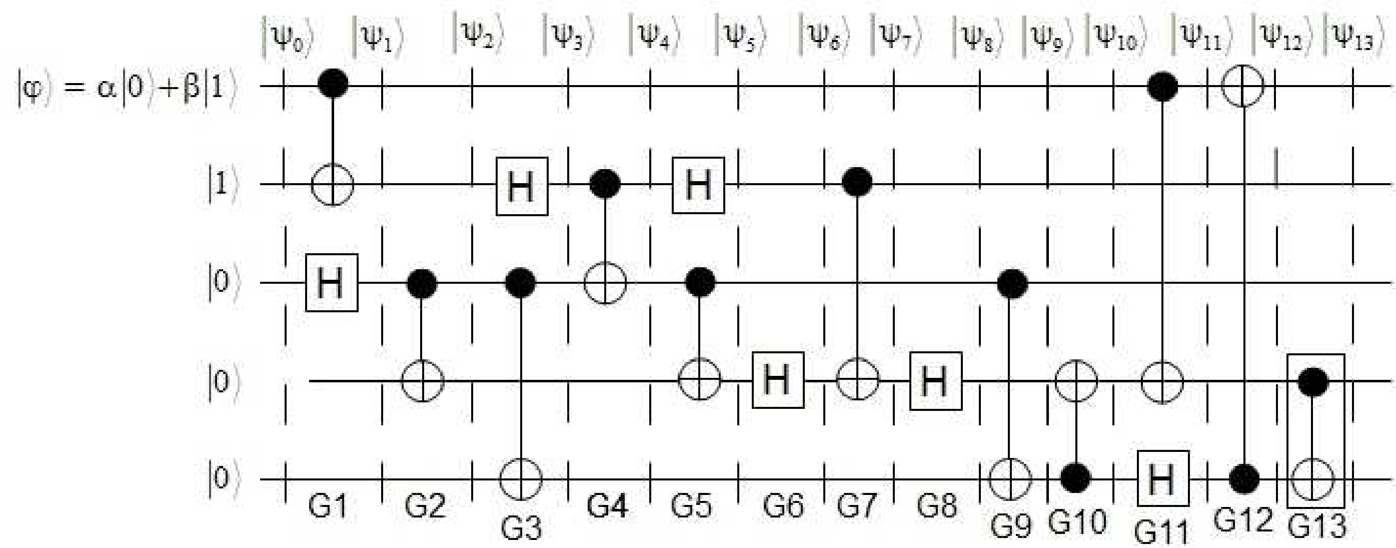
```
> restart :
> interface(warnlevel = 0) :     # Maple 12
> # interface(rtablesize = 32) :
> with(LinearAlgebra) :
> with(Bits) :
```




```
> K := proc(a, b) return KroneckerProduct(a, b)  end proc:
> T := proc(x) return Transpose(x) end proc:
> VSte := proc(n)                  # Generates a list of  computational states for  n qubits
            local i, L;            # e.g.  n= 2  ⇒ [ |00⟩ |01⟩ |10⟩ |11⟩ ]
            L := Matrix(1, 2^n);
            Settings(defaultbits = n);
            for i from 1 to 2^n do
                L[1, i] := cat(`|`, String(i − 1, msbfirst), "⟩");
            end do;
        #    print(L);
          return L;      # returns Matrix L
        end proc:
> Gxnot := proc(bit)                  #` `
             local i, srow, lrow;
             global M;
             srow := 2^(bit − 1) + 1;
             lrow := 2^bit − 1;
             for i from  srow to lrow by 2 do
                 M := RowOperation(M, [i, (i + 1) ]);
             end do;
           end proc:
```

```
> Gnotx := proc(bit)
         local i, srow, lrow;
         global M;
         srow := 2;
         lrow := 2^(bit - 1);
         for i from srow to lrow by 2 do
             M := RowOperation(M, [i, (i + lrow)]);
         end do;
     end proc:
```

Defining working matrices/operators

```
> I2 := IdentityMatrix(2); I4 := IdentityMatrix(4);
CNOT := RowOperation(I4, [3, 4]);
G23 := RowOperation(I4, [2, 3]); G24 := RowOperation(I4, [2, 4]);
H := 1/sqrt(2) · Matrix([[1, 1], [1, -1]]);
M3 := Multiply(K(G23, I2), Multiply(K(I2, CNOT), K(G23, I2))) : # swap gate
```

$$I2 := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$I4 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$CNOT := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$G23 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$G24 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$H := \begin{bmatrix} \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \\ \frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} \end{bmatrix} \tag{1}$$

The gates

```
> G1 := K(CNOT, K(H, I4)) :
> G2 := K(I4, K(CNOT, I2)) :
> G3 := K(K(I2, H), M3) :
> G4 := K(I2, K(CNOT, I4)) :
> G5 := K(I2, K(H, K(CNOT, I2))) :
> G6 := K(I2, K(I4, K(H, I2))) :
> G7 := K(I2, K(M3, I2)) :
> G8 := K(I2, K(I4, K(H, I2))) :
> G9 := K(I4, M3) :
> G10 := K(I2, K(I4, G24)) :
> M := IdentityMatrix(16) :
  G11 := K(Gxnot(4), H) :
> M := IdentityMatrix(32) :
  G12 := Gnotx(5) :
```

Initial State Functions functions

```
> Co := Matrix([[α], [β]]) :
  St := T(VSte(1)) :
  ϕ := Multiply(T(Co), St)[1, 1];   # the unknown qubit to be teleported
```

$$\phi := \alpha\,|0\rangle + \beta\,|1\rangle \tag{2}$$

```
> q1 := Matrix([[0], [1]]) :          # ancillary qubits
  q0 := Matrix([[1], [0]]) :
  St := T(VSte(5)) :
  Co0 := K(Co, K(q1, K(q0, K(q0, q0)))) :
  |ψ0⟩ := Multiply(T(Co0), St)[1, 1];        # initial state: unknown qubit with ancillary qubits
```

$$|\psi 0\rangle := \alpha\,|01000\rangle + \beta\,|11000\rangle \tag{3}$$

```
> Co1 := Multiply(G1, Co0) :
  |ψ1⟩ := factor(Multiply(T(Co1), St)[1, 1]);
```

$$|\psi 1\rangle := \frac{1}{2}\sqrt{2}\,\left(\alpha\,|01000\rangle + \alpha\,|01100\rangle + \beta\,|10000\rangle + \beta\,|10100\rangle\right) \tag{4}$$

> Co2 := Multiply(G2, Co1) :
> |ψ2⟩ := factor(Multiply(T(Co2), St)[1, 1]);

$$|\psi 2\rangle := \frac{1}{2}\sqrt{2}\left(\alpha\,|01110\rangle + \beta\,|10110\rangle + \alpha\,|01000\rangle + \beta\,|10000\rangle\right) \quad \textbf{(5)}$$

> Co3 := Multiply(G3, Co2) :
> `|ψ3⟩` := collect(Multiply(T(Co3), St)[1, 1], [α, β]);

$$|\psi 3\rangle := \left(\frac{1}{2}|00000\rangle - \frac{1}{2}|01111\rangle - \frac{1}{2}|01000\rangle + \frac{1}{2}|00111\rangle\right)\alpha + \left(\frac{1}{2}|10000\rangle + \frac{1}{2}|11111\rangle + \frac{1}{2}|11000\rangle + \frac{1}{2}|10111\rangle\right)\beta \quad \textbf{(6)}$$

> Co4 := Multiply(G4, Co3) :
> |ψ4⟩ := collect(Multiply(T(Co4), St)[1, 1], [α, β]);

$$|\psi 4\rangle := \left(-\frac{1}{2}|01100\rangle - \frac{1}{2}|01011\rangle + \frac{1}{2}|00000\rangle + \frac{1}{2}|00111\rangle\right)\alpha + \left(\frac{1}{2}|10000\rangle + \frac{1}{2}|11011\rangle + \frac{1}{2}|11100\rangle + \frac{1}{2}|10111\rangle\right)\beta \quad \textbf{(7)}$$

> Co5 := Multiply(G5, Co4) :
> |ψ5⟩ := collect(Multiply(T(Co5), St)[1, 1], [α, β]);

$$|\psi 5\rangle := \left(\frac{1}{4}\sqrt{2}|01110\rangle + \frac{1}{4}\sqrt{2}|00000\rangle + \frac{1}{4}\sqrt{2}|01011\rangle + \frac{1}{4}\sqrt{2}|00101\rangle - \frac{1}{4}\sqrt{2}|00110\rangle - \frac{1}{4}\sqrt{2}|00011\rangle + \frac{1}{4}\sqrt{2}|01101\rangle + \frac{1}{4}\sqrt{2}|01000\rangle\right)\alpha + \left(\frac{1}{4}\sqrt{2}|11000\rangle + \frac{1}{4}\sqrt{2}|10110\rangle + \frac{1}{4}\sqrt{2}|10101\rangle + \frac{1}{4}\sqrt{2}|10000\rangle - \frac{1}{4}\sqrt{2}|11011\rangle + \frac{1}{4}\sqrt{2}|10011\rangle + \frac{1}{4}\sqrt{2}|11101\rangle - \frac{1}{4}\sqrt{2}|11110\rangle\right)\beta \quad \textbf{(8)}$$

> Co6 := Multiply(G6, Co5) :
> |ψ6⟩ := collect(Multiply(T(Co6), St)[1, 1], [α, β]);

$$|\psi 6\rangle := \left(\frac{1}{4}|01111\rangle + \frac{1}{4}|00000\rangle - \frac{1}{4}|01110\rangle + \frac{1}{4}|01100\rangle + \frac{1}{4}|01000\rangle + \frac{1}{4}|01001\rangle + \frac{1}{4}|00111\rangle + \frac{1}{4}|00101\rangle + \frac{1}{4}|00110\rangle - \frac{1}{4}|01011\rangle + \frac{1}{4}|01010\rangle + \frac{1}{4}|01101\rangle - \frac{1}{4}|00001\rangle + \frac{1}{4}|00010\rangle + \frac{1}{4}|00011\rangle - \frac{1}{4}|00100\rangle\right)\alpha + \left(\frac{1}{4}|10100\rangle + \frac{1}{4}|10000\rangle - \frac{1}{4}|11001\rangle + \frac{1}{4}|11111\rangle + \frac{1}{4}|11101\rangle + \frac{1}{4}|11110\rangle - \frac{1}{4}|11100\rangle + \frac{1}{4}|10111\rangle + \frac{1}{4}|10010\rangle + \frac{1}{4}|11000\rangle - \frac{1}{4}|10011\rangle - \frac{1}{4}|10110\rangle + \frac{1}{4}|11010\rangle + \frac{1}{4}|11011\rangle + \frac{1}{4}|10101\rangle + \frac{1}{4}|10001\rangle\right)\beta \quad \textbf{(9)}$$

> $Co7 := Multiply(G7, Co6)$:
$|\psi7\rangle := collect(Multiply(T(Co7), St)[1, 1], [\alpha, \beta])$;

$$|\psi7\rangle := \Big(-\frac{1}{4}|01100\rangle + \frac{1}{4}|01000\rangle + \frac{1}{4}|01111\rangle - \frac{1}{4}|00001\rangle + \frac{1}{4}|00010\rangle + \frac{1}{4}|00011\rangle - \frac{1}{4}|00100\rangle + \frac{1}{4}|00101\rangle + \frac{1}{4}|00110\rangle + \frac{1}{4}|00111\rangle - \frac{1}{4}|01001\rangle + \frac{1}{4}|01010\rangle + \frac{1}{4}|01011\rangle + \frac{1}{4}|01101\rangle + \frac{1}{4}|01110\rangle + \frac{1}{4}|00000\rangle\Big)\alpha + \Big(\frac{1}{4}|10100\rangle + \frac{1}{4}|10000\rangle - \frac{1}{4}|11110\rangle + \frac{1}{4}|11111\rangle + \frac{1}{4}|11000\rangle + \frac{1}{4}|10001\rangle + \frac{1}{4}|10010\rangle - \frac{1}{4}|10011\rangle + \frac{1}{4}|10101\rangle + \frac{1}{4}|10111\rangle + \frac{1}{4}|11001\rangle + \frac{1}{4}|11010\rangle - \frac{1}{4}|11011\rangle + \frac{1}{4}|11100\rangle + \frac{1}{4}|11101\rangle - \frac{1}{4}|10110\rangle\Big)\beta \quad \textbf{(10)}$$

> $Co8 := Multiply(G8, Co7)$:
$|\psi8\rangle := collect(Multiply(T(Co8), St)[1, 1], [\alpha, \beta])$;

$$|\psi8\rangle := \Big(\frac{1}{4}\sqrt{2}|01000\rangle - \frac{1}{4}\sqrt{2}|01011\rangle + \frac{1}{4}\sqrt{2}|01101\rangle - \frac{1}{4}\sqrt{2}|01110\rangle + \frac{1}{4}\sqrt{2}|00000\rangle - \frac{1}{4}\sqrt{2}|00110\rangle + \frac{1}{4}\sqrt{2}|00101\rangle - \frac{1}{4}\sqrt{2}|00011\rangle\Big)\alpha + \Big(\frac{1}{4}\sqrt{2}|11011\rangle + \frac{1}{4}\sqrt{2}|10101\rangle + \frac{1}{4}\sqrt{2}|11110\rangle + \frac{1}{4}\sqrt{2}|10000\rangle + \frac{1}{4}\sqrt{2}|10110\rangle + \frac{1}{4}\sqrt{2}|10011\rangle + \frac{1}{4}\sqrt{2}|11000\rangle + \frac{1}{4}\sqrt{2}|11101\rangle\Big)\beta \quad \textbf{(11)}$$

> $Co9 := Multiply(G9, Co8)$:
$|\psi9\rangle := collect(Multiply(T(Co9), St)[1, 1], [\alpha, \beta])$;

$$|\psi9\rangle := \Big(-\frac{1}{4}\sqrt{2}|01111\rangle - \frac{1}{4}\sqrt{2}|00111\rangle + \frac{1}{4}\sqrt{2}|01100\rangle - \frac{1}{4}\sqrt{2}|01011\rangle + \frac{1}{4}\sqrt{2}|00100\rangle - \frac{1}{4}\sqrt{2}|00011\rangle + \frac{1}{4}\sqrt{2}|01000\rangle + \frac{1}{4}\sqrt{2}|00000\rangle\Big)\alpha + \Big(\frac{1}{4}\sqrt{2}|11111\rangle + \frac{1}{4}\sqrt{2}|11100\rangle + \frac{1}{4}\sqrt{2}|10100\rangle + \frac{1}{4}\sqrt{2}|10111\rangle + \frac{1}{4}\sqrt{2}|11011\rangle + \frac{1}{4}\sqrt{2}|10011\rangle + \frac{1}{4}\sqrt{2}|11000\rangle + \frac{1}{4}\sqrt{2}|10000\rangle\Big)\beta \quad \textbf{(12)}$$

> $Co10 := Multiply(G10, Co9)$:
$|\psi10\rangle := collect(Multiply(T(Co10), St)[1, 1], [\alpha, \beta])$;

$$|\psi10\rangle := \Big(-\frac{1}{4}\sqrt{2}|01001\rangle + \frac{1}{4}\sqrt{2}|01100\rangle - \frac{1}{4}\sqrt{2}|00001\rangle - \frac{1}{4}\sqrt{2}|01101\rangle - \frac{1}{4}\sqrt{2}|00101\rangle + \frac{1}{4}\sqrt{2}|01000\rangle + \frac{1}{4}\sqrt{2}|00000\rangle + \frac{1}{4}\sqrt{2}|00100\rangle\Big)\alpha + \Big(\frac{1}{4}\sqrt{2}|11100\rangle + \frac{1}{4}\sqrt{2}|10100\rangle + \frac{1}{4}\sqrt{2}|11101\rangle + \frac{1}{4}\sqrt{2}|10001\rangle + \frac{1}{4}\sqrt{2}|11001\rangle + \frac{1}{4}\sqrt{2}|11000\rangle + \frac{1}{4}\sqrt{2}|10101\rangle + \frac{1}{4}\sqrt{2}|10000\rangle\Big)\beta \quad \textbf{(13)}$$

> $Co11 := Multiply(G11, Co10):$
> $|\psi 11\rangle := collect(Multiply(T(Co11), St)[1,1], [\alpha, \beta]);$

$$|\psi 11\rangle := \left(\frac{1}{2}\,|00001\rangle + \frac{1}{2}\,|00101\rangle + \frac{1}{2}\,|01001\rangle + \frac{1}{2}\,|01101\rangle\right)\alpha + \left(\frac{1}{2}\,|10010\rangle + \frac{1}{2}\,|11010\rangle + \frac{1}{2}\,|11110\rangle + \frac{1}{2}\,|10110\rangle\right)\beta \quad \textbf{(14)}$$

> $Co12 := Multiply(G12, Co11):$
> $|\psi 12\rangle := collect(Multiply(T(Co12), St)[1,1], [\alpha, \beta]);$

$$|\psi 12\rangle := \left(\frac{1}{2}\,|10101\rangle + \frac{1}{2}\,|10001\rangle + \frac{1}{2}\,|11101\rangle + \frac{1}{2}\,|11001\rangle\right)\alpha + \left(\frac{1}{2}\,|10010\rangle + \frac{1}{2}\,|11010\rangle + \frac{1}{2}\,|11110\rangle + \frac{1}{2}\,|10110\rangle\right)\beta \quad \textbf{(15)}$$

Notice that we can re-write $|\psi 12\rangle$ as

$$\frac{1}{2}\left(|100\rangle + |101\rangle + |110\rangle + |111\rangle\right) \otimes \left(\alpha|01\rangle + \beta|10\rangle\right)$$

The last 2 qubits allows the recovery of the unknown state φ. This can be done by operating with a CNOT gate on the last 2 qubits.

> $G13 := K(K(I2, I4), CNOT);$
> $Co13 := Multiply(G13, Co12);$
> $|\psi 13\rangle := collect(Multiply(T(Co13), St)[1,1], [\alpha, \beta]);$

$$G13 := \begin{bmatrix} \textit{32 x 32 Matrix} \\ \textit{Data Type: anything} \\ \textit{Storage: rectangular} \\ \textit{Order: Fortran\_order} \end{bmatrix}$$

$$Co13 := \begin{bmatrix} \textit{32 x 1 Matrix} \\ \textit{Data Type: anything} \\ \textit{Storage: rectangular} \\ \textit{Order: Fortran\_order} \end{bmatrix}$$

$$|\psi 13\rangle := \left(\frac{1}{2}\,|10101\rangle + \frac{1}{2}\,|10001\rangle + \frac{1}{2}\,|11101\rangle + \frac{1}{2}\,|11001\rangle\right)\alpha + \left(\frac{1}{2}\,|11011\rangle + \frac{1}{2}\,|11111\rangle + \frac{1}{2}\,|10011\rangle + \frac{1}{2}\,|10111\rangle\right)\beta \quad \textbf{(16)}$$

We can re-write $|\psi 13\rangle$ as

$$\frac{1}{2}\left(|100\rangle + |101\rangle + |110\rangle + |111\rangle\right) \otimes \left(\alpha|0\rangle + \beta|1\rangle\right) \otimes |1\rangle$$