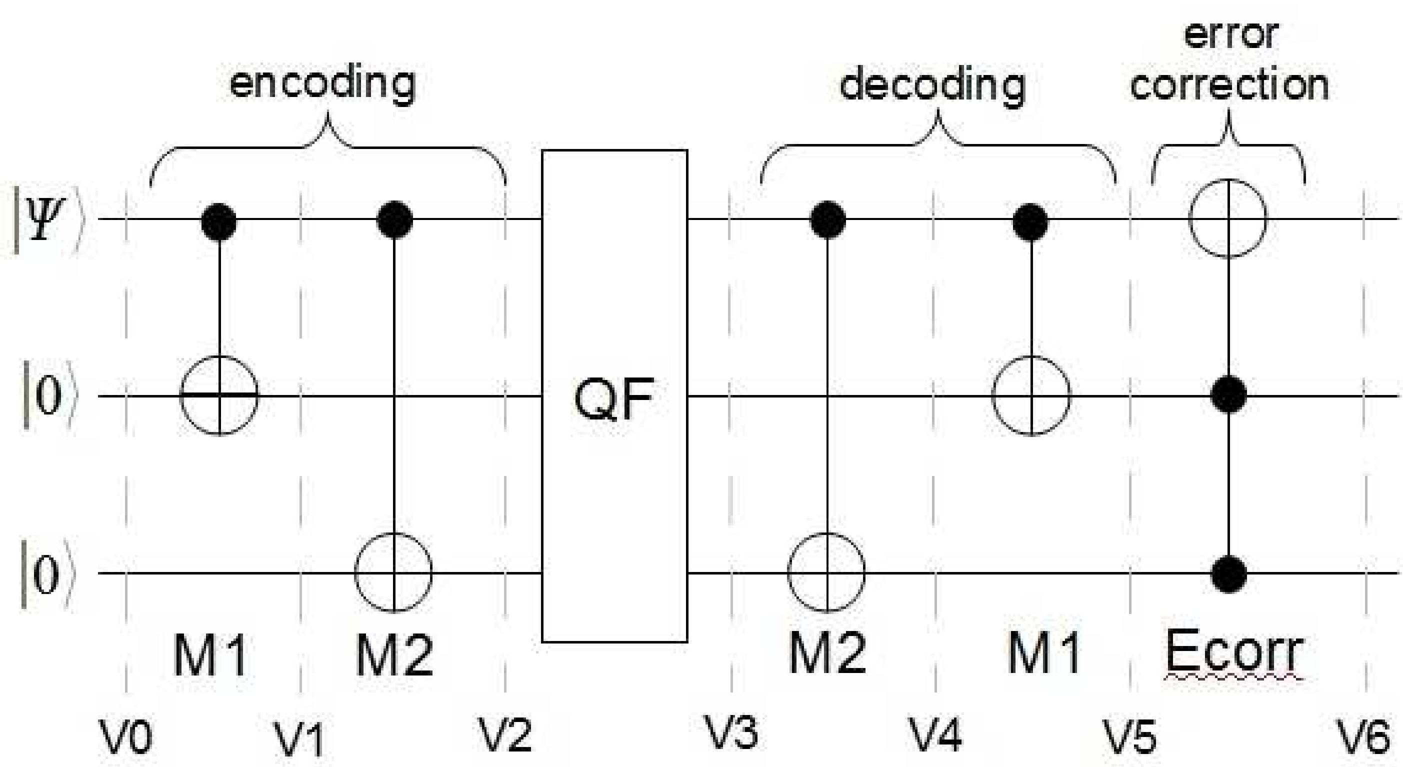
```
> restart :
> interface(warnlevel = 0) :  #
```

Maple 12

```
> with(LinearAlgebra) :
> with(Bits) :
```



Utility functions

```
> K := proc(a, b) return KroneckerProduct(a, b) end proc:
> T := proc(x) return Transpose(x) end proc:
> VSte := proc(n)                 # Generates a list of  computational states for  n qubits
          local i, L;            #  e.g.  n= 2  ⇒ [ |00⟩ |01⟩ |10⟩ |11⟩ ]
          L := Matrix(1, 2^n);
          Settings(defaultbits = n);
          for i from 1 to 2^n do
               L[1, i] := cat(`|`, String(i − 1, msbfirst), "⟩");
          end do;
     #    print(L);
        return L;       # returns Matrix L
     end proc:
```

Utility matrices

```
> I2 := IdentityMatrix(2);
```

$$I2 := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (1)$$

> $I4 := IdentityMatrix(4);$

$$I4 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

> $I8 := IdentityMatrix(8);$

$$I8 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

Utility Operators

> $Ux := RowOperation(I2, [1, 2]);$ *# qubit-flip operator*

$$Ux := \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad (4)$$

> $CNOT := RowOperation(I4, [3, 4]);$

$$CNOT := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (5)$$

> $G23 := RowOperation(I4, [2, 3]);$ *# qubit exchange operator*

$$G23 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

M1, M2, QF, and Ecorr matrices/operators

```
> M1 := K(CNOT, I2);
```

$$M1 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{7}$$

```
> M2 := Multiply(K(G23, I2), Multiply(K(I2, CNOT), K(G23, I2)));
```

$$M2 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{8}$$

```
> QF := K(Ux, I4);    # Matrix/operator for a qubit-flip error on the first qubit
```

$$QF := \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{9}$$

Ecorr matrix/operator is a variation of the Toffoli gate. The 2nd and 3rd qubits are the control qubits. When the state of both qubits equal "1", the 1st qubit is "flipped"

$$c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + \mathbf{c_3|011\rangle} + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + \mathbf{c_7|111\rangle}$$

$$c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + \mathbf{c_3|111\rangle} + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + \mathbf{c_7|011\rangle}$$

```
> Ecorr := RowOperation(I8, [8, 4]);    # Error Correction matrix
```

$$Ecorr := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \textbf{(10)}$$

$$\mathbf{V0 = (\alpha|0\rangle + \beta|1\rangle) \otimes |0\rangle \otimes |0\rangle = \alpha|000\rangle + \beta|100\rangle}$$

```
> n := 3 :
q1 := Matrix([[α], [β]]) :     #  α|0⟩ + β|1⟩
q2 := Matrix([[1], [0]]) :     #  |0⟩
q3 := Matrix([[1], [0]]) :     #  |0⟩
Co0 := K(K(q1, q2), q3) :
St := Transpose(VSte(n)) :
|V0⟩ := Multiply(T(Co0), St)[1, 1];
```

$$|V0\rangle := \alpha\ |000\rangle + \beta\ |100\rangle \quad \textbf{(11)}$$

$$\mathbf{V1 = M1 \cdot V0}$$
$$= \mathbf{M1(\alpha|000\rangle + \beta|100\rangle)}$$
$$= \mathbf{\alpha|000\rangle + \beta|110\rangle}$$

```
> Co1 := Multiply(M1, Co0) :
|V1⟩ := factor(Multiply(T(Co1), St)[1, 1]);
```

$$|V1\rangle := \alpha\ |000\rangle + \beta\ |110\rangle \quad \textbf{(12)}$$

$$\mathbf{V2} = \mathbf{M2} \cdot \mathbf{V1}$$
$$= \mathbf{M2}(\alpha|\mathbf{000}\rangle + \beta|\mathbf{110}\rangle)$$
$$= \alpha|\mathbf{000}\rangle + \beta|\mathbf{111}\rangle$$

```
> Co2 := Multiply(M2, Co1) :
  |V2⟩ := factor(Multiply(T(Co2), St)[1, 1]);
```

$$|V2\rangle := \alpha\ |000\rangle + \beta\ |111\rangle \tag{13}$$

The encoding operator/matrix

```
> Encode := Multiply(M2, M1);
  Co2 := Multiply(Encode, Co0) :
  |Ψ2⟩ := Multiply(T(Co2), St)[1, 1];
```

$$Encode := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$|\Psi 2\rangle := \alpha\ |000\rangle + \beta\ |111\rangle \tag{14}$$

A qubit-flip error on the first qubit

$$\mathbf{V3} = \alpha|\mathbf{100}\rangle + \beta|\mathbf{011}\rangle$$

```
> Co3 := simplify(Multiply(QF, Co2)) :
  |V3⟩ := Multiply(T(Co3), St)[1, 1];
```

$$|V3\rangle := \beta\ |011\rangle + \alpha\ |100\rangle \tag{15}$$

$$\mathbf{V4} = \mathbf{M2} \cdot \mathbf{V3}$$
$$= \mathbf{M2}(\alpha|\mathbf{100}\rangle + \beta|\mathbf{011}\rangle)$$
$$= \alpha|\mathbf{101}\rangle + \beta|\mathbf{011}\rangle$$

```
> Co4 := simplify(Multiply(M2, Co3)) :
  |V4⟩ := Multiply(T(Co4), St)[1, 1];
```

$$|V4\rangle := \beta\ |011\rangle + \alpha\ |101\rangle \tag{16}$$

$$\mathbf{V5} = \mathbf{M1} \cdot \mathbf{V4}$$
$$= \mathbf{M1}(\alpha|\mathbf{101}\rangle + \beta|\mathbf{011}\rangle)$$
$$= \alpha|\mathbf{111}\rangle + \beta|\mathbf{011}\rangle$$

```
> Co5 := simplify(Multiply(M1, Co4)) :
  |V5> := Multiply(T(Co5), St)[1, 1];
```

$$|V5\rangle := \beta\ |011\rangle + \alpha\ |111\rangle \tag{17}$$

The decoding operator/matrix

```
> Decode := Multiply(M1, M2);
Co5 := simplify(Multiply(Decode, Co3)) :
 `|V5>` := Multiply(T(Co5), St)[1, 1];
```

$$Decode := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$|V5\rangle := \beta\ |011\rangle + \alpha\ |111\rangle \tag{18}$$

$\mathbf{Encode} \cdot \mathbf{Decode} = \mathbf{I_8}$

```
> Multiply( Encode, Decode);
```

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{19}$$

$$\mathbf{V6} = \mathbf{Ecorr} \cdot \mathbf{V5}$$
$$= \mathbf{Ecorr}(\alpha|\mathbf{111}\rangle + \beta|\mathbf{011}\rangle)$$
$$= \alpha|\mathbf{011}\rangle + \beta|\mathbf{111}\rangle$$
$$= (\alpha|\mathbf{0}\rangle + \beta|\mathbf{1}\rangle) \otimes |\mathbf{1}\rangle \otimes |\mathbf{1}\rangle$$

```
> Co6 := simplify(Multiply(Ecorr, Co5)) :
  |V6⟩ := Multiply(T(Co6), St)[1, 1];
```

$$|V6\rangle := \alpha\ |011\rangle + \beta\ |111\rangle \tag{20}$$

Compare to V0

$$\mathbf{V0} = (\alpha|\mathbf{0}\rangle + \beta|\mathbf{1}\rangle) \otimes |\mathbf{0}\rangle \otimes |\mathbf{0}\rangle = \alpha|\mathbf{000}\rangle + \beta|\mathbf{100}\rangle$$

```
> |V0⟩ := Multiply(T(Co0), St)[1, 1];
```

$$|V0\rangle := \alpha\ |000\rangle + \beta\ |100\rangle \tag{21}$$