> *restart*;

> *interface*(*warnlevel* = 0) :   #   **Maple 12**

> *with*(*plots*) :
with(*LinearAlgebra*) :

Bloch Equation

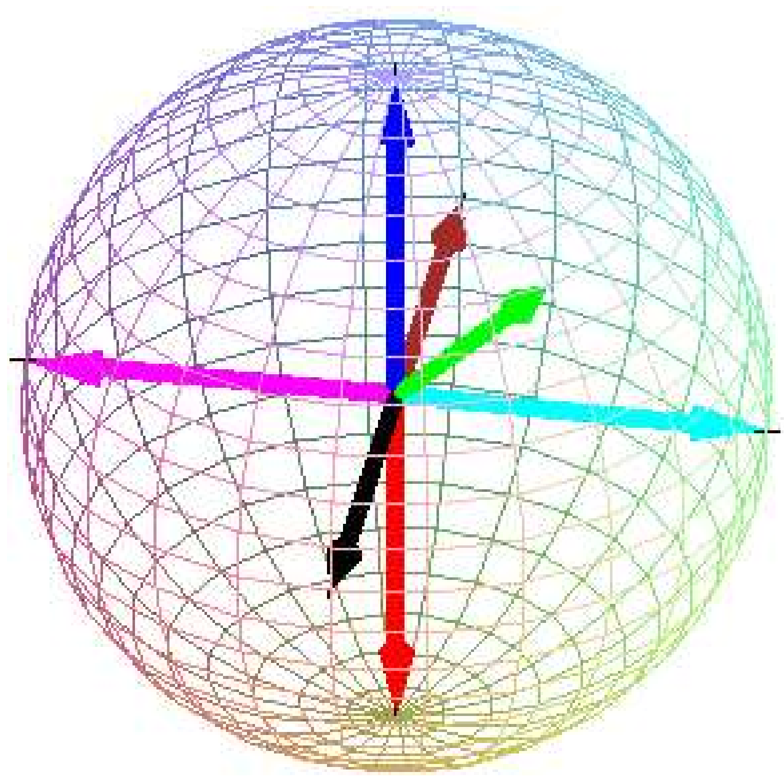
> $\psi := (\theta, \phi) \rightarrow Vector\left(\left[\left[\cos\left(\frac{\theta}{2}\right), e^{I\cdot\phi}\cdot \sin\left(\frac{\theta}{2}\right)\right]\right]\right)$:

Bloch Vector

> $r := (\theta, \phi) \rightarrow Vector([[\ \sin(\theta)\cos(\phi),\ \sin(\theta)\sin(\phi),\ \cos(\theta)\ ]])$:

Bloch Sphere

> $R := plot3d(r(\theta, \phi),\ \theta = 0..\pi, \phi = 0..2\cdot\pi)$:
$a1 := arrow(r(0, 0),\ color = blue)$: $a2 := arrow(r(\pi, 0), color = red)$:
$a3 := arrow\left(r\left(\frac{\pi}{2}, 0\right), color = black\right)$: $a4 := arrow\left(r\left(\frac{\pi}{2}, \pi\right), color = brown\right)$:
$a5 := arrow\left(r\left(\frac{\pi}{2}, \frac{\pi}{2}\right), color = cyan\right)$: $a6 := arrow\left(r\left(\frac{\pi}{2}, \frac{3\pi}{2}\right), color = magenta\right)$:
$P := arrow\left(r\left(\frac{\pi}{4}, \frac{\pi}{4}\right), color = green\right)$:
display([*R*, *a1*, *a2*, *a3*, *a4*, *a5*, *a6*, *P*], *style* = *wireframe*, *tickmarks* = [0, 0, 0],
axes = *normal*, *orientation* = [10, 58]);



Vectors $\mathbf{r}(\theta, \phi) \rightarrow \mathbf{r}(\mathbf{x}, \mathbf{y}, \mathbf{z})$

$> V[r] := \left[\mathrm{r}(0,0), \mathrm{r}(\pi,0), \mathrm{r}\left(\frac{\pi}{2}, 0\right), \mathrm{r}\left(\frac{\pi}{2}, \pi\right), \mathrm{r}\left(\frac{\pi}{2}, \frac{\pi}{2}\right), \mathrm{r}\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)\right];$

$$V_r := \left[\begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\0\\-1\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}-1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\-1\\0\end{bmatrix}\right] \tag{1}$$

Points on the Bloch Sphere

$> P[\psi] := \left[\psi(0,0), \psi(\pi,0), \psi\left(\frac{\pi}{2}, 0\right), \psi\left(\frac{\pi}{2}, \pi\right), \psi\left(\frac{\pi}{2}, \frac{\pi}{2}\right), \psi\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)\right];$

$$P_\psi := \left[\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}\frac{1}{2}\sqrt{2}\\\frac{1}{2}\sqrt{2}\end{bmatrix}, \begin{bmatrix}\frac{1}{2}\sqrt{2}\\-\frac{1}{2}\sqrt{2}\end{bmatrix}, \begin{bmatrix}\frac{1}{2}\sqrt{2}\\\frac{1}{2}\mathrm{I}\sqrt{2}\end{bmatrix}, \begin{bmatrix}\frac{1}{2}\sqrt{2}\\-\frac{1}{2}\mathrm{I}\sqrt{2}\end{bmatrix}\right] \tag{2}$$

The vector to point P and qubit at P

$> \text{'}r\text{'} = r\left(\frac{\pi}{4}, \frac{\pi}{4}\right), \text{'}\psi\text{'} = \psi\left(\frac{\pi}{4}, \frac{\pi}{4}\right);$

$$r = \begin{bmatrix}\frac{1}{2}\\\frac{1}{2}\\\frac{1}{2}\sqrt{2}\end{bmatrix}, \psi = \begin{bmatrix}\cos\left(\frac{1}{8}\pi\right)\\\left(\frac{1}{2}\sqrt{2} + \frac{1}{2}\mathrm{I}\sqrt{2}\right)\sin\left(\frac{1}{8}\pi\right)\end{bmatrix} \tag{3}$$

Projection $|\psi\rangle\langle\psi| = \rho$

$> \rho := combine\left(Multiply\left(\psi\left(\frac{\pi}{4}, \frac{\pi}{4}\right), HermitianTranspose\left(\psi\left(\frac{\pi}{4}, \frac{\pi}{4}\right)\right)\right)\right);$

$$\rho := \begin{bmatrix}\frac{1}{4}\sqrt{2} + \frac{1}{2} & \frac{1}{4} - \frac{1}{4}\mathrm{I}\\\frac{1}{4} + \frac{1}{4}\mathrm{I} & \frac{1}{2} - \frac{1}{4}\sqrt{2}\end{bmatrix} \tag{4}$$

$> L\rho := \left[Determinant(\rho), Trace(\rho), simplify\left(Trace\left(\rho^2\right)\right)\right];$

$$L\rho := [0, 1, 1] \tag{5}$$

This procedure draws a Bloch vector from ρ

```
> BVect := proc(ro)
      local x1, y1, z1, r1, θ1, ϕ1, v, T;
      z1 := simplify(ro[1, 1] − ro[2, 2]);      # determine the x, y, z coords
      x1 := simplify(ro[1, 2] + ro[2, 1]);      #  from ρ
      y1 := simplify(I· (ro[1, 2] − ro[2, 1]));
      r1 := simplify(sqrt(|x1|^2 + |y1|^2 + |z1|^2));    # determine the length of vector r
   if r1 > 0 then                                # if the length is zero; a point at the origin.
         θ1 := simplify(cos^(−1)(z1/r1));
         if (θ1 = 0) or (θ1 = π) then ϕ1 := 0
            else
          ϕ1 := simplify(cos^(−1)(x1/(r1 · sin(θ1))));
         end if;
      else
          θ1 := 0 : ϕ1 := 0 :
      end if;
      print(x = x1, y = y1, z = z1);  # print coords. angles & vector length
      print(r = r1, θ = θ1, ϕ = ϕ1);
      T := simplify(Trace(ρ^2));
      print(Tr^2 = T);
      v := arrow([x1, y1, z1], color = red, width = 0.05);  # plot sphere & vector
      display([R, v], axes = normal, style = wireframe, orientation = [36, 80]);

end proc :
```

> 'ρ'= ρ;

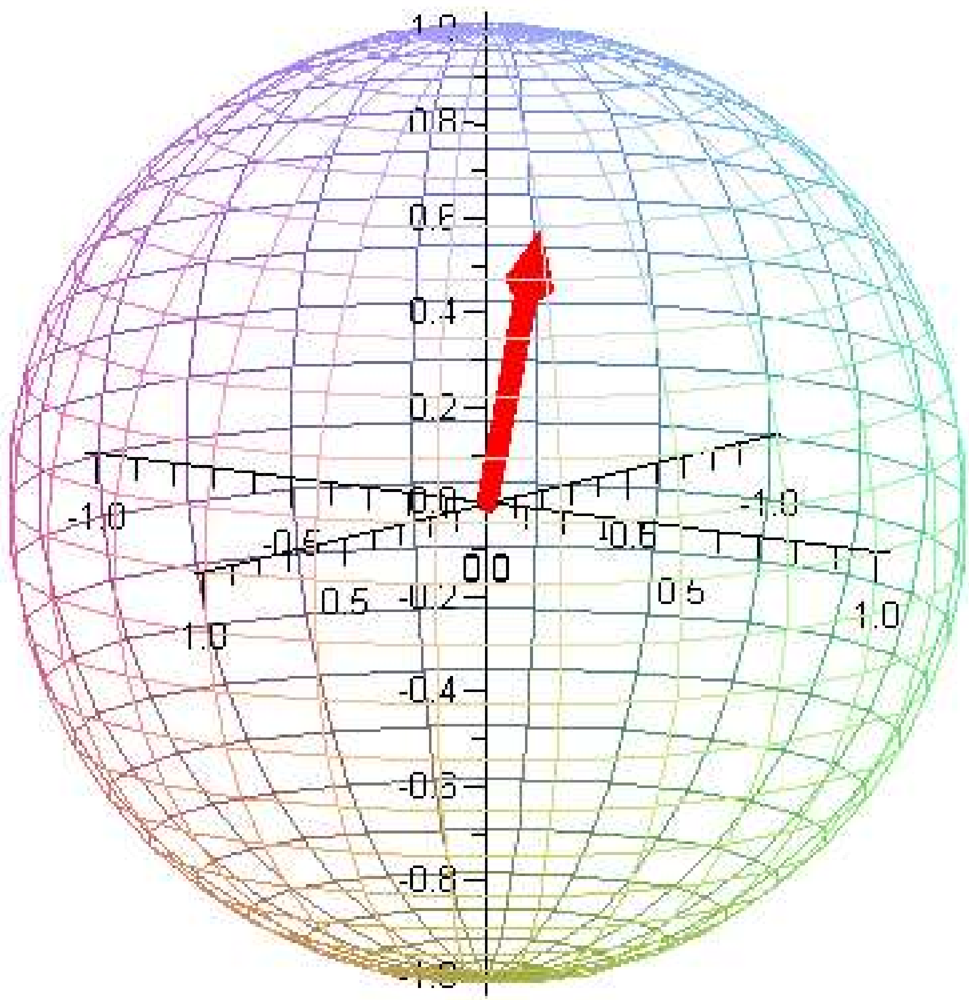
$BVect(\rho)$;

$$\rho = \begin{bmatrix} \frac{1}{4}\sqrt{2}+\frac{1}{2} & \frac{1}{4}-\frac{1}{4}\,\mathrm{I} \\ \frac{1}{4}+\frac{1}{4}\,\mathrm{I} & \frac{1}{2}-\frac{1}{4}\sqrt{2} \end{bmatrix}$$

$$x = \frac{1}{2}, y = \frac{1}{2}, z = \frac{1}{2}\sqrt{2}$$

$$r = 1, \theta = \frac{1}{4}\pi, \phi = \frac{1}{4}\pi$$

$$Tr^2 = 1$$



> $\rho := Matrix\left(\left[\left[\frac{3}{4}, 0\right], \left[0, \frac{1}{4}\right]\right]\right);$

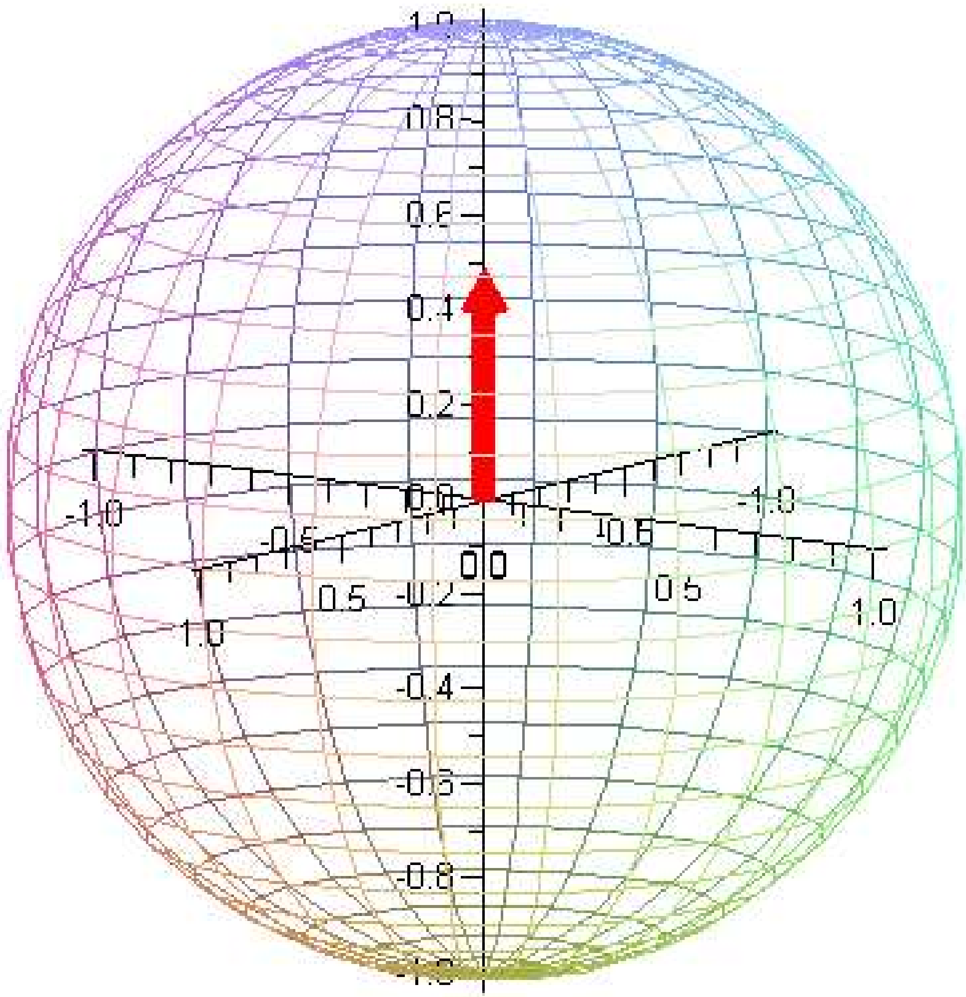
$BVect(\rho);$

$$\rho := \begin{bmatrix} \frac{3}{4} & 0 \\ 0 & \frac{1}{4} \end{bmatrix}$$

$$x = 0, y = 0, z = \frac{1}{2}$$

$$r = \frac{1}{2}, \theta = 0, \phi = 0$$

$$Tr^2 = \frac{5}{8}$$



> $\rho := Matrix\left(\left[\left[\frac{2}{3}, 0\right], \left[0, \frac{1}{3}\right]\right]\right);$

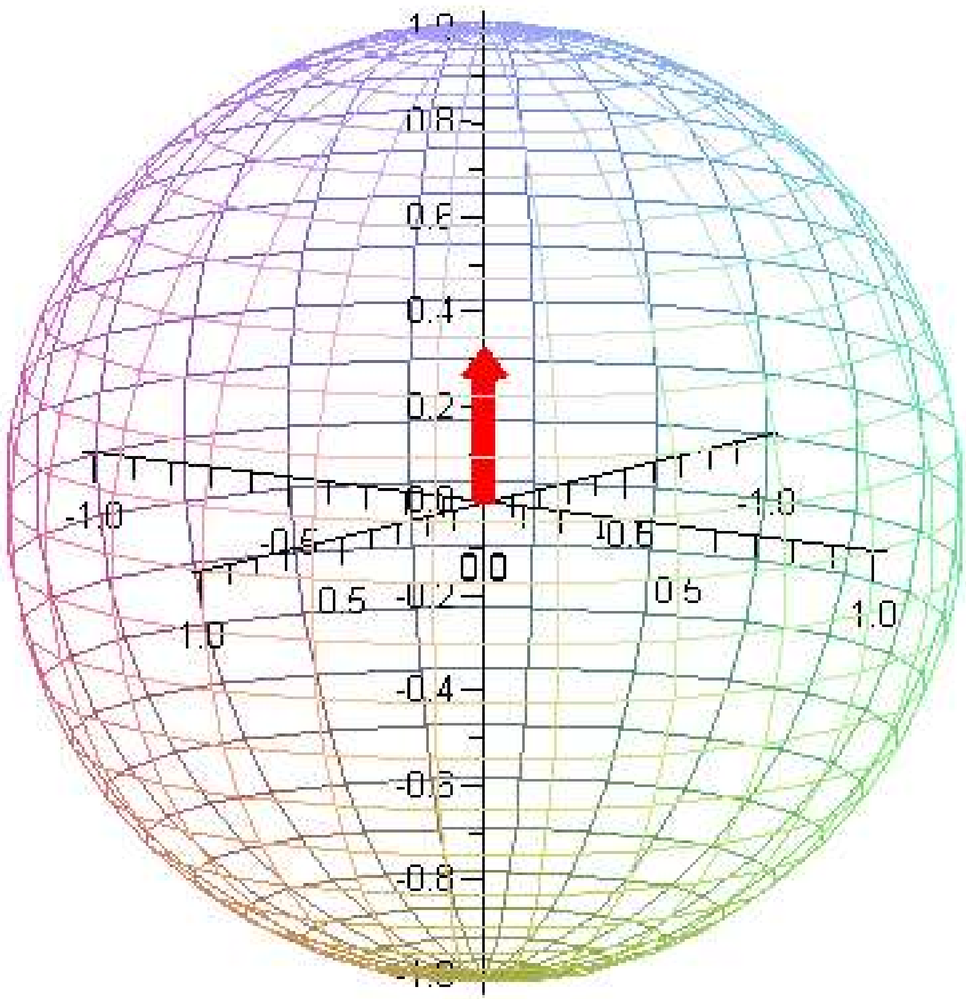
$BVect(\rho);$

$$\rho := \begin{bmatrix} \frac{2}{3} & 0 \\ 0 & \frac{1}{3} \end{bmatrix}$$

$$x = 0, y = 0, z = \frac{1}{3}$$

$$r = \frac{1}{3}, \theta = 0, \phi = 0$$

$$Tr^2 = \frac{5}{9}$$



> $\rho := Matrix\left(\left[\left[\frac{3}{7}, 0\right], \left[0, \frac{4}{7}\right]\right]\right);$

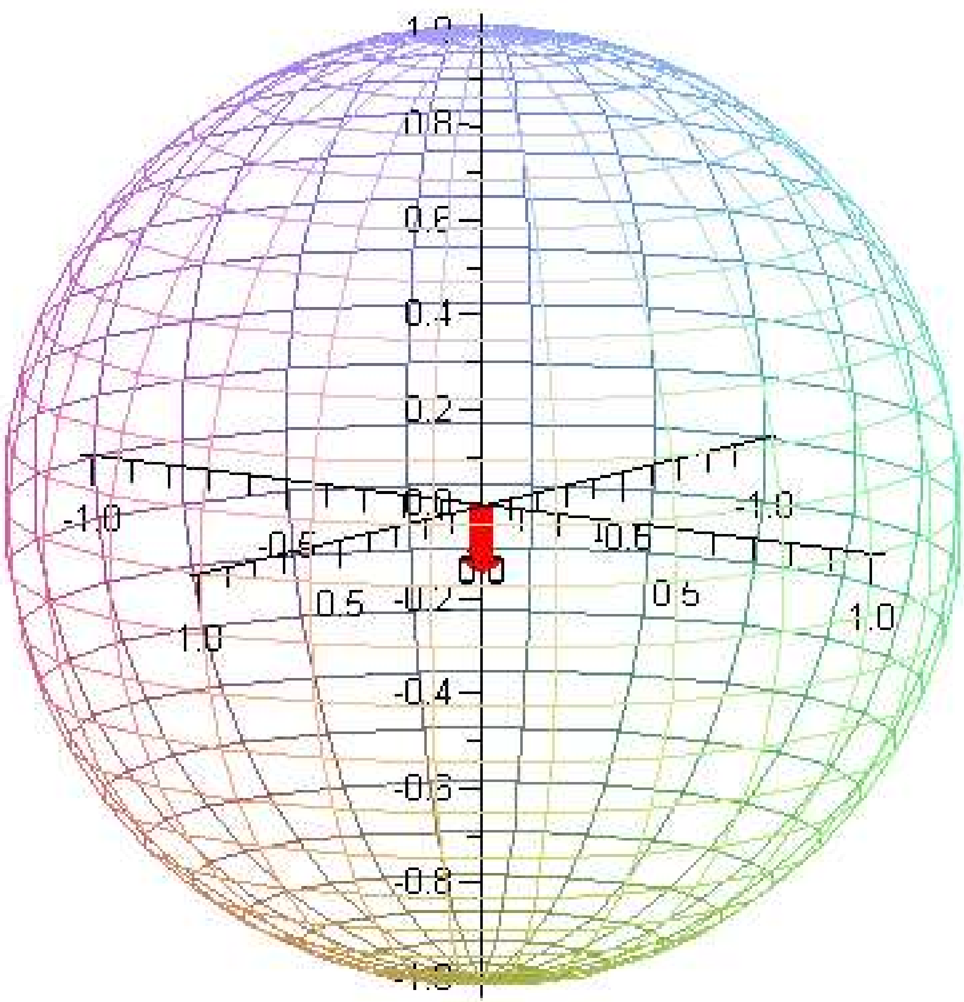
$BVect(\rho);$

$$\rho := \begin{bmatrix} \frac{3}{7} & 0 \\ 0 & \frac{4}{7} \end{bmatrix}$$

$$x = 0, y = 0, z = -\frac{1}{7}$$

$$r = \frac{1}{7}, \theta = \pi, \phi = 0$$

$$Tr^2 = \frac{25}{49}$$



> $\rho := Matrix\left(\left[\left[\frac{1}{2}, 0\right], \left[0, \frac{1}{2}\right]\right]\right);$

$BVect(\rho);$

$$\rho := \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$$

$$x = 0, y = 0, z = 0$$

$$r = 0, \theta = 0, \phi = 0$$

$$Tr^2 = \frac{1}{2}$$