

```

> restart;
> interface(warnlevel=0) :      # Maple 12
> with(LinearAlgebra) :

```

## Chapter 8 Problem 10

### Part a

```

> A :=  $\frac{1}{2}$  · IdentityMatrix(2);
CharacteristicPolynomial(A, λ);
factor(% );
solve( %=0, [λ]);

```

$$A := \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$$

$$\lambda^2 - \lambda + \frac{1}{4}$$

$$\frac{1}{4} (2\lambda - 1)^2$$

$$\left[ \left[ \lambda = \frac{1}{2} \right], \left[ \lambda = \frac{1}{2} \right] \right]$$

**(1)**

```

> L := Eigenvectors(A);
s1 :=  $\sqrt{L[1][1]}$ ;
s2 :=  $\sqrt{L[1][2]}$ ;

```

$$L := \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$s1 := \frac{1}{2} \sqrt{2}$$

$$s2 := \frac{1}{2} \sqrt{2}$$

**(2)**

**Part b**

```
> B := 1/2 * Matrix([ [1, 1], [1, 1]]);
CharacteristicPolynomial(B, λ);
factor(% );
solve( %=0, [λ]);
```

$$B := \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\lambda^2 - \lambda$$

$$\lambda (\lambda - 1)$$

$$[[\lambda = 0], [\lambda = 1]]$$

**(3)**

```
> L := Eigenvectors(B);
s1 := sqrt(L[1][1]);
s2 := sqrt(L[1][2]);
```

$$L := \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$s1 := 1$$

$$s2 := 0$$

**(4)****Part c**

```
> C := 1/3 * Matrix([ [2, 1], [1, 1]]);
CharacteristicPolynomial(C, λ);
factor(% );
solve( %=0, [λ]);
```

$$C := \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

$$\lambda^2 - \lambda + \frac{1}{9}$$

$$\lambda^2 - \lambda + \frac{1}{9}$$

$$\left[ \left[ \lambda = \frac{1}{2} + \frac{1}{6} \sqrt{5} \right], \left[ \lambda = \frac{1}{2} - \frac{1}{6} \sqrt{5} \right] \right]$$

**(5)**

>  $L := \text{Eigenvectors}(C);$

$s1 := \sqrt{L[1][1]};$

$s2 := \sqrt{L[1][2]};$

$$L := \begin{bmatrix} \frac{1}{2} + \frac{1}{6} \sqrt{5} \\ \frac{1}{2} - \frac{1}{6} \sqrt{5} \end{bmatrix}, \begin{bmatrix} \frac{1}{-\frac{1}{2} + \frac{1}{2} \sqrt{5}} & \frac{1}{-\frac{1}{2} - \frac{1}{2} \sqrt{5}} \\ 1 & 1 \end{bmatrix}$$

$$s1 := \frac{1}{6} \sqrt{15} + \frac{1}{6} \sqrt{3}$$

$$s2 := \frac{1}{6} \sqrt{15} - \frac{1}{6} \sqrt{3}$$

(6)

>  $E := \frac{1}{3} \cdot \text{Matrix}([ [1, 1], [1, 2] ]);$

$\text{CharacteristicPolynomial}(E, \lambda);$

$\text{factor}(\% );$

$\text{solve}(\% = 0, [\lambda]);$

$$E := \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

$$\lambda^2 - \lambda + \frac{1}{9}$$

$$\lambda^2 - \lambda + \frac{1}{9}$$

$$\left[ \left[ \lambda = \frac{1}{2} + \frac{1}{6} \sqrt{5} \right], \left[ \lambda = \frac{1}{2} - \frac{1}{6} \sqrt{5} \right] \right]$$

(7)

>  $L := \text{Eigenvectors}(E);$

$s1 := \sqrt{L[1][1]};$

$s2 := \sqrt{L[1][2]};$

$$L := \begin{bmatrix} \frac{1}{2} + \frac{1}{6} \sqrt{5} \\ \frac{1}{2} - \frac{1}{6} \sqrt{5} \end{bmatrix}, \begin{bmatrix} \frac{1}{\frac{1}{2} + \frac{1}{2} \sqrt{5}} & \frac{1}{\frac{1}{2} - \frac{1}{2} \sqrt{5}} \\ 1 & 1 \end{bmatrix}$$

$$s1 := \frac{1}{6} \sqrt{15} + \frac{1}{6} \sqrt{3}$$

$$s2 := \frac{1}{6} \sqrt{15} - \frac{1}{6} \sqrt{3}$$

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