```
> restart;
> interface(warnlevel = 0) :        #  Maple 12
> with(LinearAlgebra) :
> with(Bits) :
```

Chapter 8 Problem 3

```
> T :=proc(x) return Transpose(x) end proc:
> VSte := proc(n)                  # Generates a list of  computational states for  n qubits
          local i, L;              #  e.g.  n= 2  ⇒ [ |00⟩  |01⟩  |10⟩  |11⟩ ]
          L := Matrix(1, 2^n);
          Settings(defaultbits = n);
          for i from 1 to 2^n do
             L[1, i] := cat(`|`, String(i − 1, msbfirst), "⟩");
          end do;
      #   print(L);
        return L;       # returns Matrix L
      end proc:
```

Defining the vector of interest

$$|\psi\rangle = c_0|000\rangle + c_1|010\rangle + c_2|100\rangle + c_3|110\rangle$$

```
> V1 :=  Vector([c[0], [0], c[1], [0], c[2], [0], c[3], [0]]);
  St := Transpose(VSte(3)) :
  |ψ⟩ := Multiply(T(V1), St)[1];
```

$$V1 := \begin{bmatrix} c_0 \\ 0 \\ c_1 \\ 0 \\ c_2 \\ 0 \\ c_3 \\ 0 \end{bmatrix}$$

$$|\psi\rangle := c_0\,|000\rangle + c_1\,|010\rangle + c_2\,|100\rangle + c_3\,|110\rangle \qquad \textbf{(1)}$$

First Stage: Flip the target when control a= 0 and control b =1

$$|010\rangle \leftrightarrows |011\rangle$$

> $G1 := RowOperation(IdentityMatrix(8), [3, 4]);$

$$G1 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \textbf{(2)}$$

Second Stage: Flip the target when control a= 1 and control b =0

$$|100\rangle \leftrightarrows |101\rangle$$

> $G2 := RowOperation(IdentityMatrix(8), [5, 6]);$

$$G2 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \textbf{(3)}$$

The overall composition

> $G := Multiply(G2, G1);$

$$G := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

$|\psi'\rangle = G\,|\psi\rangle$

> $`|\psi\rangle` := Multiply(T(V1), St)[1];$ # compare with $|\psi'\rangle$
$V2 := Multiply(G, V1);$
$|\psi'\rangle := Multiply(T(V2), St)[1];$

$$|\psi\rangle := c_0\,|000\rangle + c_1\,|010\rangle + c_2\,|100\rangle + c_3\,|110\rangle$$

$$V2 := \begin{bmatrix} c_0 \\ 0 \\ 0 \\ c_1 \\ 0 \\ c_2 \\ c_3 \\ 0 \end{bmatrix}$$

$$|\psi'\rangle := c_0\,|000\rangle + c_1\,|011\rangle + c_2\,|101\rangle + c_3\,|110\rangle \quad (5)$$

$$|\psi'\rangle = c_0|000\rangle + c_1|011\rangle + c_2|101\rangle + c_3|110\rangle$$

$$|\psi'\rangle = c_0|00\rangle \otimes |0\rangle + c_1|01\rangle \otimes |1\rangle + c_2|10\rangle \otimes |1\rangle + c_3|11\rangle \otimes |0\rangle$$

$$c_{ab}|ab\rangle \otimes |f\rangle$$