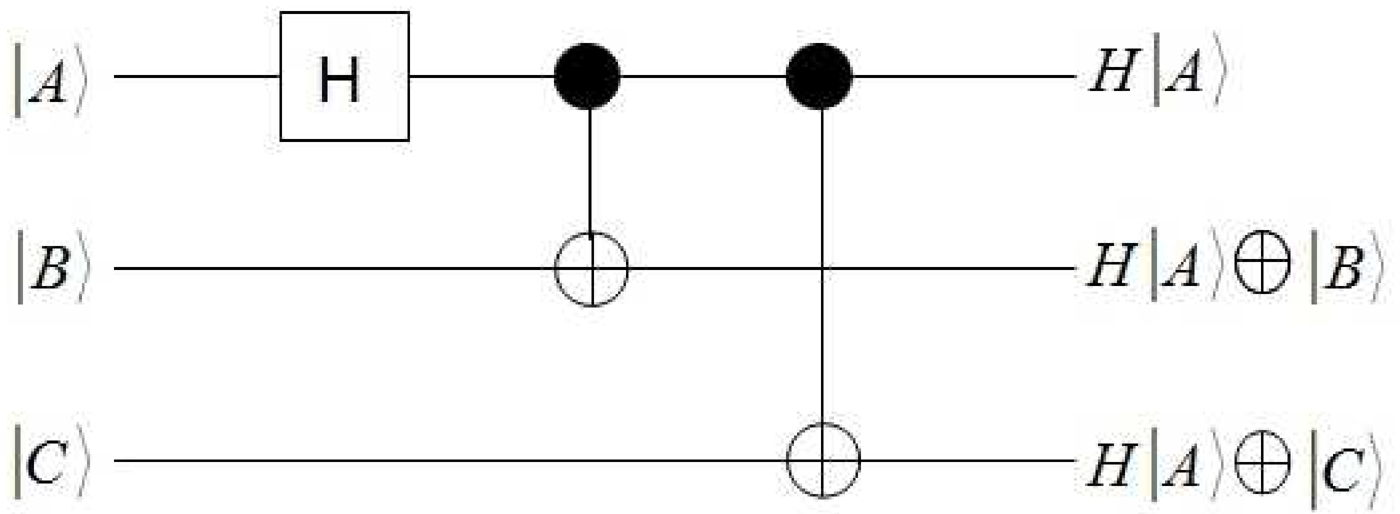
```
> restart;
> interface(warnlevel = 0) :        #   Maple 12
> with(LinearAlgebra) :
```

Chapter 7 Problem 11




```
> TP := proc(M1, M2) return KroneckerProduct(M1, M2) end proc:
```

Defining the state vector

```
> V1 := Vector([[a[0], a[1], a[2], a[3], a[4], a[5], a[6], a[7]]]);
  L := Transpose(Matrix([`|000⟩`, `|001⟩`, `|010⟩`, `|011⟩`, `|100⟩`, `|101⟩`, `|110⟩`, `|111⟩`])) :
  `|ψ⟩` = Multiply(Transpose(V1), L)[1];
```

$$V1 := \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \end{bmatrix}$$

$$|\psi\rangle = a_0\ |000\rangle + a_1\ |001\rangle + a_2\ |010\rangle + a_3\ |011\rangle + a_4\ |100\rangle + a_5\ |101\rangle + a_6\ |110\rangle + a_7\ |111\rangle \qquad \textbf{(1)}$$

Defining single-qubit and two-qubit matrices and operators

```
> I2 := IdentityMatrix(2);
X := Matrix([[0, 1], [1, 0]]);
Z :=  Matrix([[1, 0], [0, −1]]);
H := (X + Z)/'sqrt(2)';
G23 := RowOperation(IdentityMatrix(4), [2, 3]);
CNOT := RowOperation(IdentityMatrix(4), [3, 4]);
SWAP := TP(G23, I2);
```

$$I2 := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$X := \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$Z := \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$H := \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$G23 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$CNOT := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$SWAP := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2)$$

The First Stage:

```
> G1 := TP(H, TP(I2, I2));
```

$$G1 := \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} \end{bmatrix} \tag{3}$$

The Second Stage:

```
> G2 := TP(CNOT, I2) ;
```

$$G2 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{4}$$

The Third Stage:

> $G3 := Multiply(SWAP,Multiply(TP(I2, CNOT), SWAP));$

$$G3 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (5)$$

Overall Composition

> $G := Multiply(G3, Multiply(G2, G1));$

$$G := \begin{bmatrix} \frac{1}{2}\sqrt{2} & 0 & 0 & 0 & \frac{1}{2}\sqrt{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2}\sqrt{2} & 0 & 0 & 0 & \frac{1}{2}\sqrt{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2}\sqrt{2} & 0 & 0 & 0 & \frac{1}{2}\sqrt{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2}\sqrt{2} & 0 & 0 & 0 & \frac{1}{2}\sqrt{2} \\ 0 & 0 & 0 & \frac{1}{2}\sqrt{2} & 0 & 0 & 0 & -\frac{1}{2}\sqrt{2} \\ 0 & 0 & \frac{1}{2}\sqrt{2} & 0 & 0 & 0 & -\frac{1}{2}\sqrt{2} & 0 \\ 0 & \frac{1}{2}\sqrt{2} & 0 & 0 & 0 & -\frac{1}{2}\sqrt{2} & 0 & 0 \\ \frac{1}{2}\sqrt{2} & 0 & 0 & 0 & -\frac{1}{2}\sqrt{2} & 0 & 0 & 0 \end{bmatrix} \quad (6)$$

> 'V1'= V1,'G(V1)'= Multiply(G, V1);
`|ψ⟩` = Multiply(Transpose(V1), L)[1];
G|ψ⟩= Multiply(Transpose(Multiply(G, V1)), L)[1];

$$V1 = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \end{bmatrix}, G(V1) = \begin{bmatrix} \frac{1}{2}\sqrt{2}\ a_0 + \frac{1}{2}\sqrt{2}\ a_4 \\ \frac{1}{2}\sqrt{2}\ a_1 + \frac{1}{2}\sqrt{2}\ a_5 \\ \frac{1}{2}\sqrt{2}\ a_2 + \frac{1}{2}\sqrt{2}\ a_6 \\ \frac{1}{2}\sqrt{2}\ a_3 + \frac{1}{2}\sqrt{2}\ a_7 \\ \frac{1}{2}\sqrt{2}\ a_3 - \frac{1}{2}\sqrt{2}\ a_7 \\ \frac{1}{2}\sqrt{2}\ a_2 - \frac{1}{2}\sqrt{2}\ a_6 \\ \frac{1}{2}\sqrt{2}\ a_1 - \frac{1}{2}\sqrt{2}\ a_5 \\ \frac{1}{2}\sqrt{2}\ a_0 - \frac{1}{2}\sqrt{2}\ a_4 \end{bmatrix}$$

$$|\psi\rangle = a_0\ |000\rangle + a_1\ |001\rangle + a_2\ |010\rangle + a_3\ |011\rangle + a_4\ |100\rangle + a_5\ |101\rangle + a_6\ |110\rangle + a_7\ |111\rangle$$

$$G|\psi\rangle = \left(\frac{1}{2}\sqrt{2}\ a_0 + \frac{1}{2}\sqrt{2}\ a_4\right)|000\rangle + \left(\frac{1}{2}\sqrt{2}\ a_1 + \frac{1}{2}\sqrt{2}\ a_5\right)|001\rangle + \left(\frac{1}{2}\sqrt{2}\ a_2 + \frac{1}{2}\sqrt{2}\ a_6\right)|010\rangle + \left(\frac{1}{2}\sqrt{2}\ a_3 + \frac{1}{2}\sqrt{2}\ a_7\right)|011\rangle + \left(\frac{1}{2}\sqrt{2}\ a_3 - \frac{1}{2}\sqrt{2}\ a_7\right)|100\rangle + \left(\frac{1}{2}\sqrt{2}\ a_2 - \frac{1}{2}\sqrt{2}\ a_6\right)|101\rangle + \left(\frac{1}{2}\sqrt{2}\ a_1 - \frac{1}{2}\sqrt{2}\ a_5\right)|110\rangle + \left(\frac{1}{2}\sqrt{2}\ a_0 - \frac{1}{2}\sqrt{2}\ a_4\right)|111\rangle \tag{7}$$

Compare matrix G to the matrix GBell below which generates two-qubit correlated states; Bell's states

Defining the state vector

```
> V2 := Vector([[a[0], a[1], a[2], a[3]]]);
  L2 := Transpose(Matrix([|00⟩, |01⟩, |10⟩, |11⟩])) :
  `|ψ⟩` = Multiply(Transpose(V2), L2)[1];
```

$$V2 := \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$|\psi\rangle = a_0\ |00\rangle + a_1\ |01\rangle + a_2\ |10\rangle + a_3\ |11\rangle \quad \mathbf{(8)}$$

The Hadamard gate acting on a single qubit H|A⟩

```
> G1 := TP(H, I2);
```

$$G1 := \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \end{bmatrix} \quad \mathbf{(9)}$$

The composition

```
> GBell := Multiply(CNOT, G1);
```

$$GBell := \begin{bmatrix} \frac{1}{2}\sqrt{2} & 0 & \frac{1}{2}\sqrt{2} & 0 \\ 0 & \frac{1}{2}\sqrt{2} & 0 & \frac{1}{2}\sqrt{2} \\ 0 & \frac{1}{2}\sqrt{2} & 0 & -\frac{1}{2}\sqrt{2} \\ \frac{1}{2}\sqrt{2} & 0 & -\frac{1}{2}\sqrt{2} & 0 \end{bmatrix} \quad \mathbf{(10)}$$

Two-qubit quantum state

```
> 'V2'= V2,'GBell(V2)'= Multiply(GBell, V2);
  `|ψ⟩` = Multiply(Transpose(V2), L2)[1];
  G|ψ⟩= Multiply(Transpose(Multiply(GBell, V2)), L2)[1];
```

$$V2 = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}, GBell(V2) = \begin{bmatrix} \frac{1}{2}\sqrt{2}\, a_0 + \frac{1}{2}\sqrt{2}\, a_2 \\ \frac{1}{2}\sqrt{2}\, a_1 + \frac{1}{2}\sqrt{2}\, a_3 \\ \frac{1}{2}\sqrt{2}\, a_1 - \frac{1}{2}\sqrt{2}\, a_3 \\ \frac{1}{2}\sqrt{2}\, a_0 - \frac{1}{2}\sqrt{2}\, a_2 \end{bmatrix}$$

$$|\psi\rangle = a_0\ |00\rangle + a_1\ |01\rangle + a_2\ |10\rangle + a_3\ |11\rangle$$

$$G|\psi\rangle = \left(\frac{1}{2}\sqrt{2}\ a_0 + \frac{1}{2}\sqrt{2}\ a_2\right)|00\rangle + \left(\frac{1}{2}\sqrt{2}\ a_1 + \frac{1}{2}\sqrt{2}\ a_3\right)|01\rangle + \left(\frac{1}{2}\sqrt{2}\ a_1 - \frac{1}{2}\sqrt{2}\ a_3\right)|10\rangle + \left(\frac{1}{2}\sqrt{2}\ a_0 - \frac{1}{2}\sqrt{2}\ a_2\right)|11\rangle \quad (11)$$

Here is a procedure that generates a list of 3-qubit correlated states which can be easily modified for n-qubits .

```
> f:= proc( );
        local i, j, k, t1, t2, L;
         L := [ ];

         for i from 0 to 1 do        # first qubit
           for j from 0 to 1 do      # second qubit
             for k from 0 to 1 do      # third qubit

               t1 := cat(cat(|0 , 0 + j mod 2), 0 + k mod 2, `⟩`);    # first term
             t2 := cat(cat(|1 , 1 + j mod 2), 1 + k mod 2, `⟩`);     # second term

             if i = 0 then
                 L := [op(L), [cat(cat(t1, `+`), t2)]];
               else
                 L := [op(L), [cat(cat(t1, `-`), t2)]];
               end if;

              end do;
             end do;
           end do;

           for i from 1 to nops(L) do     # print the list
             print( L[i, 1] / '√2' );
           end do;
        end proc:
```

```
> f( );
```

$$\frac{|000\rangle+|111\rangle}{\sqrt{2}}$$
$$\frac{|001\rangle+|110\rangle}{\sqrt{2}}$$
$$\frac{|010\rangle+|101\rangle}{\sqrt{2}}$$
$$\frac{|011\rangle+|100\rangle}{\sqrt{2}}$$
$$\frac{|000\rangle-|111\rangle}{\sqrt{2}}$$
$$\frac{|001\rangle-|110\rangle}{\sqrt{2}}$$
$$\frac{|010\rangle-|101\rangle}{\sqrt{2}}$$
$$\frac{|011\rangle-|100\rangle}{\sqrt{2}} \tag{12}$$