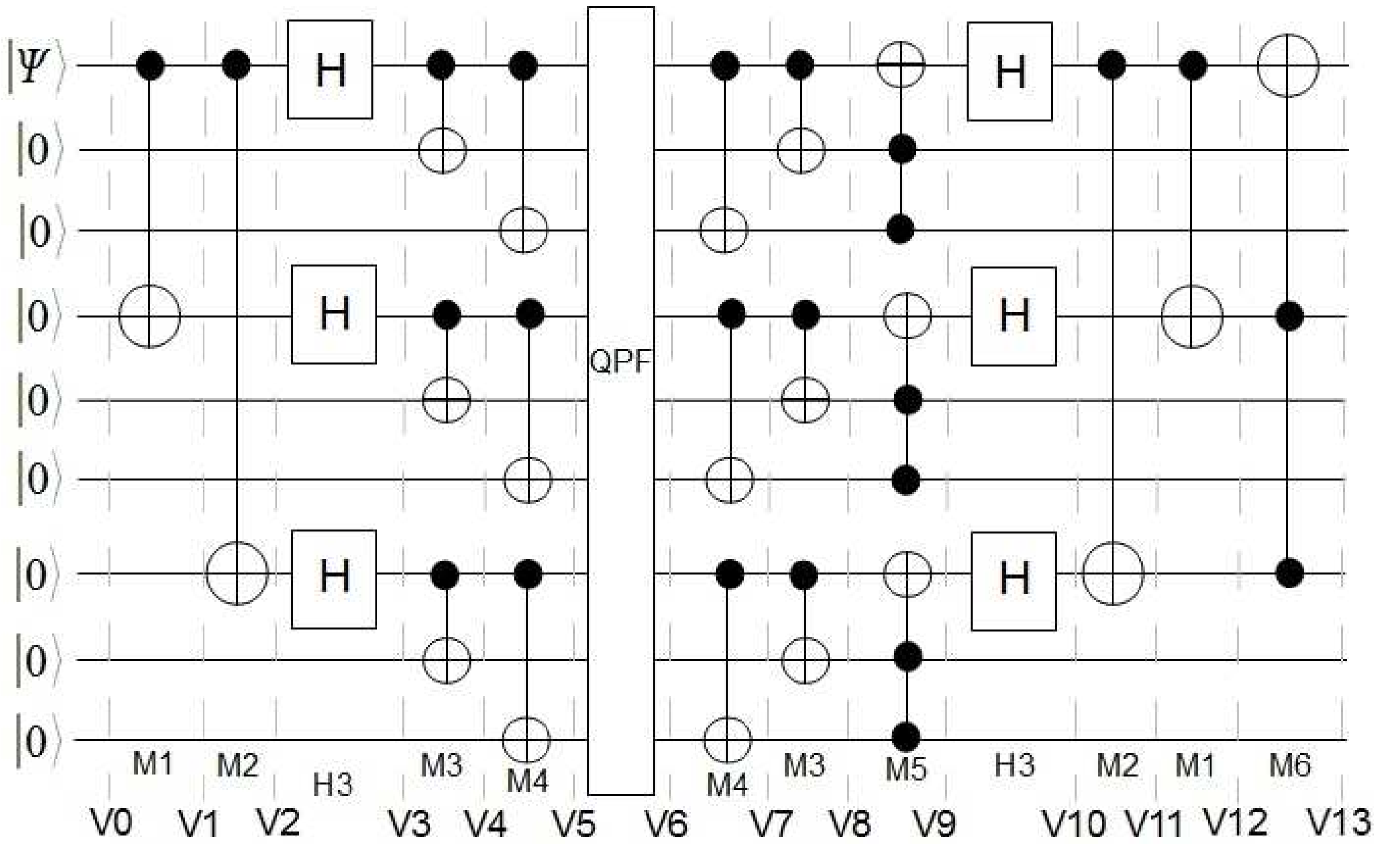
> *restart* :

> *interface*(*warnlevel* = 0) :   # **Maple 12**

> *with*(*LinearAlgebra*) :

> *with*(*Bits*) :

> *Settings*(*defaultbits* = 9) :



Error Selection

qp = 1 : phase-flip and qubit-flip at 1st qubit
= 2 : phase-flip and qubit-flip at 2
= 3 : phase-flip and qubit-flip at 3
= 4 : no phase flips, qubit-flip at 1
= 5 : no phase-flips, qubit-flip at 2
= 6 : no phase-flips, qubit-flip at 3
= 7 : phase-flip at 1, no qubit-flips
= 8 : phase-flip at 4, no qubit-flips
= 9 : phase-flip at 7, no qubit-flips
= 10 : phase-flip & qubit-flip at 6

> qp := 1 :

Utility functions

```
> K := proc(a, b) return KroneckerProduct(a, b) end proc:
> T := proc(x) return Transpose(x) end proc:
> VSte := proc(n)                    # Generates a list of  computational states for  n qubits
            local i, L;              # e.g.  n= 2  ⇒ [ |00⟩ |01⟩ |10⟩ |11⟩ ]
            L := Matrix(1, 2^n);
            Settings(defaultbits = n);
            for i from 1 to 2^n do
                L[1, i] := cat(`|`, String(i − 1, msbfirst), "⟩");
            end do;
      #   print(L);
          return L;       # returns Matrix L
        end proc:
> Gxt := proc(bit, h)                       # generates Toffoli type matrices such as M6
           local i, j, a, b, c, d, e, s;
          global M;
           a := bit - h;
           b := 2^(a + 1);
           c := (2^(bit − 1) − 2^a) + 2;
           s := 2^a + 2;
           d := b − s;
           e := 2^(bit − 1);
           for i from s to c by b do
               for j from i to i + d by 2 do
                    M := RowOperation(M, [ j , ( j + e) ])
               end do;
           end do;
        end proc:
> Gxnot := proc(bit)                          # generates xnot type matrices such as M1 and M2
             local i, srow, lrow;
             global M;
             srow := 2^(bit − 1) + 1;
             lrow := 2^bit − 1;
             for i from  srow to lrow by 2 do
                 M := RowOperation(M, [i, (i + 1) ]);
             end do;
          end proc:
```

Utility matrices

```
> I2 := IdentityMatrix(2) :
> I4 := IdentityMatrix(4) :
> I8 := IdentityMatrix(8) :
> I16 := IdentityMatrix(16) :
> I32 := IdentityMatrix(32) :
> I64 := IdentityMatrix(64) :
> I128 := IdentityMatrix(128) :
```

Utility Operators

> $Uz := RowOperation(I2, 2, -1);$ # *phase-flip operator*

$$Uz := \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{1}$$

> $Ux := RowOperation(I2, [1, 2]);$ # *qubit-flip operator*

$$Ux := \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{2}$$

> $CNOT := RowOperation(I4, [3, 4]);$

$$CNOT := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{3}$$

> $G23 := RowOperation(I4, [2, 3]);$ # *qubit-exchange operator*

$$G23 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

> $H := \frac{1}{\sqrt{2}} Matrix([[1, 1], [1, -1]]);$

$$H := \begin{bmatrix} \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \\ \frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} \end{bmatrix} \tag{5}$$

M1 - M6, H3, and QPF matrices/operators

> M := I16:
M1 := K(Gxnot(4), I32);

$$M1 := \begin{bmatrix} \textit{512 x 512 Matrix} \\ \textit{Data Type: anything} \\ \textit{Storage: rectangular} \\ \textit{Order: Fortran\_order} \end{bmatrix} \tag{6}$$

```
> M := I128:
  M2 := K(Gxnot(7), I4);
```

$$M2 := \begin{bmatrix} \textit{512 x 512 Matrix} \\ \textit{Data Type: anything} \\ \textit{Storage: rectangular} \\ \textit{Order: Fortran\_order} \end{bmatrix} \tag{7}$$

```
> H3 := K(K(H, I4), K(K(H, I4), K(H, I4))) :
  '√8 · H3' = √8 H3;
```

$$\sqrt{8}\; H3 = \begin{bmatrix} \textit{512 x 512 Matrix} \\ \textit{Data Type: anything} \\ \textit{Storage: rectangular} \\ \textit{Order: Fortran\_order} \end{bmatrix} \tag{8}$$

```
> M3 := K(K(K(CNOT, I2), K(CNOT, I2)), K(CNOT, I2));
```

$$M3 := \begin{bmatrix} \textit{512 x 512 Matrix} \\ \textit{Data Type: anything} \\ \textit{Storage: rectangular} \\ \textit{Order: Fortran\_order} \end{bmatrix} \tag{9}$$

```
> M4 := K(Multiply( K(G23, I2), Multiply(K(I2, CNOT), K(G23, I2))),
            K(Multiply( K(G23, I2), Multiply(K(I2, CNOT), K(G23, I2))),
               Multiply( K(G23, I2), Multiply(K(I2, CNOT), K(G23, I2)))));
```

$$M4 := \begin{bmatrix} \textit{512 x 512 Matrix} \\ \textit{Data Type: anything} \\ \textit{Storage: rectangular} \\ \textit{Order: Fortran\_order} \end{bmatrix} \tag{10}$$

IT matrix/operator is a variation of the Toffoli gate. The 2nd and 3rd qubits are the control qubits. When the state of both qubits equal "1" the 1st qubit is "flipped"

$$c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + \mathbf{c_3}|\mathbf{011}\rangle + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + \mathbf{c_7}|\mathbf{111}\rangle$$

$$c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + \mathbf{c_3}|\mathbf{111}\rangle + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + \mathbf{c_7}|\mathbf{011}\rangle$$

```
> IT := RowOperation(I8, [8, 4]) :
  M5 := K(K(IT, IT), IT);
```

$$M5 := \begin{bmatrix} \textit{512 x 512 Matrix} \\ \textit{Data Type: anything} \\ \textit{Storage: rectangular} \\ \textit{Order: Fortran\_order} \end{bmatrix} \quad \textbf{(11)}$$

```
> M := I128:
  M6 := K(Gxt(7, 4), I4);
```

$$M6 := \begin{bmatrix} \textit{512 x 512 Matrix} \\ \textit{Data Type: anything} \\ \textit{Storage: rectangular} \\ \textit{Order: Fortran\_order} \end{bmatrix} \quad \textbf{(12)}$$

QPF: qubit error operator

```
> if qp = 1 then
    QPF := K(Multiply(Ux, Uz), IdentityMatrix(2^8)); # phase-flip & qubit-flip at 1
  elif qp = 2 then
    QPF := K(I2, K(Multiply(Ux, Uz), IdentityMatrix(2^7))); # phase-flip  & qubit-flip at 2
  elif qp = 3 then
    QPF := K(I4, K(Multiply(Ux, Uz), IdentityMatrix(2^6))); # phase-flip  & qubit-flip at 3
  elif qp = 4 then
    QPF := K(Multiply(Ux, I2), IdentityMatrix(2^8)); # no phase-flip & qubit-flip at 1
  elif qp = 5 then
    QPF := K(I2, K(Multiply(Ux, I2), IdentityMatrix(2^7))); # no phase-flip & qubit-flip at 2
  elif qp = 6 then
    QPF := K(I4, K(Multiply(Ux, I2), IdentityMatrix(2^6))); # no phase-flip & qubit-flip at 3
  elif qp = 7 then
    QPF := K(Multiply(I2, Uz), IdentityMatrix(2^8)); #  phase-flip at 1 &  no qubit-flip
  elif qp = 8 then
    QPF := K(I8, K(Multiply(I2, Uz), IdentityMatrix(2^5))); #  phase-flip at 4 & no qubit-flip
  elif qp = 9 then
    QPF := K(I64, K(Multiply(I2, Uz), IdentityMatrix(2^2))); #  phase-flip at 7 & no qubit-flip
  elif qp = 10 then
    QPF := K(I32, K(Multiply(Ux, Uz), IdentityMatrix(2^3))); # phase-flip & qubit-flip at 6
  end if;
```

$$QPF := \begin{bmatrix} \textit{512 x 512 Matrix} \\ \textit{Data Type: anything} \\ \textit{Storage: rectangular} \\ \textit{Order: Fortran\_order} \end{bmatrix} \quad \textbf{(13)}$$

$$\mathbf{V0} = (\alpha|0\rangle + \beta|1\rangle) \otimes |00000000\rangle = \alpha|000000000\rangle + \beta|100000000\rangle$$

The octal representation: $\alpha|0\rangle_o + \beta|400\rangle_o$

```
> n := 9 :
q1 := Matrix([[α], [β]]) :    # α|0⟩ + β|1⟩
qn := Matrix([[1], [0]]) :    #  |0⟩
Co0 := K(q1, K(qn, K(qn, K(qn, K(qn, K(qn, K(qn, K(qn, qn)))))))) :
St := Transpose(VSte(n)) :
|V0⟩ := Multiply(T(Co0), St)[1, 1];
```

$$|V0\rangle := \alpha\,|000000000\rangle + \beta\,|100000000\rangle \quad (14)$$

$$\begin{aligned}\mathbf{V1} &= \mathbf{M1}\cdot\mathbf{V0}\\ &= \mathbf{M1}(\alpha|000000000\rangle + \beta|100000000\rangle)\\ &= \alpha|000000000\rangle + \beta|100100000\rangle\\ &= \alpha|0\rangle_o + \beta|440\rangle_o\end{aligned}$$

```
> Co1 := Multiply(M1, Co0) :
|V1⟩ := Multiply(T(Co1), St)[1, 1];
```

$$|V1\rangle := \beta\,|100100000\rangle + \alpha\,|000000000\rangle \quad (15)$$

$$\begin{aligned}\mathbf{V2} &= \mathbf{M2}\cdot\mathbf{V1}\\ &= \mathbf{M2}(\alpha|000000000\rangle + \beta|100100000\rangle)\\ &= \alpha|000000000\rangle + \beta|100100100\rangle\\ &= \alpha|0\rangle_o + \beta|444\rangle_o\end{aligned}$$

```
> Co2 := Multiply(M2, Co1) :
|V2⟩ := Multiply(T(Co2), St)[1, 1];
```

$$|V2\rangle := \beta\,|100100100\rangle + \alpha\,|000000000\rangle \quad (16)$$

The Hadamard H3 matrix/operator

$$H\,|0\rangle = \frac{\alpha|0\rangle + \beta|1\rangle}{\sqrt{2}};\ H\,|1\rangle = \frac{\alpha|0\rangle - \beta|1\rangle}{\sqrt{2}},$$

$$\begin{aligned}\mathbf{V3} = {} & \alpha\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\otimes\left|00\right\rangle\otimes\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\otimes\left|00\right\rangle\otimes\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\otimes\left|00\right\rangle \\ & + \beta\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\otimes\left|00\right\rangle\otimes\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\otimes\left|00\right\rangle\otimes\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\otimes\left|00\right\rangle\end{aligned}$$

$$\begin{aligned}\mathbf{V3} = {} & (\alpha+\beta)|000000000\rangle + (\alpha-\beta)|000000100\rangle + (\alpha-\beta)|000100000\rangle + (\alpha+\beta)|000100100\rangle \\ & + (\alpha-\beta)|100000000\rangle + (\alpha+\beta)|100000100\rangle + (\alpha+\beta)|100100000\rangle + (\alpha-\beta)|100100100\rangle\end{aligned}$$

$$\begin{aligned}\mathbf{V3} = {} & (\alpha+\beta)|0\rangle_o + (\alpha-\beta)|4\rangle_o + (\alpha-\beta)|40\rangle_o + (\alpha+\beta)|44\rangle_o \\ & + (\alpha-\beta)|400\rangle_o + (\alpha+\beta)|404\rangle_o + (\alpha+\beta)|440\rangle_o + (\alpha-\beta)|444\rangle_o\end{aligned}$$

$$\begin{aligned}\mathbf{V3} = {} & (\alpha+\beta)|0\rangle + (\alpha-\beta)|4\rangle + (\alpha-\beta)|32\rangle + (\alpha+\beta)|36\rangle \\ & + (\alpha-\beta)|256\rangle + (\alpha+\beta)|260\rangle + (\alpha+\beta)|288\rangle + (\alpha-\beta)|292\rangle\end{aligned}$$

> $\mathrm{Co3} := simplify\left(\sqrt{8}\ Multiply(H3, Co2)\right):$
> $|V3\rangle := Multiply(T(Co3), St)[1, 1];$

$$\begin{aligned}|V3\rangle := {} & (\alpha+\beta)\ |000000000\rangle + (\alpha-\beta)\ |000000100\rangle + (\alpha-\beta)\ |000100000\rangle + (\alpha+\beta)\ |000100100\rangle \\ & + (\alpha-\beta)\ |100000000\rangle + (\alpha+\beta)\ |100000100\rangle + (\alpha+\beta)\ |100100000\rangle + (\alpha-\beta)\ |100100100\rangle\end{aligned} \quad \mathbf{(17)}$$

$$\begin{aligned}\mathbf{V4} = {} & \mathbf{M3}\cdot\mathbf{V3} \\ = {} & (\alpha+\beta)|0\rangle_o + (\alpha-\beta)|6\rangle_o + (\alpha-\beta)|60\rangle_o + (\alpha+\beta)|66\rangle_o \\ & + (\alpha-\beta)|600\rangle_o + (\alpha+\beta)|606\rangle_o + (\alpha+\beta)|660\rangle_o + (\alpha-\beta)|666\rangle_o\end{aligned}$$

> $Co4 := Multiply(M3, Co3):$
> $|V4\rangle := Multiply(T(Co4), St)[1, 1];$

$$\begin{aligned}|V4\rangle := {} & (\alpha-\beta)\ |000000110\rangle + (\alpha-\beta)\ |000110000\rangle + (\alpha+\beta)\ |000110110\rangle + (\alpha \\ & -\beta)\ |110000000\rangle + (\alpha+\beta)\ |110000110\rangle + (\alpha+\beta)\ |110110000\rangle + (\alpha-\beta)\ |110110110\rangle + (\alpha \\ & +\beta)\ |000000000\rangle\end{aligned} \quad \mathbf{(18)}$$

$V5 = M4 \cdot V4$

$$= (\alpha + \beta)|0\rangle_o + (\alpha - \beta)|7\rangle_o + (\alpha - \beta)|70\rangle_o + (\alpha + \beta)|77\rangle_o$$
$$+ (\alpha - \beta)|700\rangle_o + (\alpha + \beta)|707\rangle_o + (\alpha + \beta)|770\rangle_o + (\alpha - \beta)|777\rangle_o$$

```
> Co5 := Multiply(M4, Co4) :
  |V5⟩ := Multiply(T(Co5), St)[1, 1];
```

$$|V5\rangle := (\alpha - \beta)\ |000000111\rangle + (\alpha - \beta)\ |000111000\rangle + (\alpha + \beta)\ |000111111\rangle + (\alpha - \beta)\ |111000000\rangle + (\alpha + \beta)\ |111000111\rangle + (\alpha + \beta)\ |111111000\rangle + (\alpha - \beta)\ |111111111\rangle + (\alpha + \beta)\ |000000000\rangle \quad (19)$$

Qubit Error: QPF

A phase-flip error on the first qubit

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|0\rangle - \beta|1\rangle$$

or

$$\alpha|+\rangle + \beta|-\rangle \rightarrow \alpha|-\rangle + \beta|+\rangle$$

A qubit-flip error on the first qubit

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|1\rangle + \beta|0\rangle$$

Both types of errors

$$\alpha\left(\frac{|000\rangle+|111\rangle}{\sqrt{2}}\right) \rightarrow \alpha\left(\frac{|100\rangle-|011\rangle}{\sqrt{2}}\right)$$
$$\beta\left(\frac{|000\rangle-|111\rangle}{\sqrt{2}}\right) \rightarrow \beta\left(\frac{|100\rangle+|011\rangle}{\sqrt{2}}\right)$$

if qp = 1

$V6 = QPF \cdot V7$

$$= (\alpha + \beta)|400\rangle_o + (\alpha - \beta)|407\rangle_o + (\alpha - \beta)|470\rangle_o + (\alpha + \beta)|477\rangle_o$$
$$+ (-\alpha + \beta)|300\rangle_o + (-\alpha - \beta)|307\rangle_o - (\alpha - \beta)|370\rangle_o + (-\alpha + \beta)|377\rangle_o$$

```
> Co6 := Multiply(QPF, Co5) :
  |V6⟩ := Multiply(T(Co6), St)[1, 1];
```

$$|V6\rangle := (-\alpha + \beta)\ |011000000\rangle + (-\alpha - \beta)\ |011000111\rangle + (-\alpha - \beta)\ |011111000\rangle + (-\alpha + \beta)\ |011111111\rangle + (\alpha + \beta)\ |100000000\rangle + (\alpha - \beta)\ |100000111\rangle + (\alpha - \beta)\ |100111000\rangle + (\alpha + \beta)\ |100111111\rangle \quad (20)$$

if qp = 1

V7 = M4 · V6

$= (\alpha+\beta)|500\rangle_o + (\alpha-\beta)|506\rangle_o + (\alpha-\beta)|560\rangle_o + (\alpha+\beta)|566\rangle_o$
$+ (-\alpha+\beta)|300\rangle_o + (-\alpha-\beta)|306\rangle_o + (-\alpha-\beta)|360\rangle_o + (-\alpha+\beta)|366\rangle_o$

> $Co7 := Multiply(M4, Co6):$
> $|V7\rangle := Multiply(T(Co7), St)[1, 1];$

$$|V7\rangle := (-\alpha+\beta)\,|011000000\rangle + (-\alpha-\beta)\,|011000110\rangle + (-\alpha-\beta)\,|011110000\rangle + (-\alpha+\beta)\,|011110110\rangle + (\alpha+\beta)\,|101000000\rangle + (\alpha-\beta)\,|101000110\rangle + (\alpha-\beta)\,|101110000\rangle + (\alpha+\beta)\,|101110110\rangle \quad \textbf{(21)}$$

if qp = 1

V8 = M3 · V7

$= (\alpha+\beta)|700\rangle_o + (\alpha-\beta)|704\rangle_o + (\alpha-\beta)|740\rangle_o + (\alpha+\beta)|744\rangle_o$
$+ (-\alpha+\beta)|300\rangle_o + (-\alpha-\beta)|304\rangle_o + (-\alpha-\beta)|340\rangle_o + (\alpha-\beta)|344\rangle_o$

> $Co8 := Multiply(M3, Co7):$
> $|V8\rangle := Multiply(T(Co8), St)[1, 1];$

$$|V8\rangle := (-\alpha+\beta)\,|011000000\rangle + (-\alpha-\beta)\,|011000100\rangle + (-\alpha-\beta)\,|011100000\rangle + (-\alpha+\beta)\,|011100100\rangle + (\alpha+\beta)\,|111000000\rangle + (\alpha-\beta)\,|111000100\rangle + (\alpha-\beta)\,|111100000\rangle + (\alpha+\beta)\,|111100100\rangle \quad \textbf{(22)}$$

if qp = 1

V10 = M5 · V9

$= (\alpha+\beta)|300\rangle_o + (\alpha-\beta)|304\rangle_o + (\alpha-\beta)|340\rangle_o + (\alpha+\beta)|344\rangle_o$
$+ (-\alpha+\beta)|700\rangle_o + (-\alpha-\beta)|704\rangle_o + (-\alpha-\beta)|740\rangle_o + (-\alpha+\beta)|744\rangle_o$

> $Co9 := Multiply(M5, Co8):$
> $|V9\rangle := Multiply(T(Co9), St)[1, 1];$

$$|V9\rangle := (\alpha+\beta)\,|011100100\rangle + (-\alpha+\beta)\,|111000000\rangle + (-\alpha-\beta)\,|111000100\rangle + (-\alpha-\beta)\,|111100000\rangle + (-\alpha+\beta)\,|111100100\rangle + (\alpha+\beta)\,|011000000\rangle + (\alpha-\beta)\,|011000100\rangle + (\alpha-\beta)\,|011100000\rangle \quad \textbf{(23)}$$

When qb = 1: phase-flip and qubit-flip errors at q1

$$V9 = \alpha\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\otimes\left|11\right\rangle\otimes\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\otimes\left|00\right\rangle\otimes\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\otimes\left|00\right\rangle$$
$$+\ \beta\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\otimes\left|11\right\rangle\otimes\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\otimes\left|00\right\rangle\otimes\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\otimes\left|00\right\rangle$$

$$= \alpha(q1 \otimes|11\rangle\otimes q4 \otimes|00\rangle\otimes q7 \otimes|00\rangle) + \beta(q1 \otimes|11\rangle\otimes q4 \otimes|00\rangle\otimes q7 \otimes|00\rangle)$$

Apply the H3 operator

$$V10 = \alpha(|1\rangle\otimes|11\rangle\otimes |0\rangle\otimes|00\rangle\otimes |0\rangle\otimes|00\rangle) + \beta(|0\rangle\otimes|11\rangle\otimes|1\rangle \otimes|00\rangle\otimes |1\rangle\otimes|00\rangle)$$
$$= \alpha|111000000\rangle + \beta|011100100\rangle$$
$$= \alpha|700\rangle_o + \beta|344\rangle_o$$

> $Co10 := simplify\left(\frac{1}{\sqrt{8}} Multiply(H3, Co9)\right):$
$|V10\rangle := Multiply(T(Co10), St)[1, 1];$

$$|V10\rangle := \alpha\ |111000000\rangle + \beta\ |011100100\rangle \tag{24}$$

if qp = 1
V11 = M2 · V10
$$= M2(\alpha|111000000\rangle+\beta|011100100\rangle$$
$$= \alpha|111000100\rangle+\beta|011100100\rangle$$
$$= \alpha|704\rangle_o + \beta|344\rangle_o$$

> $Co11 := Multiply(M2, Co10):$
$|V11\rangle := Multiply(T(Co11), St)[1, 1];$

$$|V11\rangle := \beta\ |011100100\rangle + \alpha\ |111000100\rangle \tag{25}$$

if qp = 1
V12 = M1 · V11
$$= M1(\alpha|111000100\rangle+\beta|011100100\rangle)$$
$$= \alpha|111100100\rangle+\beta|011100100\rangle$$
$$= \alpha|744\rangle_o + \beta|344\rangle_o$$

> $Co12 := Multiply(M1, Co11):$
$|V12\rangle := Multiply(T(Co12), St)[1, 1];$

$$|V12\rangle := \beta\ |011100100\rangle + \alpha\ |111100100\rangle \tag{26}$$

if qp = 1

$V13 = M6 \cdot V12$

$= M6(\alpha|111100100\rangle+\beta|011100100\rangle$

$= \alpha|011100100\rangle+\beta|111100100\rangle$

$= \alpha|344\rangle_o + \beta|744\rangle_o$

which can be re-written as

$V13 = (\alpha|0\rangle+\beta|1\rangle) \otimes|11100100\rangle$

if qp = 2 then $V13 = \alpha|010100100\rangle+\beta|110100100\rangle = \alpha|244\rangle_o + \beta|644\rangle_o$

if qp = 3 then $V13 = \alpha|001100100\rangle+\beta|101100100\rangle = \alpha|144\rangle_o + \beta|544\rangle_o$

```
> Co13 := Multiply(M6, Co12) :
  |V13> := Multiply(T(Co13), St)[1, 1];
```

$$|V13\rangle := \beta\,|111100100\rangle + \alpha\,|011100100\rangle \quad (27)$$

The 9-qubit code:

$|q_1\, q_2\, q_3\, q_4\, q_5\, q_6\, q_7\, q_8\, q_9\rangle$:

qubit-flips: q_2q_3 = 11 1st qubit q_5q_6 = 11 4th qubit q_8q_9 = 11 7th qubit
= 10 2nd = 10 5th = 10 8th
= 01 3rd = 01 6th = 01 9th

phase-flip codes: q_4q_7 = 11 at 1st qubit
= 10 at 4th
= 01 at 7th

Examples:

$|q1\rangle\otimes|$ 11 00 00 00 $\rangle$: no phase-flips, qubit-flip at 1
$|q1\rangle\otimes|$ 10 00 00 00 $\rangle$: no phase-flips, qubit-flip at 2
$|q1\rangle\otimes|$ 01 00 00 00 $\rangle$: no phase-flips, qubit-flip at 3
$|q1\rangle\otimes|$ 00 10 01 00 $\rangle$: phase-flip at 1, no qubit-flips
$|q1\rangle\otimes|$ 00 10 00 00 $\rangle$: phase-flip at 4 no qubit-flips
$|q1\rangle\otimes|$ 00 00 01 00 $\rangle$: phase-flip at 7 no qubit-flips
$|q1\rangle\otimes|$ 11 10 01 00 $\rangle$: phase-flip & qubit flip at 1
$|q1\rangle\otimes|$ 00 10 10 00 $\rangle$: phase-flip & qubit-flip at 6