```
> restart;
> interface(warnlevel = 0) :  #
```

Maple 12

```
> with(LinearAlgebra) :
> with(Bits) :
```

Grover's Algorithm

```
> VSte := proc(n)                 # Generates a list of  computational states for  n qubits
            local i, L;            #  e.g.  n= 2  ⇒ [ |00〉 |01〉 |10〉 |11〉 ]
            L := Matrix(1, 2^n);
            Settings(defaultbits = n);
            for i from 1 to 2^n do
                L[1, i] := cat(`|`, String(i − 1, msbfirst), "〉");
            end do;
        #   print(L);
          return L;        # returns Matrix L
        end proc:
```

1 out 4 search; n= 2 and N=4

```
> n := 2 :
```

Implementing the Grover operator (I - 2 |ψ〉〈ψ|)*Uf*

ρ is the Projection operator |ψ〉〈ψ|

```
> ρ :=  ConstantMatrix(1, 2^n) :
  '|ψ〉〈ψ|' = ρ;
```

$$|\psi\rangle\langle\psi| = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad \textbf{(1)}$$

4 x 4 Identity Matrix

```
> I[4] :=  IdentityMatrix(2^n);
```

$$I_4 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \textbf{(2)}$$

The Inversion operator (I − 2 |ψ⟩⟨ψ|)

> $\mathbb{M} := I[4] - \dfrac{2}{2^n} \cdot \rho;$

$$\mathbb{M} := \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad (3)$$

A 2 qubit Linear Computational Basis

> $Co0 := \dfrac{1}{\sqrt{2^n}} \cdot Vector([[1, 1, 1, 1]]) :$
> $St := VSte(\mathrm{n}) :$
> $|\Psi0\rangle := factor(Multiply(St, Co0)[1]);$

$$|\Psi0\rangle := \frac{1}{2}\,|00\rangle + \frac{1}{2}\,|01\rangle + \frac{1}{2}\,|10\rangle + \frac{1}{2}\,|11\rangle \quad (4)$$

After an Oracle query *Uf* |ψ⟩

> $Co1 := \dfrac{1}{\sqrt{2^n}} \cdot Vector([[1, -1, 1, 1]]) :$
> $|\Psi1\rangle := simplify(Multiply(St, Co1)[1]);$

$$|\Psi1\rangle := \frac{1}{2}\,|00\rangle - \frac{1}{2}\,|01\rangle + \frac{1}{2}\,|10\rangle + \frac{1}{2}\,|11\rangle \quad (5)$$

One iteration of the operator [I - 2 |ψ⟩⟨ψ|]*Uf*|ψ⟩

> $\mathrm{Co2} := Multiply(\mathbb{M}, \mathrm{Co1}) :$
> $|\Psi2\rangle := simplify(Multiply(St, Co2)[1]);$

$$|\Psi2\rangle := -|01\rangle \quad (6)$$

1 out 8 search; n= 3 and N=8

> $n := 3 :$

ρ is the Projection operator |ψ⟩⟨ψ|

> $\rho := \mathit{ConstantMatrix}\left(1, 2^n\right) :$
> $'|\psi\rangle\langle\psi|' = \rho;$

$$|\psi\rangle\langle\psi| = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \tag{7}$$

8 x 8 Identity Matrix

> $I[8] := \mathit{IdentityMatrix}\left(2^n\right) :$

The Inversion operator (I - 2 |ψ⟩⟨ψ|)

> $\mathbb{M} := I[8] - \frac{2}{2^n} \cdot \rho;$

$$\mathbb{M} := \begin{bmatrix} \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{3}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{3}{4} \end{bmatrix} \tag{8}$$

Linear Computational basis of 3 qubits

$$> \; Co0 := \frac{1}{\sqrt{2^n}} \cdot Vector([[1, 1, 1, 1, 1, 1, 1, 1]]) :$$

$$St := VSte(n) :$$

$$|\Psi 0\rangle := factor(Multiply(St, Co0)[1]);$$

$$|\Psi 0\rangle := \frac{1}{4}\sqrt{2}\,(|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle) \qquad \textbf{(9)}$$

After an Oracle query $Uf\,|\psi\rangle$

$$> \; Co1 := \frac{1}{\sqrt{2^n}} \cdot Vector([[1, 1, 1, 1, 1, -1, 1, 1]]) :$$

$$|\Psi 1\rangle := factor(Multiply(St, Co1)[1]);$$

$$|\Psi 1\rangle := \frac{1}{4}\sqrt{2}\,(|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle - |101\rangle + |110\rangle + |111\rangle) \qquad \textbf{(10)}$$

One iteration of the Grover operator $[(I - 2\,|\psi\rangle\langle\psi|\,)]Uf|\psi\rangle$

$$> \; Co2 := Multiply(\mathbb{M}, Co1) :$$

$$|\Psi 2\rangle := factor(Multiply(St, Co2)[1]);$$

$$|\Psi 2\rangle := -\frac{1}{8}\sqrt{2}\,(|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + 5\,|101\rangle + |110\rangle + |111\rangle) \qquad \textbf{(11)}$$

A second query to the Oracle

$$> \; Co3 := -\frac{\sqrt{2}}{8} \cdot Vector([[1, 1, 1, 1, 1, -5, 1, 1]]) :$$

$$|\Psi 3\rangle := factor(Multiply(St, Co3)[1]);$$

$$|\Psi 3\rangle := -\frac{1}{8}\sqrt{2}\,(|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle - 5\,|101\rangle + |110\rangle + |111\rangle) \qquad \textbf{(12)}$$

Second iteration of the Grover operator $[(I - 2\,|\psi\rangle\langle\psi|\,)]Uf|\psi\rangle$

$$> \; Co4 := Multiply(\mathbb{M}, Co3) :$$

$$|\Psi 4\rangle := factor(Multiply(St, Co4)[1]);$$

$$|\Psi 4\rangle := -\frac{1}{16}\sqrt{2}\,(|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle - 11\,|101\rangle + |110\rangle + |111\rangle) \qquad \textbf{(13)}$$

See Grover.mw where the Inversion Operator is $2\,|\psi\rangle\langle\psi| - I$