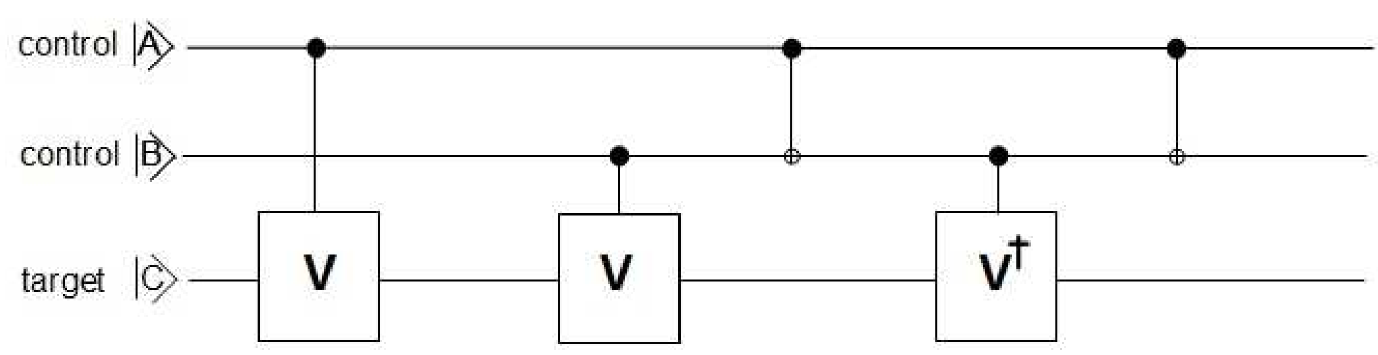
```
> restart;
> interface(warnlevel = 0) :        # Maple 12``
> with(LinearAlgebra) :
> with(Bits) :
```




```
> TP := proc(M1, M2) return KroneckerProduct(M1, M2) end proc:
> VSte := proc(n)                    # Generates a list of  computational states for  n qubits
            local i, L;              # e.g.  n= 2  ⇒ [ |00⟩ |01⟩ |10⟩ |11⟩ ]
            L := Matrix(1, 2^n);
            Settings(defaultbits = n);
            for i from 1 to 2^n do
                L[1, i] := cat(`|`, String(i − 1, msbfirst), "⟩");
            end do;
        #   print(L);
          return L;        # returns Matrix L
        end proc:

> Ceff :=proc(n)
            local i, L;              #  Generates a list of  computational state coefficients for  n qubits
             L := Matrix(1, 2^n);              #  e.g.  n= 2 ⇒ [c0 c1 c2 c3]
            for i from 1 to 2^n do
               L[1, i] := c[i − 1];          # c[i-1] represents the coefficient c_i
            end do;
        #   print(L);
          return L;        # returns Matrix L
          end proc:
```

**Defining single-qubit and two-qubit matrices and operators**

> $\mathbb{I}2 := Matrix([[1, 0], [0, 1]]);$
> $\mathbb{V} := Matrix([[x, y], [z, w]]);$
> $\mathbb{G} := Matrix([[1, 0, 0, 0], [0, 1, 0, 0], [0, 0, x, y], [0, 0, z, w]]);$
> $\mathbb{G}23 := Matrix([[1, 0, 0, 0], [0, 0, 1, 0], [0, 1, 0, 0], [0, 0, 0, 1]]);$
> $CNOT := Matrix([[1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 0, 1], [0, 0, 1, 0]]);$

$$\mathbb{I}2 := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathbb{V} := \begin{bmatrix} x & y \\ z & w \end{bmatrix}$$

$$\mathbb{G} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & x & y \\ 0 & 0 & z & w \end{bmatrix}$$

$$\mathbb{G}23 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$CNOT := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \textbf{(1)}$$

**Defining the SWAP operator/ Matrix**

> $SWAP := TP(\mathbb{G}23, \mathbb{I}2);$

$$SWAP := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \textbf{(2)}$$

**The Identity, SWAP and $\mathbb{G}$ matrices**

> $\mathrm{TP}(\mathbb{I}2, \mathcal{G})$;
> $\mathrm{Multiply}(TP(\mathbb{I}2, \mathcal{G}), SWAP)$;
> $Multiply(SWAP, \%)$;

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x & y & 0 & 0 & 0 & 0 \\ 0 & 0 & z & w & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x & y \\ 0 & 0 & 0 & 0 & 0 & 0 & z & w \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & x & y & 0 & 0 \\ 0 & 0 & 0 & 0 & z & w & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x & y \\ 0 & 0 & 0 & 0 & 0 & 0 & z & w \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & x & y & 0 & 0 \\ 0 & 0 & 0 & 0 & z & w & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x & y \\ 0 & 0 & 0 & 0 & 0 & 0 & z & w \end{bmatrix} \quad (3)$$

**The first stage**

> $U1 := Multiply(SWAP,\ Multiply(TP(\mathbb{I}2,\ \mathbb{G}), SWAP));$

$$U1 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & x & y & 0 & 0 \\ 0 & 0 & 0 & 0 & z & w & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x & y \\ 0 & 0 & 0 & 0 & 0 & 0 & z & w \end{bmatrix} \qquad (4)$$

**The second stage**

> $U2 := \mathrm{TP}(\mathbb{I}2,\ \mathbb{G});$

$$U2 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x & y & 0 & 0 & 0 & 0 \\ 0 & 0 & z & w & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x & y \\ 0 & 0 & 0 & 0 & 0 & 0 & z & w \end{bmatrix} \qquad (5)$$

**At the end of the second state; the $U_2U_1$ composition**

> $Multiply(U2, U1), '\mathbb{V}' = \mathbb{V}, '\mathbb{V}^2' = \mathbb{V}2;$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x & y & 0 & 0 & 0 & 0 \\ 0 & 0 & z & w & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & x & y & 0 & 0 \\ 0 & 0 & 0 & 0 & z & w & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x^2 + y z & x y + y w \\ 0 & 0 & 0 & 0 & 0 & 0 & z x + w z & y z + w^2 \end{bmatrix}, \mathbb{V} = \begin{bmatrix} x & y \\ z & w \end{bmatrix}, \mathbb{V}2 = \begin{bmatrix} x^2 + y z & x y + y w \\ z x + w z & y z + w^2 \end{bmatrix} \qquad (6)$$

**The third stage**

> $U3 := TP(CNOT, \mathbb{I}2);$

$$U3 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{7}$$

**At the end of the third stage; the $U_3U_2U_1$ composition**

> $Multiply(U3, Multiply(U2, U1));$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x & y & 0 & 0 & 0 & 0 \\ 0 & 0 & z & w & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x^2 + y z & x y + y w \\ 0 & 0 & 0 & 0 & 0 & 0 & z x + w z & y z + w^2 \\ 0 & 0 & 0 & 0 & x & y & 0 & 0 \\ 0 & 0 & 0 & 0 & z & w & 0 & 0 \end{bmatrix} \tag{8}$$

**The conjugate transpose**

> $\mathcal{G} := HermitianTranspose(\mathbb{G});$

$$\mathcal{G} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \overline{x} & \overline{z} \\ 0 & 0 & \overline{y} & \overline{w} \end{bmatrix} \tag{9}$$

**The fourth stage**

> $U4 := \mathrm{TP}(\mathbb{I}2, \mathcal{G});$

$$U4 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{x} & \overline{z} & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{y} & \overline{w} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \overline{x} & \overline{z} \\ 0 & 0 & 0 & 0 & 0 & 0 & \overline{y} & \overline{w} \end{bmatrix} \tag{10}$$

**Stages 3 and 4; the $U_4U_3$ composition**

> $Multiply(U4, U3);$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{x} & \overline{z} & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{y} & \overline{w} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & \overline{x} & \overline{z} & 0 & 0 \\ 0 & 0 & 0 & 0 & \overline{y} & \overline{w} & 0 & 0 \end{bmatrix} \tag{11}$$

**At the end of the fourth stage; the $U_4U_3U_2U_1$ composition**

> $Multiply(Multiply(U4, U3), Multiply(U2, U1));$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{x}\,x + \overline{z}\,z & \overline{x}\,y + \overline{z}\,w & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{y}\,x + \overline{w}\,z & \overline{y}\,y + \overline{w}\,w & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x^2 + y\,z & x\,y + y\,w \\ 0 & 0 & 0 & 0 & 0 & 0 & z\,x + w\,z & y\,z + w^2 \\ 0 & 0 & 0 & 0 & \overline{x}\,x + \overline{z}\,z & \overline{x}\,y + \overline{z}\,w & 0 & 0 \\ 0 & 0 & 0 & 0 & \overline{y}\,x + \overline{w}\,z & \overline{y}\,y + \overline{w}\,w & 0 & 0 \end{bmatrix} \tag{12}$$

**The final stage**

> $U5 := TP(CNOT, I2);$

$$U5 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (13)$$

**At the end of the fifth and final stage; the $U_5U_4U_3U_2U_1$ composition**

> $U := simplify(Multiply(U5, Multiply(U4, Multiply(U3, Multiply(U2, U1)))));$

$$U := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & |x|^2 + |z|^2 & \overline{x}\, y + \overline{z}\, w & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{y}\, x + \overline{w}\, z & |y|^2 + |w|^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & |x|^2 + |z|^2 & \overline{x}\, y + \overline{z}\, w & 0 & 0 \\ 0 & 0 & 0 & 0 & \overline{y}\, x + \overline{w}\, z & |y|^2 + |w|^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x^2 + y\, z & x\, y + y\, w \\ 0 & 0 & 0 & 0 & 0 & 0 & z\, x + w\, z & y\, z + w^2 \end{bmatrix} \quad (14)$$

**Determining the square root of $\mathcal{U}_x$ using the MatrixPower() function**

$$\mathbb{V} = (\mathcal{U}_x)^{1/2}$$

> $Ux := Matrix([[0, 1], [1, 0]]);$
> $\mathbb{V} := MatrixPower\left(Ux, \frac{1}{2}\right);$

$$Ux := \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\mathbb{V} := \begin{bmatrix} \frac{1}{2} + \frac{1}{2} I & \frac{1}{2} - \frac{1}{2} I \\ \frac{1}{2} - \frac{1}{2} I & \frac{1}{2} + \frac{1}{2} I \end{bmatrix} \quad (15)$$

**The values of x, y, z, w**

> $x := \mathbb{V}[1, 1] : y := \mathbb{V}[1, 2] : z := \mathbb{V}[2, 1] : w := \mathbb{V}[2, 2] :$
> $'U' = U;$

$$U = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (16)$$

**Defining the 3-qubit state vector $|\psi_0\rangle$ and $|\psi_1\rangle$**

$$|\psi_1\rangle = \mathbf{U}\,|\psi_0\rangle$$

> $CoEff := Ceff(3) :$
> $States := VSte(3) :$
> $|\psi 0\rangle := Multiply(Ceff(3), Transpose(VSte(3)))[1, 1];$
> $States := Multiply(U, Transpose(States)) :$
> $|\psi 1\rangle := Multiply(CoEff, States)[1, 1];$

$$|\psi 0\rangle := c_0\,|000\rangle + c_1\,|001\rangle + c_2\,|010\rangle + c_3\,|011\rangle + c_4\,|100\rangle + c_5\,|101\rangle + c_6\,|110\rangle + c_7\,|111\rangle$$

$$|\psi 1\rangle := c_0\,|000\rangle + c_1\,|001\rangle + c_2\,|010\rangle + c_3\,|011\rangle + c_4\,|100\rangle + c_5\,|101\rangle + c_6\,|111\rangle + c_7\,|110\rangle \quad (17)$$