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> restart;
  interface(warnlevel = 0) :   #   Maple 12
  with(LinearAlgebra) :
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The rotation operator $\mathscr{R}(\alpha,\beta,\gamma)$ when $\alpha = \gamma$.

The R matrices; Rz(α) and Ry(β)

$$> \ Rz := \alpha \rightarrow Matrix\left(\left[\left[e^{-I\cdot\frac{\alpha}{2}}, 0\right], \left[0, e^{+I\cdot\frac{\alpha}{2}}\right]\right]\right) :$$
$$'Rz(\alpha)' = Rz(\alpha);$$

$$Rz(\alpha) = \begin{bmatrix} e^{-\frac{1}{2} I\alpha} & 0 \\ 0 & e^{\frac{1}{2} I\alpha} \end{bmatrix} \tag{1}$$

$$> \ Ry := \beta \rightarrow Matrix\left(\left[\left[\cos\left(\frac{\beta}{2}\right), -\sin\left(\frac{\beta}{2}\right)\right], \left[\sin\left(\frac{\beta}{2}\right), \cos\left(\frac{\beta}{2}\right)\right]\right]\right) :$$
$$'Ry(\beta)' = Ry(\beta);$$

$$Ry(\beta) = \begin{bmatrix} \cos\left(\frac{1}{2}\beta\right) & -\sin\left(\frac{1}{2}\beta\right) \\ \sin\left(\frac{1}{2}\beta\right) & \cos\left(\frac{1}{2}\beta\right) \end{bmatrix} \tag{2}$$

The M1, M2, and M3 matrices

$$> \ M1 := (\alpha, \beta) \rightarrow \left(simplify\left(Multiply\left(Rz(\alpha), Ry\left(\frac{\beta}{2}\right)\right)\right)\right) :$$
$$'M1(\alpha, \beta)' = M1(\alpha, \beta);$$

$$M1(\alpha, \beta) = \begin{bmatrix} e^{-\frac{1}{2} I\alpha} \cos\left(\frac{1}{4}\beta\right) & -e^{-\frac{1}{2} I\alpha} \sin\left(\frac{1}{4}\beta\right) \\ e^{\frac{1}{2} I\alpha} \sin\left(\frac{1}{4}\beta\right) & e^{\frac{1}{2} I\alpha} \cos\left(\frac{1}{4}\beta\right) \end{bmatrix} \tag{3}$$

$$> \ M2 := (\alpha, \beta, \delta) \rightarrow \left(simplify\left(Multiply\left(Ry\left(-\frac{\beta}{2}\right), Rz\left(-\frac{(\alpha+\delta)}{2}\right)\right)\right)\right) :$$
$$'M2(\alpha, \beta, \delta)' = M2(\alpha, \beta, \delta);$$

$$M2(\alpha, \beta, \delta) = \begin{bmatrix} \cos\left(\frac{1}{4}\beta\right) e^{\frac{1}{4} I(\alpha+\delta)} & \sin\left(\frac{1}{4}\beta\right) e^{-\frac{1}{4} I(\alpha+\delta)} \\ -\sin\left(\frac{1}{4}\beta\right) e^{\frac{1}{4} I(\alpha+\delta)} & \cos\left(\frac{1}{4}\beta\right) e^{-\frac{1}{4} I(\alpha+\delta)} \end{bmatrix} \tag{4}$$

> $M3 := (\alpha, \delta) \rightarrow \left(simplify\left(Rz\left(\frac{(\delta - \alpha)}{2} \right) \right) \right):$
> $'M3(\alpha, \delta)' = M3(\alpha, \delta);$

$$M3(\alpha, \delta) = \begin{bmatrix} e^{\frac{1}{4} I(-\delta + \alpha)} & 0 \\ 0 & e^{-\frac{1}{4} I(-\delta + \alpha)} \end{bmatrix} \tag{5}$$

The $\mathcal{U}_x$ operator/matrix

> $\mathcal{U}x := Matrix([[0, 1], [1, 0]]);$

$$\mathcal{U}x := \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{6}$$

Condition 1. Showing that $M_1 M_2 = I_2$

> $'M1(\alpha, \beta) M2(\alpha, \beta, \alpha)' = simplify(Multiply(M1(\alpha, \beta), M2(\alpha, \beta, \alpha)));$

$$M1(\alpha, \beta)\ M2(\alpha, \beta, \alpha) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{7}$$

Condition 2. Showing that $M_1 \mathcal{U}_x M_2 \mathcal{U}_x = \mathcal{R}(\alpha,\beta,\alpha)$
First calculate $\mathcal{R}(\alpha,\beta,\alpha)$

> $\mathcal{R} := (\alpha, \beta, \delta) \rightarrow simplify(Multiply(Rz(\alpha), Multiply(Ry(\beta), Rz(\delta)))):$
> $'\mathcal{R}(\alpha, \beta, \alpha)' = \mathcal{R}(\alpha, \beta, \alpha);$

$$\mathcal{R}(\alpha, \beta, \alpha) = \begin{bmatrix} \cos\left(\frac{1}{2}\beta\right) e^{-I\alpha} & -\sin\left(\frac{1}{2}\beta\right) \\ \sin\left(\frac{1}{2}\beta\right) & \cos\left(\frac{1}{2}\beta\right) e^{I\alpha} \end{bmatrix} \tag{8}$$

Now we calculate $M_1 \mathcal{U}_x M_2 \mathcal{U}_x$

> $'M1\,\mathcal{U}x\ M2\,\mathcal{U}x' = combine(Multiply(M1(\alpha, \beta), Multiply(\mathcal{U}x, Multiply(M2(\alpha, \beta, \alpha), \mathcal{U}x))));$

$$M1\,\mathcal{U}x\ M2\,\mathcal{U}x = \begin{bmatrix} \cos\left(\frac{1}{2}\beta\right) e^{-I\alpha} & -\sin\left(\frac{1}{2}\beta\right) \\ \sin\left(\frac{1}{2}\beta\right) & \cos\left(\frac{1}{2}\beta\right) e^{I\alpha} \end{bmatrix} \tag{9}$$

Define the rotation operator V

\> $V := (\alpha, \beta, \delta) \rightarrow \mathit{combine}(\mathit{Multiply}(\mathit{M1}(\alpha, \beta), \mathit{Multiply}(\mathcal{U}x, \mathit{M2}(\alpha, \beta, \delta)))) :$
$'V(\alpha, \beta, \alpha,)' = V(\alpha, \beta, \alpha);$

$$V(\alpha, \beta, \alpha) = \begin{bmatrix} -\sin\left(\frac{1}{2}\beta\right) & \cos\left(\frac{1}{2}\beta\right) e^{-I\alpha} \\ \cos\left(\frac{1}{2}\beta\right) e^{I\alpha} & \sin\left(\frac{1}{2}\beta\right) \end{bmatrix} \tag{10}$$

Calculate V$\mathcal{U}_x$

\> $'V \cdot \mathcal{U}x' = \mathit{simplify}(\mathit{Multiply}(V(\alpha, \beta, \alpha), \mathcal{U}x));$

$$V\ \mathcal{U}x = \begin{bmatrix} \cos\left(\frac{1}{2}\beta\right) e^{-I\alpha} & -\sin\left(\frac{1}{2}\beta\right) \\ \sin\left(\frac{1}{2}\beta\right) & \cos\left(\frac{1}{2}\beta\right) e^{I\alpha} \end{bmatrix} \tag{11}$$

Calculating V$_i$ as a function of $\mathcal{R}$(α,β,α)$\mathcal{U}_x$

\> $Vy := \mathit{simplify}\left(\mathit{Multiply}\left(\mathcal{R}\left(\frac{\pi}{2}, 0, \frac{\pi}{2}\right), \mathcal{U}x\right)\right) : 'Vy' = Vy;$

$$Vy = \begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix} \tag{12}$$

\> $Vz := -\mathit{simplify}(\mathit{Multiply}(\mathcal{R}(0, \pi, 0), \mathcal{U}x)) : 'Vz' = Vz;$

$$Vz = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{13}$$

\> $Vx := \mathit{simplify}(\mathit{Multiply}(\mathcal{R}(0, 0, 0), \mathcal{U}x)) : 'Vx' = Vx;$

$$Vx = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{14}$$

Calculating V$_i$ as a function of V(α,β,α)

\> $'Vx' = \mathit{simplify}(V(0, 0, 0));$

$$Vx = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{15}$$

\> $'Vz' = -\mathit{simplify}(V(0, \pi, 0));$

$$Vz = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{16}$$

\> $'Vy' = \mathit{simplify}\left(V\left(\frac{\pi}{2}, 0, \frac{\pi}{2}\right)\right);$

$$Vy = \begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix} \tag{17}$$