```
> restart;
> interface(warnlevel = 0) :   # Maple 12
> with(LinearAlgebra) :
> with(Bits) :
```

Grover's Algorithm

```
> VSte := proc(n)                  # Generates a list of  computational states for  n qubits
          local i, L;              # e.g.  n= 2  ⇒ [ |00〉 |01〉 |10〉 |11〉 ]
          L := Matrix(1, 2^n);
          Settings(defaultbits = n);
          for i from 1 to 2^n do
              L[1, i] := cat(`|`, String(i − 1, msbfirst), "〉");
          end do;
     #    print(L);
        return L;        # returns Matrix L
      end proc:
```

1 out 4 search; n= 2 and N=4

```
> n := 2 :
```

Implementing the Grover operator (2 |ψ〉〈ψ| - I)*Uf*

ρ is the Projection operator |ψ〉〈ψ|

```
> ρ := ConstantMatrix(1, 2^n) :
  '|ψ〉〈ψ|' = ρ;
```

$$|\psi\rangle\langle\psi| = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \qquad (1)$$

4 x 4 Identity Matrix

```
> I[4] := IdentityMatrix(2^n);
```

$$I_4 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2)$$

The Inversion operator (2 |ψ⟩⟨ψ| - I)

> $\mathbb{M} := \frac{2}{2^n} \cdot \rho - \mathrm{I}[4];$

$$\mathbb{M} := \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix} \tag{3}$$

A 2 qubit Linear Computational Basis

> $Co0 := \frac{1}{\sqrt{2^n}} \cdot Vector([[1, 1, 1, 1]]):$

$St := VSte(\mathrm{n}):$

$|\Psi 0\rangle := factor(Multiply(St, Co0)[1]);$

$$|\Psi 0\rangle := \frac{1}{2}\,|00\rangle + \frac{1}{2}\,|01\rangle + \frac{1}{2}\,|10\rangle + \frac{1}{2}\,|11\rangle \tag{4}$$

After an Oracle query *Uf* |ψ⟩

> $Co1 := \frac{1}{\sqrt{2^n}} \cdot Vector([[1, -1, 1, 1]]):$

$|\Psi 1\rangle := simplify(Multiply(St, Co1)[1]);$

$$|\Psi 1\rangle := \frac{1}{2}\,|00\rangle - \frac{1}{2}\,|01\rangle + \frac{1}{2}\,|10\rangle + \frac{1}{2}\,|11\rangle \tag{5}$$

One iteration of the Grover operator [(2 |ψ⟩⟨ψ| - I)]*Uf*|ψ⟩

> $\mathrm{Co2} := Multiply(\mathbb{M}, \mathrm{Co1}):$

$|\Psi 2\rangle := simplify(Multiply(St, Co2)[1]);$

$$|\Psi 2\rangle := |01\rangle \tag{6}$$

1 out 8 search; n= 3 and N=8

> $n := 3:$

ρ is the Projection operator |ψ⟩⟨ψ|

> $\rho := ConstantMatrix(1, 2^n):$
> $'|\psi\rangle\langle\psi|' = \rho;$

$$|\psi\rangle\langle\psi| = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \quad (7)$$

8 x 8 Identity Matrix

> $I[8] := IdentityMatrix(2^n):$

The Inversion operator (2 |ψ⟩⟨ψ| - I)

> $\mathcal{M} := \frac{2}{2^n} \cdot \rho - I[8];$

$$\mathcal{M} := \begin{bmatrix} -\frac{3}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{3}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & -\frac{3}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{3}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{3}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{3}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{3}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{3}{4} \end{bmatrix} \quad (8)$$

Linear Computational basis of 3 qubits

$$> \; Co0 := \frac{1}{\sqrt{2^n}} \cdot Vector([[1, 1, 1, 1, 1, 1, 1, 1]]) :$$

$St := VSte(\mathrm{n}) :$

$|\Psi0\rangle := factor(Multiply(St, Co0)[1]);$

$$|\Psi0\rangle := \frac{1}{4}\sqrt{2}\,(|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle) \qquad \textbf{(9)}$$

After an Oracle query $Uf\,|\psi\rangle$

$$> \; Co1 := \frac{1}{\sqrt{2^n}} \cdot Vector([[1, 1, 1, 1, 1, -1, 1, 1]]) :$$

$|\Psi1\rangle := factor(Multiply(St, Co1)[1]);$

$$|\Psi1\rangle := \frac{1}{4}\sqrt{2}\,(|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle - |101\rangle + |110\rangle + |111\rangle) \qquad \textbf{(10)}$$

One iteration of the Grover operator $[(2\,|\psi\rangle\langle\psi| - \mathbf{I})]Uf|\psi\rangle$

$> \; \mathrm{Co2} := Multiply(\mathcal{M}, \mathrm{Co1}) :$

$|\Psi2\rangle := factor(Multiply(St, Co2)[1]);$

$$|\Psi2\rangle := \frac{1}{8}\sqrt{2}\,(|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + 5\,|101\rangle + |110\rangle + |111\rangle) \qquad \textbf{(11)}$$

A second query to the Oracle

$$> \; \mathrm{Co3} := \frac{\sqrt{2}}{8} \cdot Vector([[1, 1, 1, 1, 1, -5, 1, 1]]) :$$

$|\Psi3\rangle := factor(Multiply(St, Co3)[1]);$

$$|\Psi3\rangle := \frac{1}{8}\sqrt{2}\,(|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle - 5\,|101\rangle + |110\rangle + |111\rangle) \qquad \textbf{(12)}$$

Second iteration of the Grover operator $[(2\,|\psi\rangle\langle\psi| - \mathbf{I})]Uf|\psi\rangle$

$> \; \mathrm{Co4} := Multiply(\mathcal{M}, \mathrm{Co3}) :$

$|\Psi4\rangle := factor(Multiply(St, Co4)[1]);$

$$|\Psi4\rangle := -\frac{1}{16}\sqrt{2}\,(|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle - 11\,|101\rangle + |110\rangle + |111\rangle) \qquad \textbf{(13)}$$

See Grover2.mw where the Inversion Operator is $\mathbf{I} - 2\,|\psi\rangle\langle\psi|$
Also see problem # 9.