```
> restart;
> interface(warnlevel = 0) :        #  Maple 12
> with(LinearAlgebra) :
> with(Bits) :
```

Chapter 8 Problem 1

```
> TP :=proc(M1, M2) return  KroneckerProduct(M1, M2) end proc:
> T := proc(x) return Transpose(x) end proc:
> VSte := proc(n)                 # Generates a list of  computational states for  n qubits
        local i, L;               #  e.g.  n= 2  ⇒ [ |00⟩  |01⟩  |10⟩  |11⟩ ]
        L := Matrix(1, 2^n);
        Settings(defaultbits = n);
        for i from 1 to 2^n do
            L[1, i] := cat(`|`, String(i − 1, msbfirst), "⟩");
        end do;
    #   print(L);
      return L;       # returns Matrix L
    end proc:
```

Defining matrices

```
> I2 := IdentityMatrix(2);
  X := Matrix([[0, 1], [1, 0]]);
  Z :=  Matrix([[1, 0], [0, −1]]);
  H := 1/'√2' (X + Z);
  CNOT := RowOperation(IdentityMatrix(4), [3, 4]) :
```

$$I2 := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$X := \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$Z := \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$H := \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \tag{1}$$

The input vectors
$|A\rangle = a_0|0\rangle + a_1|1\rangle$ and $|B\rangle = b_0|0\rangle + b_1|1\rangle$

$V0 = |A\rangle \otimes |B\rangle$ when $a_1 = b_1 = 0$

```
> V0 := Vector([[1,0, 0, 0]]) :  # |00⟩
  V1 := Vector([[0, 1,0, 0, ]]) :  # |01⟩
  V2 := Vector([[0, 0, 1,0]]) :  # |10⟩
  V3 := Vector([[0, 0, 0, 1]]) :  # |11⟩
  St := Transpose(VSte(2)) :
```

Bell-Meter - Read-0ut is in terms of the Bell's states β_{00}, β_{01}, β_{10}, and β_{11}

$$\beta_0 = \frac{(|00\rangle + |11\rangle)}{\sqrt{2}}$$

```
> M := Multiply(CNOT, TP(H, I2));
  B00 := Multiply(M, V0);
  β[0] := factor(Multiply(T(B00), St)[1]);
```

$$M := \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

$$B00 := \begin{bmatrix} \frac{1}{2}\sqrt{2} \\ 0 \\ 0 \\ \frac{1}{2}\sqrt{2} \end{bmatrix}$$

$$\beta_0 := \frac{1}{2}\sqrt{2}\,(|00\rangle + |11\rangle) \quad (2)$$

$$\beta_1 = \frac{(|01\rangle + |10\rangle)}{\sqrt{2}}$$

```
> B01 := Multiply(M, V1);
  β[1] := factor(Multiply(T(B01), St)[1]);
```

$$B01 := \begin{bmatrix} 0 \\ \frac{1}{2}\sqrt{2} \\ \frac{1}{2}\sqrt{2} \\ 0 \end{bmatrix}$$

$$\beta_1 := \frac{1}{2}\sqrt{2}\,(|01\rangle + |10\rangle) \tag{3}$$

$$\beta_2 = \frac{(|00\rangle - |11\rangle)}{\sqrt{2}}$$

```
> B10 := Multiply(M, V2);
  β[2] := factor( Multiply(T(B10), St)[1]);
```

$$B10 := \begin{bmatrix} \frac{1}{2}\sqrt{2} \\ 0 \\ 0 \\ -\frac{1}{2}\sqrt{2} \end{bmatrix}$$

$$\beta_2 := \frac{1}{2}\sqrt{2}\,(|00\rangle - |11\rangle) \tag{4}$$

$$\beta_3 = \frac{(|01\rangle - |10\rangle)}{\sqrt{2}}$$

```
> B11 := Multiply(M, V3);
  β[3] := factor( Multiply(T(B11), St)[1]);
```

$$B11 := \begin{bmatrix} 0 \\ \frac{1}{2}\sqrt{2} \\ -\frac{1}{2}\sqrt{2} \\ 0 \end{bmatrix}$$

$$\beta_3 := \frac{1}{2}\sqrt{2}\,(|01\rangle - |10\rangle) \tag{5}$$

Bell's Projection Operators

> $PB00 := Multiply(\mathrm{B00},\ Transpose(\mathrm{B00}));$

$$PB00 := \begin{bmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{bmatrix} \tag{6}$$

> $PB01 := Multiply(\mathrm{B01},\ Transpose(\mathrm{B01}));$

$$PB01 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{7}$$

> $PB10 := Multiply(\mathrm{B10},\ Transpose(\mathrm{B10}));$

$$PB10 := \begin{bmatrix} \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & \frac{1}{2} \end{bmatrix} \tag{8}$$

> $PB11 := Multiply(\mathrm{B11},\ Transpose(\mathrm{B11}));$

$$PB11 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{9}$$

$\psi = |A\rangle \otimes |B\rangle$

> $V0 := Vector([[a[0], a[1], a[2], a[3]]]);$
> `` `|\psi\rangle` = Multiply(Transpose(V0), St)[1]; ``

$$V0 := \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$|\psi\rangle = a_0\,|00\rangle + a_1\,|01\rangle + a_2\,|10\rangle + a_3\,|11\rangle \tag{10}$$

> $G := Multiply(CNOT, TP(H, I2)) :$ # Bell's State Gate
> $V := Multiply(G, V0);$
> $|\psi\rangle = Multiply(Transpose(V), St)[1];$

$$V := \begin{bmatrix} \frac{1}{2}\sqrt{2}\,a_0 + \frac{1}{2}\sqrt{2}\,a_2 \\ \frac{1}{2}\sqrt{2}\,a_1 + \frac{1}{2}\sqrt{2}\,a_3 \\ \frac{1}{2}\sqrt{2}\,a_1 - \frac{1}{2}\sqrt{2}\,a_3 \\ \frac{1}{2}\sqrt{2}\,a_0 - \frac{1}{2}\sqrt{2}\,a_2 \end{bmatrix}$$

$$|\psi\rangle = \left(\frac{1}{2}\sqrt{2}\,a_0 + \frac{1}{2}\sqrt{2}\,a_2\right)|00\rangle + \left(\frac{1}{2}\sqrt{2}\,a_1 + \frac{1}{2}\sqrt{2}\,a_3\right)|01\rangle + \left(\frac{1}{2}\sqrt{2}\,a_1 - \frac{1}{2}\sqrt{2}\,a_3\right)|10\rangle + \left(\frac{1}{2}\sqrt{2}\,a_0 - \frac{1}{2}\sqrt{2}\,a_2\right)|11\rangle \tag{11}$$

Projections

> $Multiply(PB00, V);$
> $`|\psi\rangle` = \dfrac{factor\left(\sqrt{2}\,Multiply(T(Multiply(PB00, V)), St)[1]\right)}{'\sqrt{2}'};$

$$\begin{bmatrix} \frac{1}{2}\sqrt{2}\,a_0 \\ 0 \\ 0 \\ \frac{1}{2}\sqrt{2}\,a_0 \end{bmatrix}$$

$$|\psi\rangle = \frac{(|00\rangle + |11\rangle)\,a_0}{\sqrt{2}} \tag{12}$$

> $Multiply(PB01, V);$
$`|\psi\rangle` = \frac{factor\left(\sqrt{2}\,Multiply(T(Multiply(PB01, V)), St)[1]\right)}{'\sqrt{2}'};$

$$\begin{bmatrix} 0 \\ \frac{1}{2}\sqrt{2}\,a_1 \\ \frac{1}{2}\sqrt{2}\,a_1 \\ 0 \end{bmatrix}$$

$$|\psi\rangle = \frac{(|01\rangle + |10\rangle)\,a_1}{\sqrt{2}} \tag{13}$$

> $Multiply(PB10, V);$
$`|\psi\rangle` = \frac{factor\left(\sqrt{2}\,Multiply(T(Multiply(PB10, V)), St)[1]\right)}{'\sqrt{2}'};$

$$\begin{bmatrix} \frac{1}{2}\sqrt{2}\,a_2 \\ 0 \\ 0 \\ -\frac{1}{2}\sqrt{2}\,a_2 \end{bmatrix}$$

$$|\psi\rangle = \frac{(|00\rangle - |11\rangle)\,a_2}{\sqrt{2}} \tag{14}$$

> $Multiply(PB11, V);$
$`|\psi\rangle` = \frac{factor\left(\sqrt{2}\,Multiply(T(Multiply(PB11, V)), St)[1]\right)}{'\sqrt{2}'};$

$$\begin{bmatrix} 0 \\ \frac{1}{2}\sqrt{2}\,a_3 \\ -\frac{1}{2}\sqrt{2}\,a_3 \\ 0 \end{bmatrix}$$

$$|\psi\rangle = \frac{(|01\rangle - |10\rangle)\,a_3}{\sqrt{2}} \tag{15}$$