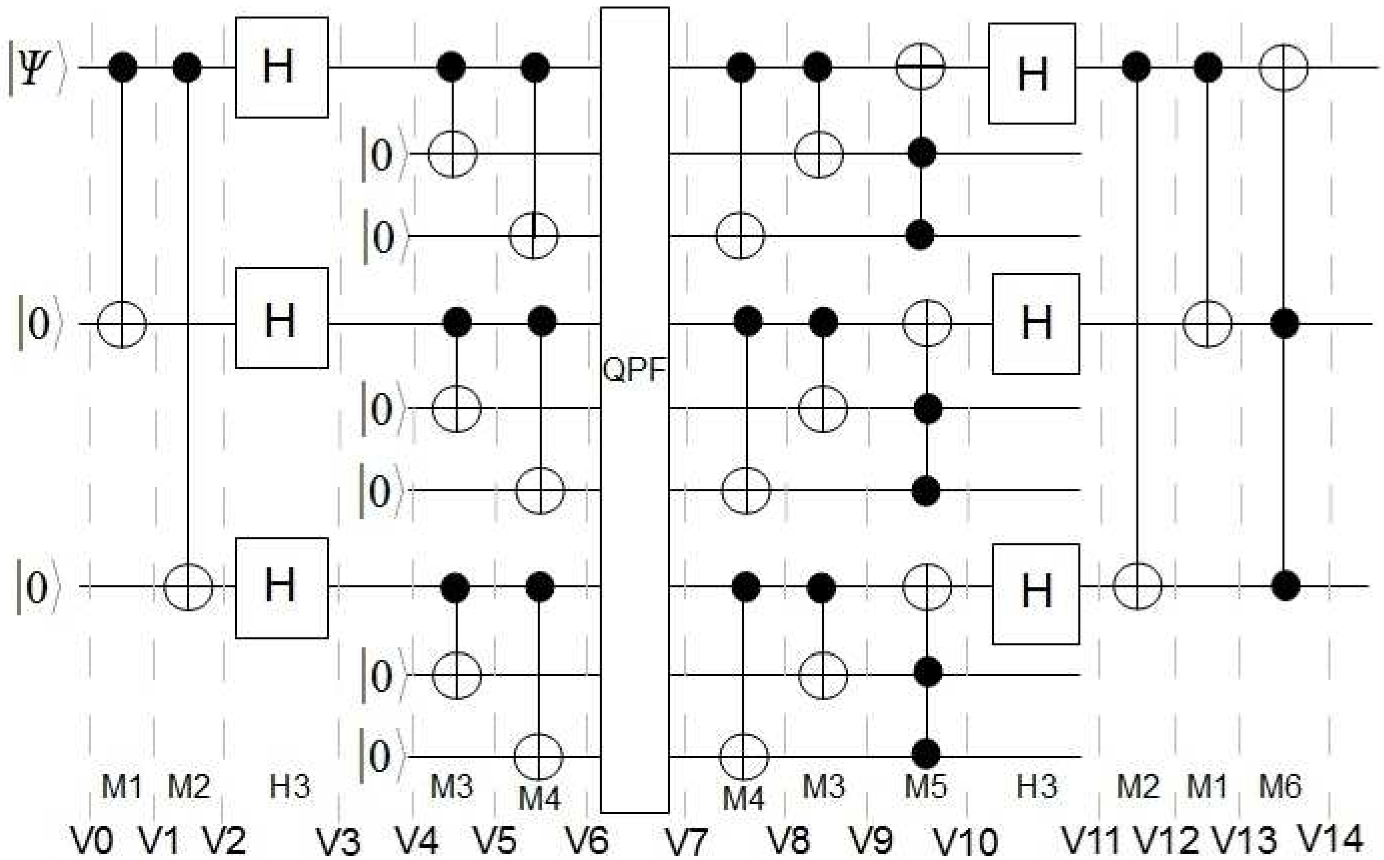
```
> restart :
> interface(warnlevel = 0) :      # Maple 12
> with(LinearAlgebra) :
> with(Bits) :
> Settings(defaultbits = 9) :
```



Execution flag

qp = 1 : qubit-flip and phase-flip at 1st qubit
= 2 : qubit-flip and phase-flip at 2
= 3 : qubit-flip and phase-flip at 3

```
> qp := 1 :
```

Utility functions

```
> K := proc(a, b) return KroneckerProduct(a, b) end proc:
> T := proc(x) return Transpose(x) end proc:
```

```
> VSte := proc(n)                # Generates a list of  computational states for  n qubits
          local i, L;            # e.g.  n= 2  ⇒ [ |00⟩  |01⟩  |10⟩  |11⟩ ]
          L := Matrix(1, 2^n);
          Settings(defaultbits = n);
          for i from 1 to 2^n do
              L[1, i] := cat(`|`, String(i − 1, msbfirst), "⟩");
          end do;
      #   print(L);
        return L;       # returns Matrix L
      end proc:
```

Utility matrices

```
> I2 := IdentityMatrix(2) :
> I4 := IdentityMatrix(4) :
> I8 := IdentityMatrix(8) :
```

Utility Operators

```
> Uz := RowOperation(I2, 2, -1);         # phase-flip operator
```

$$Uz := \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad (1)$$

```
> Ux := RowOperation(I2, [1, 2]);        # qubit-flip operator
```

$$Ux := \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad (2)$$

```
> CNOT := RowOperation(I4, [3, 4]);
```

$$CNOT := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad (3)$$

```
> G23 := RowOperation(I4, [2, 3]);      # qubit-exchange operator
```

$$G23 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (4)$$

$$> H := \frac{1}{\sqrt{2}} Matrix([[1, 1], [1, -1]]);$$

$$H := \begin{bmatrix} \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \\ \frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} \end{bmatrix} \qquad (5)$$

M1 - M6, H3, and QPF matrices/operators

> M1 := $K(CNOT, I2)$;

$$M1 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{6}$$

> M2 := $Multiply(\ K(G23, I2), Multiply(K(I2, CNOT),\ K(G23, I2)))$;

$$M2 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{7}$$

> H3 := K(H, K(H, H)) :
' $\sqrt{8}\cdot$H3 ' = $\sqrt{8}$ H3;

$$\sqrt{8}\ H3 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix} \tag{8}$$

```
> M3 := K(K(K(CNOT, I2), K(CNOT, I2)), K(CNOT, I2));   #  large matrix 2^9 by 2^9
```

$$M3 := \begin{bmatrix} \textit{512 x 512 Matrix} \\ \textit{Data Type: anything} \\ \textit{Storage: rectangular} \\ \textit{Order: Fortran\_order} \end{bmatrix} \tag{9}$$

```
> M4 := K(Multiply( K(G23, I2), Multiply(K(I2, CNOT),  K(G23, I2))),
       K(Multiply( K(G23, I2), Multiply(K(I2, CNOT),  K(G23, I2))),
         Multiply( K(G23, I2), Multiply(K(I2, CNOT),  K(G23, I2)))));
```

$$M4 := \begin{bmatrix} \textit{512 x 512 Matrix} \\ \textit{Data Type: anything} \\ \textit{Storage: rectangular} \\ \textit{Order: Fortran\_order} \end{bmatrix} \tag{10}$$

M6 matrix/operator is a variation of the Toffoli gate. The 2nd and 3rd qubits are the control qubits. When the state of both qubits equal "1" the 1st qubit is "flipped"

$$c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + \mathbf{c_3}|\mathbf{011}\rangle + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + \mathbf{c_7}|\mathbf{111}\rangle$$

$$c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + \mathbf{c_3}|\mathbf{111}\rangle + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + \mathbf{c_7}|\mathbf{011}\rangle$$

```
> M6 := RowOperation(I8, [8, 4]);
```

$$M6 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{11}$$

```
> M5 := K(K(M6, M6), M6);
```

$$M5 := \begin{bmatrix} \textit{512 x 512 Matrix} \\ \textit{Data Type: anything} \\ \textit{Storage: rectangular} \\ \textit{Order: Fortran\_order} \end{bmatrix} \tag{12}$$

QPF: qubit error operator

```
> if qp = 1 then
    QPF := K(Multiply(Ux, Uz), IdentityMatrix(2^8));
      # phase-flip & qubit-flip errors on the first qubit
  elif qp = 2 then
      QPF := K(I2, K(Multiply(Ux, Uz), IdentityMatrix(2^7))); # phase-flip & qubit-flip at 2
  elif qp = 3 then
      QPF := K(I4, K(Multiply(Ux, Uz), IdentityMatrix(2^6))); # phase-flip  & qubit-flip at 3
  end if;
```

$$QPF := \left[\begin{array}{c} \textit{512 x 512 Matrix} \\ \textit{Data Type: anything} \\ \textit{Storage: rectangular} \\ \textit{Order: Fortran\_order} \end{array}\right] \quad \textbf{(13)}$$

$$\mathbf{V0 = (\alpha|0\rangle + \beta|1\rangle) \otimes |0\rangle \otimes |0\rangle = \alpha|000\rangle + \beta|100\rangle}$$

The decimal representation: $\alpha|000\rangle + \beta|100\rangle \Rightarrow \alpha|0\rangle + \beta|4\rangle$

```
> n := 3 :
  q1 := Matrix([[α], [β]]) :     #  α|0⟩ + β|1⟩
  q2 := Matrix([[1], [0]]) :     #   |0⟩
  q3 := Matrix([[1], [0]]) :     #   |0⟩
  Co0 := K(K(q1, q2), q3) :
  St := Transpose(VSte(n)) :
  |V0⟩ := Multiply(T(Co0), St)[1, 1];
```

$$|V0\rangle := \alpha\,|000\rangle + \beta\,|100\rangle \quad \textbf{(14)}$$

$$\begin{aligned} \mathbf{V1} &= \mathbf{M1 \cdot V0} \\ &= \mathbf{M1(\alpha|000\rangle + \beta|100\rangle)} \\ &= \alpha|\mathbf{000}\rangle + \beta|\mathbf{110}\rangle \\ &= \alpha|\mathbf{0}\rangle + \beta|\mathbf{6}\rangle \end{aligned}$$

```
> Co1 := Multiply(M1, Co0) :
  |V1⟩ := Multiply(T(Co1), St)[1, 1];
```

$$|V1\rangle := \alpha\,|000\rangle + \beta\,|110\rangle \quad \textbf{(15)}$$

$$
\begin{aligned}
\mathbf{V2} &= \mathbf{M2 \cdot V1} \\
&= \mathbf{M2}(\alpha|\mathbf{000}\rangle + \beta|\mathbf{110}\rangle) \\
&= \alpha|\mathbf{000}\rangle + \beta|\mathbf{111}\rangle \\
&= \alpha|\mathbf{0}\rangle + \beta|\mathbf{7}\rangle \\
&= \alpha|\mathbf{0}\rangle|\mathbf{0}\rangle|\mathbf{0}\rangle + \beta|\mathbf{1}\rangle|\mathbf{1}\rangle|\mathbf{1}\rangle
\end{aligned}
$$

```
> Co2 := Multiply(M2, Co1) :
  |V2⟩ := Multiply(T(Co2), St)[1, 1];
```

$$|V2\rangle := \alpha\,|000\rangle + \beta\,|111\rangle \tag{16}$$

The Hadamard H3 matrix/operator

$$\mathrm{H}\,|\mathbf{0}\rangle = \frac{\alpha|\mathbf{0}\rangle + \beta|\mathbf{1}\rangle}{\sqrt{2}};\ \mathrm{H}\,|\mathbf{1}\rangle = \frac{\alpha|\mathbf{0}\rangle - \beta|\mathbf{1}\rangle}{\sqrt{2}},$$

$$\mathbf{V3 = H3 \cdot V2}$$

$$= \alpha\left(\mathbf{H}|\mathbf{0}\rangle \otimes \mathbf{H}|\mathbf{0}\rangle \otimes \mathbf{H}|\mathbf{0}\rangle\right) + \beta\left(\mathbf{H}|\mathbf{1}\rangle \otimes \mathbf{H}|\mathbf{1}\rangle \otimes \mathbf{H}|\mathbf{1}\rangle\right.$$

$$\mathbf{V3} = \alpha\left(\frac{|\mathbf{0}\rangle+|\mathbf{1}\rangle}{\sqrt{\mathbf{2}}}\right) \otimes \left(\frac{|\mathbf{0}\rangle+|\mathbf{1}\rangle}{\sqrt{\mathbf{2}}}\right) \otimes \left(\frac{|\mathbf{0}\rangle+|\mathbf{1}\rangle}{\sqrt{\mathbf{2}}}\right)$$

$$+ \beta \otimes \left(\frac{|\mathbf{0}\rangle-|\mathbf{1}\rangle}{\sqrt{\mathbf{2}}}\right) \otimes \left(\frac{|\mathbf{0}\rangle-|\mathbf{1}\rangle}{\sqrt{\mathbf{2}}}\right) \otimes \left(\frac{|\mathbf{0}\rangle-|\mathbf{1}\rangle}{\sqrt{\mathbf{2}}}\right)$$

$$\mathbf{V3} = \alpha|\mathbf{000}\rangle + \alpha|\mathbf{001}\rangle + \alpha|\mathbf{010}\rangle + \alpha|\mathbf{011}\rangle + \alpha|\mathbf{100}\rangle + \alpha|\mathbf{101}\rangle + \alpha|\mathbf{110}\rangle + \alpha|\mathbf{111}\rangle$$
$$+ \beta|\mathbf{000}\rangle - \beta|\mathbf{001}\rangle - \beta|\mathbf{010}\rangle + \beta|\mathbf{011}\rangle - \beta|\mathbf{100}\rangle + \beta|\mathbf{101}\rangle + \beta|\mathbf{110}\rangle - \beta|\mathbf{111}\rangle$$

$$\mathbf{V3} = (\alpha + \beta)|\mathbf{0}\rangle + (\alpha - \beta)|\mathbf{1}\rangle + (\alpha - \beta)|\mathbf{2}\rangle + (\alpha + \beta)|\mathbf{3}\rangle + (\alpha - \beta)|\mathbf{4}\rangle + (\alpha + \beta)|\mathbf{5}\rangle + (\alpha + \beta)\mathbf{6}\rangle + (\alpha - \beta)|\mathbf{7}\rangle$$

```
> Co3 := simplify(√8 Multiply(H3, Co2)) :
  |V3⟩ := Multiply(T(Co3), St)[1, 1];
```

$$|V3\rangle := (\alpha+\beta)\,|000\rangle + (\alpha-\beta)\,|001\rangle + (\alpha-\beta)\,|010\rangle + (\alpha+\beta)\,|011\rangle + (\alpha-\beta)\,|100\rangle + (\alpha+\beta)\,|101\rangle + (\alpha+\beta)\,|110\rangle + (\alpha-\beta)\,|111\rangle \tag{17}$$

9-qubit code $\Rightarrow 2^9$ states. Large one column matrix: 512 by 1

$$|000000000\rangle \rightarrow |0\rangle$$
$$|000000001\rangle \rightarrow |1\rangle$$
$$\downarrow$$
$$|111111111\rangle \rightarrow |511\rangle \text{ or octal } |777\rangle_o$$

$$V4 = \alpha\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\otimes|00\rangle\otimes\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\otimes|00\rangle\otimes\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\otimes|00\rangle$$
$$+\beta\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\otimes|00\rangle\otimes\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\otimes|00\rangle\otimes\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\otimes|00\rangle$$

$$V4 = (\alpha+\beta)|000000000\rangle + (\alpha-\beta)|000000100\rangle + (\alpha-\beta)|000100000\rangle + (\alpha+\beta)|000100100\rangle$$
$$+ (\alpha-\beta)|100000000\rangle + (\alpha+\beta)|100000100\rangle + (\alpha+\beta)|100100000\rangle + (\alpha-\beta)|100100100\rangle$$

$$V4 = (\alpha+\beta)|0\rangle_o + (\alpha-\beta)|4\rangle_o + (\alpha-\beta)|40\rangle_o + (\alpha+\beta)|44\rangle_o$$
$$+ (\alpha-\beta)|400\rangle_o + (\alpha+\beta)|404\rangle_o + (\alpha+\beta)|440\rangle_o + (\alpha-\beta)|444\rangle_o$$

$$V4 = (\alpha+\beta)|0\rangle + (\alpha-\beta)|4\rangle + (\alpha-\beta)|32\rangle + (\alpha+\beta)|36\rangle$$
$$+ (\alpha-\beta)|256\rangle + (\alpha+\beta)|260\rangle + (\alpha+\beta)|288\rangle + (\alpha-\beta)|292\rangle$$

```
> n = 9 :
  qa := 1/sqrt(2) Matrix([[1], [1]]) :      #  |0⟩ + |1⟩
  qb := 1/sqrt(2) Matrix([[1], [-1]]) :     #  |0⟩ - |1⟩
  q2 := Matrix([[1], [0]]) :                #  |0⟩
  q3 := Matrix([[1], [0]]) :                #  |0⟩

  A := α·K(K(qa, K(q2, q3)), K(K(qa, K(q2, q3)), K(qa, K(q2, q3)))) :
  B := β·K(K(qb, K(q2, q3)), K(K(qb, K(q2, q3)), K(qb, K(q2, q3)))) :
  Co4 := simplify(sqrt(8) (A + B)) :
  St := Transpose(VSte(9)) :
  |V4⟩ := Multiply(T(Co4), St)[1, 1];
```

$$|V4\rangle := (\alpha+\beta)\,|000000000\rangle + (\alpha-\beta)\,|000000100\rangle + (\alpha-\beta)\,|000100000\rangle + (\alpha+\beta)\,|000100100\rangle$$
$$+ (\alpha-\beta)\,|100000000\rangle + (\alpha+\beta)\,|100000100\rangle + (\alpha+\beta)\,|100100000\rangle + (\alpha-\beta)\,|100100100\rangle \quad \textbf{(18)}$$

V5 = M3 · V4

$$= (\alpha+\beta)|0\rangle_o + (\alpha-\beta)|6\rangle_o + (\alpha-\beta)|60\rangle_o + (\alpha+\beta)|66\rangle_o$$
$$+ (\alpha-\beta)|600\rangle_o + (\alpha+\beta)|606\rangle_o + (\alpha+\beta)|660\rangle_o + (\alpha-\beta)|666\rangle_o$$

```
> Co5 := Multiply(M3, Co4) :
  |V5⟩ := Multiply(T(Co5), St)[1, 1];
```

$$|V5\rangle := (\alpha+\beta)\ |110000110\rangle + (\alpha+\beta)\ |110110000\rangle + (\alpha-\beta)\ |110110110\rangle + (\alpha-\beta)\ |000000110\rangle + (\alpha-\beta)\ |000110000\rangle + (\alpha+\beta)\ |000110110\rangle + (\alpha-\beta)\ |110000000\rangle + (\alpha+\beta)\ |000000000\rangle \quad (19)$$

V6 = M4 · V5

$$= (\alpha+\beta)|0\rangle_o + (\alpha-\beta)|7\rangle_o + (\alpha-\beta)|70\rangle_o + (\alpha+\beta)|77\rangle_o$$
$$+ (\alpha-\beta)|700\rangle_o + (\alpha+\beta)|707\rangle_o + (\alpha+\beta)|770\rangle_o + (\alpha-\beta)|777\rangle_o$$

```
> Co6 := Multiply(M4, Co5) :
  |V6⟩ := Multiply(T(Co6), St)[1, 1];
```

$$|V6\rangle := (\alpha-\beta)\ |000000111\rangle + (\alpha-\beta)\ |000111000\rangle + (\alpha+\beta)\ |000111111\rangle + (\alpha-\beta)\ |111000000\rangle + (\alpha+\beta)\ |111000111\rangle + (\alpha+\beta)\ |111111000\rangle + (\alpha-\beta)\ |111111111\rangle + (\alpha+\beta)\ |000000000\rangle \quad (20)$$

Qubit Error: QPF

A phase-flip error on the first qubit

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|0\rangle - \beta|1\rangle$$

or

$$\alpha|+\rangle + \beta|-\rangle \rightarrow \alpha|-\rangle + \beta|+\rangle$$

A qubit-flip error on the first qubit

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|1\rangle + \beta|0\rangle$$

Both types of errors

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|1\rangle - \beta|0\rangle$$

$$\alpha\left(\frac{|000\rangle+|111\rangle}{\sqrt{2}}\right) \rightarrow \alpha\left(\frac{|100\rangle-|011\rangle}{\sqrt{2}}\right)$$
$$\beta\left(\frac{|000\rangle-|111\rangle}{\sqrt{2}}\right) \rightarrow \beta\left(\frac{|100\rangle+|011\rangle}{\sqrt{2}}\right)$$

if qp = 1

$V7 = QPF \cdot V6$

$$= (\alpha + \beta)|400\rangle_o + (\alpha - \beta)|407\rangle_o + (\alpha - \beta)|470\rangle_o + (\alpha + \beta)|477\rangle_o + (-\alpha + \beta)|300\rangle_o + (-\alpha - \beta)|307\rangle_o - (\alpha - \beta)|370\rangle_o + (-\alpha + \beta)|377\rangle_o$$

\> $Co7 := Multiply(QPF, Co6)$:
$|V7\rangle := Multiply(T(Co7), St)[1, 1]$;

$$|V7\rangle := (-\alpha + \beta)\,|011000000\rangle + (-\alpha - \beta)\,|011000111\rangle + (-\alpha - \beta)\,|011111000\rangle + (-\alpha + \beta)\,|011111111\rangle + (\alpha + \beta)\,|100000000\rangle + (\alpha - \beta)\,|100000111\rangle + (\alpha - \beta)\,|100111000\rangle + (\alpha + \beta)\,|100111111\rangle \quad \textbf{(21)}$$

if qp = 1

$V8 = M4 \cdot V7$

$$= (\alpha + \beta)|500\rangle_o + (\alpha - \beta)|506\rangle_o + (\alpha - \beta)|560\rangle_o + (\alpha + \beta)|566\rangle_o + (-\alpha + \beta)|300\rangle_o + (-\alpha - \beta)|306\rangle_o + (-\alpha - \beta)|360\rangle_o + (-\alpha + \beta)|366\rangle_o$$

\> $Co8 := Multiply(M4, Co7)$:
$|V8\rangle := Multiply(T(Co8), St)[1, 1]$;

$$|V8\rangle := (-\alpha - \beta)\,|011000110\rangle + (-\alpha - \beta)\,|011110000\rangle + (-\alpha + \beta)\,|011110110\rangle + (\alpha + \beta)\,|101000000\rangle + (\alpha - \beta)\,|101000110\rangle + (\alpha - \beta)\,|101110000\rangle + (\alpha + \beta)\,|101110110\rangle + (-\alpha + \beta)\,|011000000\rangle \quad \textbf{(22)}$$

if qp = 1

$V9 = M3 \cdot V8$

$$= (\alpha + \beta)|700\rangle_o + (\alpha - \beta)|704\rangle_o + (\alpha - \beta)|740\rangle_o + (\alpha + \beta)|744\rangle_o + (-\alpha + \beta)|300\rangle_o + (-\alpha - \beta)|304\rangle_o + (-\alpha - \beta)|340\rangle_o + (\alpha - \beta)|344\rangle_o$$

\> $Co9 := Multiply(M3, Co8)$:
$|V9\rangle := Multiply(T(Co9), St)[1, 1]$;

$$|V9\rangle := (-\alpha + \beta)\,|011000000\rangle + (-\alpha - \beta)\,|011000100\rangle + (-\alpha - \beta)\,|011100000\rangle + (-\alpha + \beta)\,|011100100\rangle + (\alpha + \beta)\,|111000000\rangle + (\alpha - \beta)\,|111000100\rangle + (\alpha - \beta)\,|111100000\rangle + (\alpha + \beta)\,|111100100\rangle \quad \textbf{(23)}$$

if qp = 1

V10 = M5 · V9

$$= (\alpha + \beta)|300\rangle_o + (\alpha - \beta)|304\rangle_o + (\alpha - \beta)|340\rangle_o + (\alpha + \beta)|344\rangle_o$$
$$+ (-\alpha + \beta)|700\rangle_o + (-\alpha - \beta)|704\rangle_o + (-\alpha - \beta)|740\rangle_o + (-\alpha + \beta)|744\rangle_o$$

```
> Co10 := Multiply(M5, Co9) :
  |V10> := Multiply(T(Co10), St)[1, 1];
```

$$|V10\rangle := (\alpha+\beta)\,|011000000\rangle + (\alpha-\beta)\,|011000100\rangle + (\alpha-\beta)\,|011100000\rangle + (\alpha+\beta)\,|011100100\rangle + (-\alpha+\beta)\,|111000000\rangle + (-\alpha-\beta)\,|111000100\rangle + (-\alpha-\beta)\,|111100000\rangle + (-\alpha+\beta)\,|111100100\rangle \quad \textbf{(24)}$$

if qp = 1

$$V10 = \alpha\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\otimes\left|11\right\rangle\otimes\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\otimes\left|00\right\rangle\otimes\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\otimes\left|00\right\rangle$$
$$+\ \beta\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\otimes\left|11\right\rangle\otimes\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\otimes\left|00\right\rangle\otimes\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\otimes\left|00\right\rangle$$

$$= \alpha(q1 \otimes|11\rangle\otimes q4 \otimes|00\rangle\otimes q7 \otimes|00\rangle) + \beta(q1 \otimes|11\rangle\otimes q4 \otimes|00\rangle\otimes q7 \otimes|00\rangle)$$

Now working with the 1^{st}, 4^{th} & 7^{th} qubits

$$V10 = \alpha\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\otimes\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\otimes\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)+\beta\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\otimes\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\Big\rangle\otimes\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)$$

Apply the H3 operator

V11 = H3 · V10

$$= \alpha|100\rangle+\beta|011\rangle$$
$$= \alpha|4\rangle+\beta|3\rangle$$

```
> if qp = 1 then
     simplify((α·K(qb, K(qa, qa)) + β·K(qa, K(qb, qb)))) :
   elif qp = 2 then
     simplify((α·K(qa, K(qb, qa)) + β·K(qb, K(qa, qb)))) :
    elif qp = 3 then
     simplify((α·K(qa, K(qa, qb)) + β·K(qb, K(qb, qa)))) :
  end if :
  Co11 := simplify(Multiply(H3, %)) :
   St := Transpose(VSte(3)) :
  |V11> := Multiply(T(Co11), St)[1, 1];
```

$$|V11\rangle := \beta\,|011\rangle + \alpha\,|100\rangle \quad \textbf{(25)}$$

if qp = 1

$V12 = M2 \cdot V11$

$= M2(\alpha|100\rangle+\beta|011\rangle$

$= \alpha|101\rangle+\beta|011\rangle$

$= \alpha|5\rangle+\beta|3\rangle$

```
> Co12 := Multiply(M2, Co11) :
  |V12⟩ := Multiply(T(Co12), St)[1, 1];
```

$$|V12\rangle := \beta\,|011\rangle + \alpha\,|101\rangle \quad (26)$$

if qp = 1

$V13 = M1 \cdot V12$

$= M1(\alpha|101\rangle+\beta|011\rangle$

$= \alpha|111\rangle+\beta|011\rangle$

$= \alpha|7\rangle+\beta|3\rangle$

```
> Co13 := Multiply(M1, Co12) :
  |V13⟩ := Multiply(T(Co13), St)[1, 1];
```

$$|V13\rangle := \beta\,|011\rangle + \alpha\,|111\rangle \quad (27)$$

if qp = 1

$V14 = M6 \cdot V13$

$= M6(\alpha|111\rangle+\beta|011\rangle$

$= \alpha|011\rangle+\beta|111\rangle$

$= \alpha|3\rangle+\beta|7\rangle$

which can be re-written as

$V14 = (\alpha|0\rangle+\beta|1\rangle) \otimes|1\rangle\otimes|1\rangle$

if qp = 2 then $V14 = \alpha|010\rangle+\beta|110\rangle = \alpha|2\rangle+\beta|6\rangle$

if qp = 3 then $V14 = \alpha|001\rangle+\beta|101\rangle = \alpha|1\rangle+\beta|5\rangle$

```
> Co14 := Multiply(M6, Co13) :
  |V14⟩ := Multiply(T(Co14), St)[1, 1];
```

$$|V14\rangle := \alpha\,|011\rangle + \beta\,|111\rangle \quad (28)$$

Compare to V0

$\mathbf{V0} = (\alpha|\mathbf{0}\rangle + \beta|\mathbf{1}\rangle) \otimes|\mathbf{0}\rangle\otimes|\mathbf{0}\rangle$

$= \alpha|\mathbf{000}\rangle + \beta|\mathbf{100}\rangle$

$= \alpha|\mathbf{0}\rangle+\beta|\mathbf{4}\rangle$

```
> |V0⟩ := Multiply(T(Co0), St)[1, 1];
```

$$|V0\rangle := \alpha\,|000\rangle + \beta\,|100\rangle \quad (29)$$