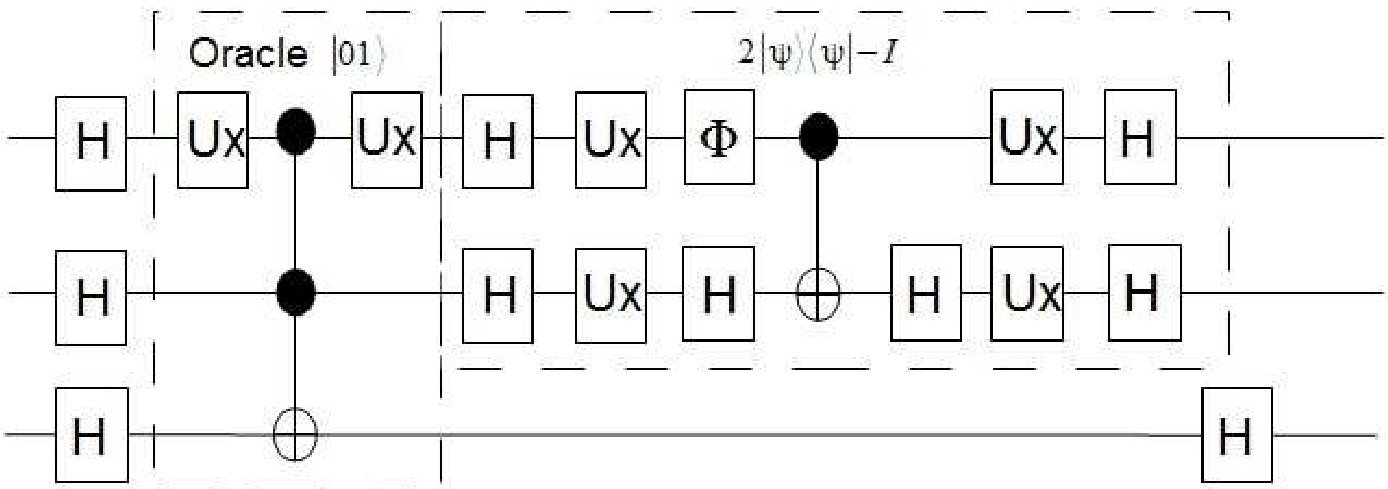
```
> restart;
> interface(warnlevel = 0) :   #   Maple 12
> interface(rtablesize = 32) :
> with(LinearAlgebra) :
> with(Bits) :
```

Grover's Algorithm



1 out of 4 search. Need 2 qubits to generate the computational basis

```
> n := 2 :
```

Goal is the state of interest. For example, 1 ⇒ |01⟩

```
> g := 1 :
```

```
> TP := proc(M1, M2)
        KroneckerProduct(M1, M2);
      end proc:
> VSte := proc(n)                # Generates a list of  computational states for  n qubits
          local i, L;            #  e.g.  n= 2  ⇒ [ |00⟩  |01⟩  |10⟩  |11⟩ ]
          L := Matrix(1, 2^n);
          Settings(defaultbits = n);
          for i from 1 to 2^n do
             L[1, i] := cat(`|`, String(i − 1, msbfirst), "⟩");
          end do;
     #   print(L);
        return L;       # returns Matrix L
      end proc:
```

```
> NumCoe := proc(n, s)                    # generates the initial state coefficients
           local p, m, i, `1`, `0`;
           `0` := Matrix([[1], [0]]) :
           `1` := Matrix([[0], [1]]) :
           p := Multiply(H, `0`) :
           m := Multiply(H, `1`) :
           if s = 0 then t := p else t := TP(p, m) end if;
           for i from 1 to n − 1 do
               t := TP(p, t) :
           end do;
           return t;
          end proc:
```

```
> MaxCo := proc(L, n)               # returns the location of largest coefficient
          local x, y, i, N, loc;
          x := 0 :
          N := 2^(n+1);
          for i from 1 to N do
             y := abs(evalf( L[i, 1]));
            if x < y then  loc := i : x := y end if
         end do;
         return loc :
        end proc:
```

Operators/Matrices

```
> I2 := IdentityMatrix(2) :
I4 := IdentityMatrix(4) :
I8 := IdentityMatrix(8) :
```

```
> Ux := RowOperation(I2, [1, 2]);
H := 1/sqrt(2) Matrix([[1, 1], [1,-1]]);    # Hadamard
 Φ := e^(I·π) I2;    # phase
 #  Φ:=I2;
 CNOT := RowOperation(I4, [3, 4]);
Gt := RowOperation(I8, [7, 8]); # Toffoli gate
CU := (Multiply(TP(Φ, H), Multiply(CNOT, TP(I2, H))));
Ux2 := TP(Ux, Ux);
H2 := TP(H, H);
```

$$Ux := \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$H := \begin{bmatrix} \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \\ \frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} \end{bmatrix}$$

$$\Phi := \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$CNOT := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$Gt := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$CU := \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Ux2 := \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$H2 := \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{1}$$

The Oracles

```
> if g = 0 then
        Oracle := Multiply(TP(Ux, TP(Ux, I2)), Multiply(Gt, TP(Ux, TP(Ux, I2)))); # goal is |00⟩
  end if;
  if g = 1 then
        Oracle := Multiply(TP(Ux, TP(I2, I2)), Multiply(Gt, TP(Ux, TP(I2, I2)))); # goal is |01⟩
  end if;
  if g = 2 then
        Oracle := Multiply(TP(I2, TP(Ux, I2)), Multiply(Gt, TP(I2, TP(Ux, I2)))); # goal is |10⟩
  end if;
  if g = 3 then
        Oracle := Gt   # goal is |11⟩
  end if;
```

$$Oracle := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

The 2|Ψ⟩⟨Ψ| - I Operator

> \# $N := 2^n$:
> \# $\rho := ConstantMatrix(1, N)$:
> \# $\mathbb{P} := \frac{2}{N} \cdot \rho - IdentityMatrix(N)$;
> \# $\mathbb{M} := TP(\mathbb{P}, I2)$;
>
> $\mathbb{P} := Multiply(H2, Multiply(Ux2, Multiply(CU, Multiply(Ux2, H2))))$;
> $\mathbb{M} := TP(\mathbb{P}, I2)$;

$$\mathbb{P} := \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

$$\mathbb{M} := \begin{bmatrix} -\frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix} \tag{3}$$

Computational basis

> $St := VSte(\mathrm{n}):$
> $Co0 := NumCoe(n, 0):$
> $|\Psi 0\rangle := factor(Multiply(St, Co0)[1, 1]);$

$$|\Psi 0\rangle := \frac{1}{2}\,|00\rangle + \frac{1}{2}\,|01\rangle + \frac{1}{2}\,|10\rangle + \frac{1}{2}\,|11\rangle \tag{4}$$

Computational basis with the "Oracle qubit"

> $St := VSte(\mathrm{n}+1):$
> $Co1 := NumCoe(n, 1):$
> $|\Psi 1\rangle := factor(Multiply(St, Co1)[1, 1]);$

$$|\Psi 1\rangle := \frac{1}{4}\sqrt{2}\,(|000\rangle - |001\rangle + |010\rangle - |011\rangle + |100\rangle - |101\rangle + |110\rangle - |111\rangle) \tag{5}$$

First Oracle query

> $Co2 := Multiply(Oracle, Co1):$
> $|\Psi 2\rangle := factor(\ Multiply(St, Co2)[1, 1]);$

$$|\Psi 2\rangle := \frac{1}{4}\sqrt{2}\,(|000\rangle - |001\rangle - |010\rangle + |011\rangle + |100\rangle - |101\rangle + |110\rangle - |111\rangle) \tag{6}$$

```
> Co3 := Multiply(M, Co2) :
|Ψ3⟩ := factor( Multiply(St, Co3)[1, 1]);
```

$$|\Psi 3\rangle := \frac{1}{2}\sqrt{2}\,(|010\rangle - |011\rangle) \quad \textbf{(7)}$$

Recover the Oracle qubit

```
> # Co4 :=Multiply(TP(I2, TP(I2, TP(I2, H))), Co3) :
Co4 := Multiply(TP(I2, TP(I2, H)), Co3) :
|Ψ4⟩ := factor( Multiply(St, Co4)[1, 1]);
```

$$|\Psi 4\rangle := |011\rangle \quad \textbf{(8)}$$

Preview result of the first pass

```
> l := MaxCo(Co4, n) :
State := St[1, l];
Probability := (evalf(Co4[l, 1])^2)·100;
```

$$State := |011\rangle$$

$$Probability := 100. \quad \textbf{(9)}$$