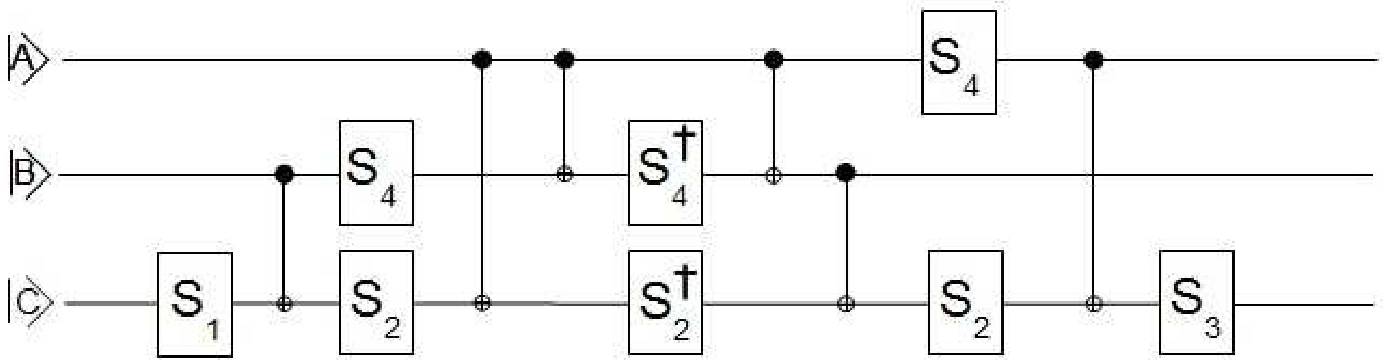
```
> restart;
> interface(warnlevel = 0) :   #   Maple 12
> with(LinearAlgebra) :
> with(Bits) :
```




```
> TP := proc(M1, M2) return KroneckerProduct(M1, M2) end proc:
```

```
> VSte := proc(n)                 # Generates a list of  computational states for  n qubits
            local i, L;           #  e.g.  n= 2  ⇒ [ |00⟩ |01⟩ |10⟩ |11⟩ ]
            L := Matrix(1, 2^n);
            Settings(defaultbits = n);
            for i from 1 to 2^n do
                L[1, i] := cat(`|`, String(i − 1, msbfirst), "⟩");
            end do;
       #    print(L);
          return L;        # returns Matrix L
       end proc:
```

```
> Ceff :=proc(n)
            local i, L;           #  Generates a list of  computational state coefficients for  n qubits
             L := Matrix(1, 2^n);          # e.g.  n= 2 ⇒ [c0 c1 c2 c3]
            for i from 1 to 2^n do
               L[1, i] := c[i − 1];        # c[i-1] represents the coefficient ci
            end do;
       #   print(L);
          return L;        # returns Matrix L
          end proc:
```

Defining Matrices and Gates

> $\mathbb{I}2 := IdentityMatrix(2)$:

> $\mathbb{I}4 := IdentityMatrix(4)$:

> $\mathbb{G}23 := \text{RowOperation}(\mathbb{I}4, [2, 3]);$

$$\mathbb{G}23 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

> $\text{CNOT} := RowOperation(\mathbb{I}4, [3, 4]);$

$$CNOT := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (2)$$

> $\text{M1c} := Matrix(2, 2, symbol = S1);$

$$M1c := \begin{bmatrix} S1_{1,1} & S1_{1,2} \\ S1_{2,1} & S1_{2,2} \end{bmatrix} \quad (3)$$

> $M2c := Matrix(2, 2, symbol = S2);$
> $\mathcal{M}2c := \text{Matrix}(2, 2, symbol = \mathcal{S}2);$

$$M2c := \begin{bmatrix} S2_{1,1} & S2_{1,2} \\ S2_{2,1} & S2_{2,2} \end{bmatrix}$$

$$\mathcal{M}2c := \begin{bmatrix} \mathcal{S}2_{1,1} & \mathcal{S}2_{1,2} \\ \mathcal{S}2_{2,1} & \mathcal{S}2_{2,2} \end{bmatrix} \quad (4)$$

> $\text{M3c} := \text{Matrix}(2, 2, symbol = S3);$

$$M3c := \begin{bmatrix} S3_{1,1} & S3_{1,2} \\ S3_{2,1} & S3_{2,2} \end{bmatrix} \quad (5)$$

> $\text{M4b} := \text{Matrix}(2, 2, symbol = S4);$
> $\mathcal{M}4\text{b} := \text{Matrix}(2, 2, symbol = \mathcal{S}4);$

$$M4b := \begin{bmatrix} S4_{1,1} & S4_{1,2} \\ S4_{2,1} & S4_{2,2} \end{bmatrix}$$

$$\mathcal{M}4b := \begin{bmatrix} \mathcal{S}4_{1,1} & \mathcal{S}4_{1,2} \\ \mathcal{S}4_{2,1} & \mathcal{S}4_{2,2} \end{bmatrix} \quad (6)$$

Stages 1 - 11

Stage 1

> U1 := TP(I4, M1c) : 'U1'= U1;

$$U1 = \begin{bmatrix} S1_{1,1} & S1_{1,2} & 0 & 0 & 0 & 0 & 0 & 0 \\ S1_{2,1} & S1_{2,2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & S1_{1,1} & S1_{1,2} & 0 & 0 & 0 & 0 \\ 0 & 0 & S1_{2,1} & S1_{2,2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & S1_{1,1} & S1_{1,2} & 0 & 0 \\ 0 & 0 & 0 & 0 & S1_{2,1} & S1_{2,2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S1_{1,1} & S1_{1,2} \\ 0 & 0 & 0 & 0 & 0 & 0 & S1_{2,1} & S1_{2,2} \end{bmatrix} \tag{7}$$

Stage 2

> U2 := TP(I2, CNOT) : 'U2'= U2;

$$U2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{8}$$

Stage 3

```
> TP(I2, TP(M4b, I2));
  TP(I4, M2c);
  U3 := Multiply(TP(I2, TP(M4b, I2)), TP(I4, M2c)) : 'U3'= U3;
```

$$\begin{bmatrix} S4_{1,1} & 0 & S4_{1,2} & 0 & 0 & 0 & 0 & 0 \\ 0 & S4_{1,1} & 0 & S4_{1,2} & 0 & 0 & 0 & 0 \\ S4_{2,1} & 0 & S4_{2,2} & 0 & 0 & 0 & 0 & 0 \\ 0 & S4_{2,1} & 0 & S4_{2,2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & S4_{1,1} & 0 & S4_{1,2} & 0 \\ 0 & 0 & 0 & 0 & 0 & S4_{1,1} & 0 & S4_{1,2} \\ 0 & 0 & 0 & 0 & S4_{2,1} & 0 & S4_{2,2} & 0 \\ 0 & 0 & 0 & 0 & 0 & S4_{2,1} & 0 & S4_{2,2} \end{bmatrix}$$

$$\begin{bmatrix} S2_{1,1} & S2_{1,2} & 0 & 0 & 0 & 0 & 0 & 0 \\ S2_{2,1} & S2_{2,2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & S2_{1,1} & S2_{1,2} & 0 & 0 & 0 & 0 \\ 0 & 0 & S2_{2,1} & S2_{2,2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & S2_{1,1} & S2_{1,2} & 0 & 0 \\ 0 & 0 & 0 & 0 & S2_{2,1} & S2_{2,2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S2_{1,1} & S2_{1,2} \\ 0 & 0 & 0 & 0 & 0 & 0 & S2_{2,1} & S2_{2,2} \end{bmatrix}$$

$$U3 = \begin{bmatrix} S4_{1,1}S2_{1,1} & S4_{1,1}S2_{1,2} & S4_{1,2}S2_{1,1} & S4_{1,2}S2_{1,2} & 0 & 0 & 0 & 0 \\ S4_{1,1}S2_{2,1} & S4_{1,1}S2_{2,2} & S4_{1,2}S2_{2,1} & S4_{1,2}S2_{2,2} & 0 & 0 & 0 & 0 \\ S4_{2,1}S2_{1,1} & S4_{2,1}S2_{1,2} & S4_{2,2}S2_{1,1} & S4_{2,2}S2_{1,2} & 0 & 0 & 0 & 0 \\ S4_{2,1}S2_{2,1} & S4_{2,1}S2_{2,2} & S4_{2,2}S2_{2,1} & S4_{2,2}S2_{2,2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & S4_{1,1}S2_{1,1} & S4_{1,1}S2_{1,2} & S4_{1,2}S2_{1,1} & S4_{1,2}S2_{1,2} \\ 0 & 0 & 0 & 0 & S4_{1,1}S2_{2,1} & S4_{1,1}S2_{2,2} & S4_{1,2}S2_{2,1} & S4_{1,2}S2_{2,2} \\ 0 & 0 & 0 & 0 & S4_{2,1}S2_{1,1} & S4_{2,1}S2_{1,2} & S4_{2,2}S2_{1,1} & S4_{2,2}S2_{1,2} \\ 0 & 0 & 0 & 0 & S4_{2,1}S2_{2,1} & S4_{2,1}S2_{2,2} & S4_{2,2}S2_{2,1} & S4_{2,2}S2_{2,2} \end{bmatrix} \tag{9}$$

Stage 4

```
> TP(𝕀2, CNOT);
  TP(𝔾23, 𝕀2);
  U4 := Multiply(TP(𝔾23, 𝕀2), Multiply(TP(𝕀2, CNOT), TP(𝔾23, 𝕀2))) :
  'U4'= U4;
```

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$U4 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{10}$$

Stage 5

```
> U5 := TP(CNOT, 𝕀2) :'U5'= U5;
```

$$U5 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{11}$$

Stage 6

```
> TP(I2, TP(m4b, I2));
  TP(I4, m2c);
  U6 := Multiply(TP(I2, TP(m4b, I2)), TP(I4, m2c)) :'U6'= U6;
```

$$\begin{bmatrix} \mathcal{S}4_{1,1} & 0 & \mathcal{S}4_{1,2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathcal{S}4_{1,1} & 0 & \mathcal{S}4_{1,2} & 0 & 0 & 0 & 0 \\ \mathcal{S}4_{2,1} & 0 & \mathcal{S}4_{2,2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathcal{S}4_{2,1} & 0 & \mathcal{S}4_{2,2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathcal{S}4_{1,1} & 0 & \mathcal{S}4_{1,2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathcal{S}4_{1,1} & 0 & \mathcal{S}4_{1,2} \\ 0 & 0 & 0 & 0 & \mathcal{S}4_{2,1} & 0 & \mathcal{S}4_{2,2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathcal{S}4_{2,1} & 0 & \mathcal{S}4_{2,2} \end{bmatrix}$$

$$\begin{bmatrix} \mathcal{S}2_{1,1} & \mathcal{S}2_{1,2} & 0 & 0 & 0 & 0 & 0 & 0 \\ \mathcal{S}2_{2,1} & \mathcal{S}2_{2,2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathcal{S}2_{1,1} & \mathcal{S}2_{1,2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathcal{S}2_{2,1} & \mathcal{S}2_{2,2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathcal{S}2_{1,1} & \mathcal{S}2_{1,2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathcal{S}2_{2,1} & \mathcal{S}2_{2,2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathcal{S}2_{1,1} & \mathcal{S}2_{1,2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathcal{S}2_{2,1} & \mathcal{S}2_{2,2} \end{bmatrix}$$

$$U6 = \begin{bmatrix} \mathcal{S}4_{1,1}\mathcal{S}2_{1,1} & \mathcal{S}4_{1,1}\mathcal{S}2_{1,2} & \mathcal{S}4_{1,2}\mathcal{S}2_{1,1} & \mathcal{S}4_{1,2}\mathcal{S}2_{1,2} & 0 & 0 & 0 & 0 \\ \mathcal{S}4_{1,1}\mathcal{S}2_{2,1} & \mathcal{S}4_{1,1}\mathcal{S}2_{2,2} & \mathcal{S}4_{1,2}\mathcal{S}2_{2,1} & \mathcal{S}4_{1,2}\mathcal{S}2_{2,2} & 0 & 0 & 0 & 0 \\ \mathcal{S}4_{2,1}\mathcal{S}2_{1,1} & \mathcal{S}4_{2,1}\mathcal{S}2_{1,2} & \mathcal{S}4_{2,2}\mathcal{S}2_{1,1} & \mathcal{S}4_{2,2}\mathcal{S}2_{1,2} & 0 & 0 & 0 & 0 \\ \mathcal{S}4_{2,1}\mathcal{S}2_{2,1} & \mathcal{S}4_{2,1}\mathcal{S}2_{2,2} & \mathcal{S}4_{2,2}\mathcal{S}2_{2,1} & \mathcal{S}4_{2,2}\mathcal{S}2_{2,2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathcal{S}4_{1,1}\mathcal{S}2_{1,1} & \mathcal{S}4_{1,1}\mathcal{S}2_{1,2} & \mathcal{S}4_{1,2}\mathcal{S}2_{1,1} & \mathcal{S}4_{1,2}\mathcal{S}2_{1,2} \\ 0 & 0 & 0 & 0 & \mathcal{S}4_{1,1}\mathcal{S}2_{2,1} & \mathcal{S}4_{1,1}\mathcal{S}2_{2,2} & \mathcal{S}4_{1,2}\mathcal{S}2_{2,1} & \mathcal{S}4_{1,2}\mathcal{S}2_{2,2} \\ 0 & 0 & 0 & 0 & \mathcal{S}4_{2,1}\mathcal{S}2_{1,1} & \mathcal{S}4_{2,1}\mathcal{S}2_{1,2} & \mathcal{S}4_{2,2}\mathcal{S}2_{1,1} & \mathcal{S}4_{2,2}\mathcal{S}2_{1,2} \\ 0 & 0 & 0 & 0 & \mathcal{S}4_{2,1}\mathcal{S}2_{2,1} & \mathcal{S}4_{2,1}\mathcal{S}2_{2,2} & \mathcal{S}4_{2,2}\mathcal{S}2_{2,1} & \mathcal{S}4_{2,2}\mathcal{S}2_{2,2} \end{bmatrix} \tag{12}$$

Stage 7

```
> U7 := U5 :'U7'= U7;
```

$$U7 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (13)$$

Stage 8

```
> U8 := U2 :'U8'= U8;
```

$$U8 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (14)$$

Stage 9

```
> TP(M4b, I4);
  TP(I4, M2c);
  U9 := Multiply(TP(M4b, I4), TP(I4, M2c)) :'U9'= U9;
```

$$\begin{bmatrix} S4_{1,1} & 0 & 0 & 0 & S4_{1,2} & 0 & 0 & 0 \\ 0 & S4_{1,1} & 0 & 0 & 0 & S4_{1,2} & 0 & 0 \\ 0 & 0 & S4_{1,1} & 0 & 0 & 0 & S4_{1,2} & 0 \\ 0 & 0 & 0 & S4_{1,1} & 0 & 0 & 0 & S4_{1,2} \\ S4_{2,1} & 0 & 0 & 0 & S4_{2,2} & 0 & 0 & 0 \\ 0 & S4_{2,1} & 0 & 0 & 0 & S4_{2,2} & 0 & 0 \\ 0 & 0 & S4_{2,1} & 0 & 0 & 0 & S4_{2,2} & 0 \\ 0 & 0 & 0 & S4_{2,1} & 0 & 0 & 0 & S4_{2,2} \end{bmatrix}$$

$$\begin{bmatrix} S2_{1,1} & S2_{1,2} & 0 & 0 & 0 & 0 & 0 & 0 \\ S2_{2,1} & S2_{2,2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & S2_{1,1} & S2_{1,2} & 0 & 0 & 0 & 0 \\ 0 & 0 & S2_{2,1} & S2_{2,2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & S2_{1,1} & S2_{1,2} & 0 & 0 \\ 0 & 0 & 0 & 0 & S2_{2,1} & S2_{2,2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S2_{1,1} & S2_{1,2} \\ 0 & 0 & 0 & 0 & 0 & 0 & S2_{2,1} & S2_{2,2} \end{bmatrix}$$

$$U9 = \begin{bmatrix} S4_{1,1} S2_{1,1} & S4_{1,1} S2_{1,2} & 0 & 0 & S4_{1,2} S2_{1,1} & S4_{1,2} S2_{1,2} & 0 & 0 \\ S4_{1,1} S2_{2,1} & S4_{1,1} S2_{2,2} & 0 & 0 & S4_{1,2} S2_{2,1} & S4_{1,2} S2_{2,2} & 0 & 0 \\ 0 & 0 & S4_{1,1} S2_{1,1} & S4_{1,1} S2_{1,2} & 0 & 0 & S4_{1,2} S2_{1,1} & S4_{1,2} S2_{1,2} \\ 0 & 0 & S4_{1,1} S2_{2,1} & S4_{1,1} S2_{2,2} & 0 & 0 & S4_{1,2} S2_{2,1} & S4_{1,2} S2_{2,2} \\ S4_{2,1} S2_{1,1} & S4_{2,1} S2_{1,2} & 0 & 0 & S4_{2,2} S2_{1,1} & S4_{2,2} S2_{1,2} & 0 & 0 \\ S4_{2,1} S2_{2,1} & S4_{2,1} S2_{2,2} & 0 & 0 & S4_{2,2} S2_{2,1} & S4_{2,2} S2_{2,2} & 0 & 0 \\ 0 & 0 & S4_{2,1} S2_{1,1} & S4_{2,1} S2_{1,2} & 0 & 0 & S4_{2,2} S2_{1,1} & S4_{2,2} S2_{1,2} \\ 0 & 0 & S4_{2,1} S2_{2,1} & S4_{2,1} S2_{2,2} & 0 & 0 & S4_{2,2} S2_{2,1} & S4_{2,2} S2_{2,2} \end{bmatrix} \tag{15}$$

Stage 10

$> \ U10 := U4 : 'U10' = U10;$

$$U10 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{16}$$

Stage 11 - last stage

$> \ U11 := TP(I4, M3c) : 'U11' = U11;$

$$U11 = \begin{bmatrix} S3_{1,1} & S3_{1,2} & 0 & 0 & 0 & 0 & 0 & 0 \\ S3_{2,1} & S3_{2,2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & S3_{1,1} & S3_{1,2} & 0 & 0 & 0 & 0 \\ 0 & 0 & S3_{2,1} & S3_{2,2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & S3_{1,1} & S3_{1,2} & 0 & 0 \\ 0 & 0 & 0 & 0 & S3_{2,1} & S3_{2,2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S3_{1,1} & S3_{1,2} \\ 0 & 0 & 0 & 0 & 0 & 0 & S3_{2,1} & S3_{2,2} \end{bmatrix} \tag{17}$$

Defining the single-gates Mi

$> \ Rz := \alpha \rightarrow Matrix\left(\left[\left[e^{-I \cdot \frac{\alpha}{2}}, 0\right], \left[0, e^{I \cdot \frac{\alpha}{2}}\right]\right]\right) :$

$> \ Ry := \beta \rightarrow Matrix\left(\left[\left[\cos\left(\frac{\beta}{2}\right), -\sin\left(\frac{\beta}{2}\right)\right], \left[\sin\left(\frac{\beta}{2}\right), \cos\left(\frac{\beta}{2}\right)\right]\right]\right) :$

$> \ S1 := simplify\left(Multiply\left(Ry\left(-\frac{\beta}{2}\right), Rz\left(-\frac{(\alpha+\delta)}{2}\right)\right)\right);$

$$S1 := \begin{bmatrix} \cos\left(\frac{1}{4}\beta\right) e^{\frac{1}{4} I(\alpha+\delta)} & \sin\left(\frac{1}{4}\beta\right) e^{-\frac{1}{4} I(\alpha+\delta)} \\ -\sin\left(\frac{1}{4}\beta\right) e^{\frac{1}{4} I(\alpha+\delta)} & \cos\left(\frac{1}{4}\beta\right) e^{-\frac{1}{4} I(\alpha+\delta)} \end{bmatrix} \tag{18}$$

$> S2 := simplify\left(Multiply\left(Rz(\alpha), Ry\left(\frac{\beta}{2}\right)\right)\right) : \mathcal{S}2 := HermitianTranspose(S2);$

$$\mathcal{S}2 := \begin{bmatrix} \overline{e^{-\frac{1}{2} I\alpha} \cos\left(\frac{1}{4}\beta\right)} & \overline{e^{\frac{1}{2} I\alpha} \sin\left(\frac{1}{4}\beta\right)} \\ \overline{-e^{-\frac{1}{2} I\alpha} \sin\left(\frac{1}{4}\beta\right)} & \overline{e^{\frac{1}{2} I\alpha} \cos\left(\frac{1}{4}\beta\right)} \end{bmatrix} \tag{19}$$

$> S3 := simplify\left(Rz\left(\frac{(\delta-\alpha)}{2}\right)\right);$

$$S3 := \begin{bmatrix} e^{\frac{1}{4} I(-\delta+\alpha)} & 0 \\ 0 & e^{-\frac{1}{4} I(-\delta+\alpha)} \end{bmatrix} \tag{20}$$

$> S4 := \mathrm{Matrix}\left(\left[[1, 0], \left[0, e^{I\cdot\frac{\phi}{2}}\right]\right]\right); \mathcal{S}4 := \mathrm{HermitianTranspose}(S4);$

$$S4 := \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{1}{2} I\phi} \end{bmatrix}$$

$$\mathcal{S}4 := \begin{bmatrix} 1 & 0 \\ 0 & e^{-\frac{1}{2} I\overline{\phi}} \end{bmatrix} \tag{21}$$

$> \alpha := 0 : \beta := \frac{\pi}{2} : \delta := \pi : \phi := \frac{\pi}{2} :$

$'S1' = S1; 'S2' = S2, '\mathcal{S}2' = \mathcal{S}2;$

$'S3' = S3; 'S4' = S4, '\mathcal{S}4' = \mathcal{S}4;$

$$S1 = \begin{bmatrix} \cos\left(\frac{1}{8}\pi\right)\left(\frac{1}{2}\sqrt{2} + \frac{1}{2} I\sqrt{2}\right) & \sin\left(\frac{1}{8}\pi\right)\left(\frac{1}{2}\sqrt{2} - \frac{1}{2} I\sqrt{2}\right) \\ -\sin\left(\frac{1}{8}\pi\right)\left(\frac{1}{2}\sqrt{2} + \frac{1}{2} I\sqrt{2}\right) & \cos\left(\frac{1}{8}\pi\right)\left(\frac{1}{2}\sqrt{2} - \frac{1}{2} I\sqrt{2}\right) \end{bmatrix}$$

$$S2 = \begin{bmatrix} \cos\left(\frac{1}{8}\pi\right) & -\sin\left(\frac{1}{8}\pi\right) \\ \sin\left(\frac{1}{8}\pi\right) & \cos\left(\frac{1}{8}\pi\right) \end{bmatrix}, \mathcal{S}2 = \begin{bmatrix} \cos\left(\frac{1}{8}\pi\right) & \sin\left(\frac{1}{8}\pi\right) \\ -\sin\left(\frac{1}{8}\pi\right) & \cos\left(\frac{1}{8}\pi\right) \end{bmatrix}$$

$$S3 = \begin{bmatrix} \frac{1}{2}\sqrt{2} - \frac{1}{2} I\sqrt{2} & 0 \\ 0 & \frac{1}{2}\sqrt{2} + \frac{1}{2} I\sqrt{2} \end{bmatrix}$$

$$S4 = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2}\sqrt{2} + \frac{1}{2} I\sqrt{2} \end{bmatrix}, \mathcal{S}4 = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2}\sqrt{2} - \frac{1}{2} I\sqrt{2} \end{bmatrix} \tag{22}$$

Determining U

```
> U := simplify(Multiply(U11, Multiply(U10, Multiply(U9, Multiply(U8, Multiply(U7, Multiply(U6,
      Multiply(U5,
            Multiply(U4, Multiply(U3, Multiply(U2, U1))))))))))) :'U'= U;
```

$$U = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{23}$$

Defining the 3-qubit state vector $|\psi_0\rangle$ and $|\psi_1\rangle$

$|\psi_1\rangle = U\,|\psi_0\rangle$

```
> CoEff := Ceff(3) :
  States := VSte(3) :
  |ψ0⟩ := Multiply(Ceff(3), Transpose(States))[1, 1];
  States := Multiply(U, Transpose(States)) :
  |ψ1⟩ := Multiply(CoEff, States)[1, 1];
```

$$|\psi 0\rangle := c_0\,|000\rangle + c_1\,|001\rangle + c_2\,|010\rangle + c_3\,|011\rangle + c_4\,|100\rangle + c_5\,|101\rangle + c_6\,|110\rangle + c_7\,|111\rangle$$
$$|\psi 1\rangle := c_0\,|000\rangle + c_1\,|001\rangle + c_2\,|010\rangle + c_3\,|011\rangle + c_4\,|100\rangle + c_5\,|101\rangle + c_6\,|111\rangle + c_7\,|110\rangle \tag{24}$$

>