

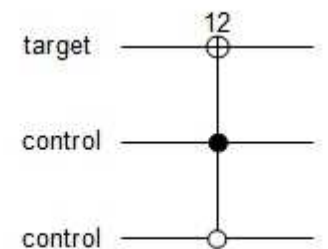
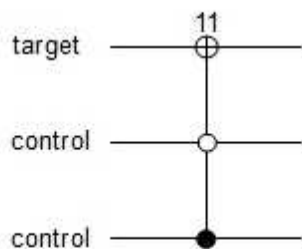
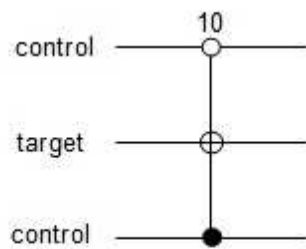
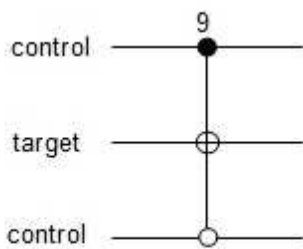
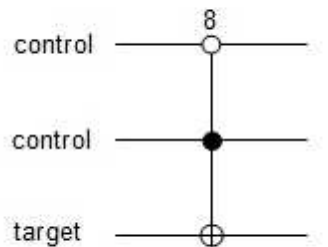
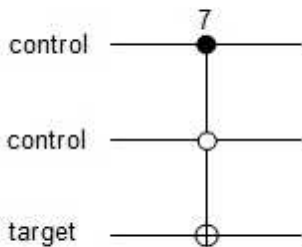
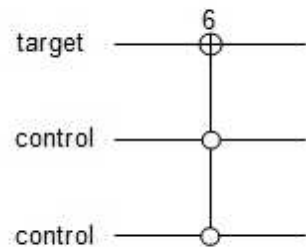
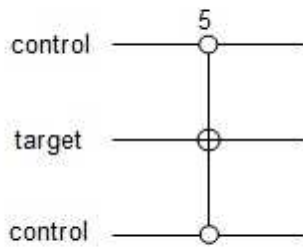
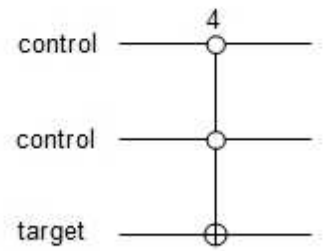
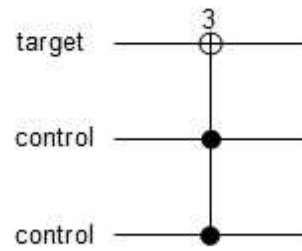
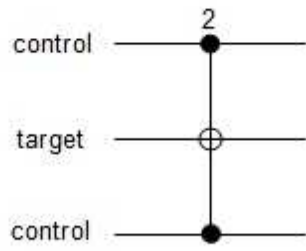
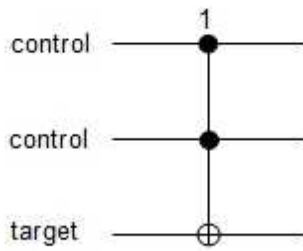
```
> restart :
```

```
> interface(warnlevel=0) :
```

## # Maple 12

```
> with(LinearAlgebra) :
```

```
> with(Bits) :
```



```
> TP := proc(M1, M2) return KroneckerProduct(M1, M2) end proc:
```

```
> VSt := proc(n) # Generates a list of computational states for n qubits
```

```
    local i, L; # e.g. n=2 => [ |00> |01> |10> |11> ]
```

```
    L := Matrix(1, 2^n);
```

```
    Settings(defaultbits = n);
```

```
    for i from 1 to 2^n do
```

```
        L[1, i] := cat(`\`, String(i-1, msbfirst), "`");
```

```
    end do;
```

```
    # print(L);
```

```
    return L; # returns Matrix L
```

```
end proc:
```

```
> Ceff := proc(n)
```

```
    local i, L; # Generates a list of computational state coefficients for n qubits
```

```
    L := Matrix(1, 2^n); # e.g. n=2 => [c0 c1 c2 c3]
```

```
    for i from 1 to 2^n do
```

```
        L[1, i] := c[i-1]; # c[i-1] represents the coefficient c_i
```

```
    end do;
```

```
    # print(L);
```

```
    return L; # returns Matrix L
```

```
end proc:
```

**Defining the 3-qubit state vector  $|\psi_0\rangle$  and  $|\psi_1\rangle$**

$$|\psi_1\rangle = U |\psi_0\rangle$$

>  $CoEff := Ceff(3) :$   
 $States := VSte(3) :$   
 $|\psi_0\rangle := Multiply(Ceff(3), Transpose(VSte(3)))[1, 1];$

$$|\psi_0\rangle := c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + c_3|011\rangle + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + c_7|111\rangle \quad (1)$$

>  $\mathbb{D} := IdentityMatrix(2);$  # generates a 2 by 2 identity matrix

$$\mathbb{D} := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (2)$$

>  $\mathbb{B} := IdentityMatrix(8);$  # generates a 8 by 8 identity matrix

$$\mathbb{B} := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

>  $Ux := RowOperation(\mathbb{D}, [1, 2]);$

$$Ux := \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad (4)$$

>  $T1 := RowOperation(\mathbb{B}, [7, 8]);$  # swaps rows 7 and 8  
 $S := Multiply(T1, Transpose(States)) :$   
 $|\psi_0\rangle := |\psi_0\rangle$   
 $|\psi_1\rangle := Multiply(CoEff, S)[1, 1];$

$$T1 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$|\psi_0\rangle := c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + c_3|011\rangle + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + c_7|111\rangle$$

$$|\psi_1\rangle := c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + c_3|011\rangle + c_4|100\rangle + c_5|101\rangle + c_6|111\rangle + c_7|110\rangle \quad (5)$$

> T2 := RowOperation( $\mathcal{B}$ , [6, 8]); # swaps rows 6 and 8  
 $S := \text{Multiply}(T2, \text{Transpose}(\text{States})) : |\psi 0\rangle := |\psi 0\rangle; |\psi 1\rangle := \text{Multiply}(\text{CoEff}, S)[1, 1];$

$$T2 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$|\psi 0\rangle := c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + c_3|011\rangle + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + c_7|111\rangle$$

$$|\psi 1\rangle := c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + c_3|011\rangle + c_4|100\rangle + c_5|111\rangle + c_6|110\rangle + c_7|101\rangle$$

(6)

> T3 := RowOperation( $\mathcal{B}$ , [4, 8]); # swaps rows 4 and 8  
 $S := \text{Multiply}(T3, \text{Transpose}(\text{States})) : |\psi 0\rangle := |\psi 0\rangle; |\psi 1\rangle := \text{Multiply}(\text{CoEff}, S)[1, 1];$

$$T3 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$|\psi 0\rangle := c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + c_3|011\rangle + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + c_7|111\rangle$$

$$|\psi 1\rangle := c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + c_3|111\rangle + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + c_7|011\rangle$$

(7)

> T4 := RowOperation( $\mathcal{B}$ , [1, 2]); # swaps rows 1 and 2  
 $S := \text{Multiply}(T4, \text{Transpose}(\text{States})) : |\psi 0\rangle := |\psi 0\rangle; |\psi 1\rangle := \text{Multiply}(\text{CoEff}, S)[1, 1];$

$$T4 := \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$|\psi 0\rangle := c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + c_3|011\rangle + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + c_7|111\rangle$$

$$|\psi 1\rangle := c_0|001\rangle + c_1|000\rangle + c_2|010\rangle + c_3|011\rangle + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + c_7|111\rangle$$

(8)

> T5 := RowOperation( $\mathcal{B}$ , [1, 3]); # swaps rows 1 and 3  
 $S := \text{Multiply}(T5, \text{Transpose}(\text{States})) : |\psi_0\rangle := |\psi_0\rangle; |\psi_1\rangle := \text{Multiply}(\text{CoEff}, S)[1, 1];$

$$T5 := \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$|\psi_0\rangle := c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + c_3|011\rangle + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + c_7|111\rangle$$

$$|\psi_1\rangle := c_0|010\rangle + c_1|001\rangle + c_2|000\rangle + c_3|011\rangle + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + c_7|111\rangle \quad (9)$$

> T6 := RowOperation( $\mathcal{B}$ , [1, 5]); # swaps rows 1 and 5  
 $S := \text{Multiply}(T6, \text{Transpose}(\text{States})) : |\psi_0\rangle := |\psi_0\rangle; |\psi_1\rangle := \text{Multiply}(\text{CoEff}, S)[1, 1];$

$$T6 := \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$|\psi_0\rangle := c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + c_3|011\rangle + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + c_7|111\rangle$$

$$|\psi_1\rangle := c_0|100\rangle + c_1|001\rangle + c_2|010\rangle + c_3|011\rangle + c_4|000\rangle + c_5|101\rangle + c_6|110\rangle + c_7|111\rangle \quad (10)$$

> T7 := RowOperation( $\mathcal{B}$ , [5, 6]); # swaps rows 5 and 6  
 $S := \text{Multiply}(T7, \text{Transpose}(\text{States})) : |\psi_0\rangle := |\psi_0\rangle; |\psi_1\rangle := \text{Multiply}(\text{CoEff}, S)[1, 1];$

$$T7 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$|\psi_0\rangle := c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + c_3|011\rangle + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + c_7|111\rangle$$

$$|\psi_1\rangle := c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + c_3|011\rangle + c_4|101\rangle + c_5|100\rangle + c_6|110\rangle + c_7|111\rangle \quad (11)$$

> T8 := RowOperation( $\mathcal{B}$ , [3, 4]); # swaps rows 3 and 4  
 $S := \text{Multiply}(T8, \text{Transpose}(\text{States})) : |\psi 0\rangle := |\psi 0\rangle; |\psi 1\rangle := \text{Multiply}(\text{CoEff}, S)[1, 1];$

$$T8 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$|\psi 0\rangle := c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + c_3|011\rangle + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + c_7|111\rangle$$

$$|\psi 1\rangle := c_0|000\rangle + c_1|001\rangle + c_2|011\rangle + c_3|010\rangle + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + c_7|111\rangle \quad (12)$$

> T9 := RowOperation( $\mathcal{B}$ , [5, 7]); # swaps rows 5 and 7  
 $S := \text{Multiply}(T9, \text{Transpose}(\text{States})) : |\psi 0\rangle := |\psi 0\rangle; |\psi 1\rangle := \text{Multiply}(\text{CoEff}, S)[1, 1];$

$$T9 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$|\psi 0\rangle := c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + c_3|011\rangle + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + c_7|111\rangle$$

$$|\psi 1\rangle := c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + c_3|011\rangle + c_4|110\rangle + c_5|101\rangle + c_6|100\rangle + c_7|111\rangle \quad (13)$$

> T10 := RowOperation( $\mathcal{B}$ , [2, 4]); # swaps rows 2 and 4  
 $S := \text{Multiply}(T10, \text{Transpose}(\text{States})) : |\psi 0\rangle := |\psi 0\rangle; |\psi 1\rangle := \text{Multiply}(\text{CoEff}, S)[1, 1];$

$$T10 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$|\psi 0\rangle := c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + c_3|011\rangle + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + c_7|111\rangle$$

$$|\psi 1\rangle := c_0|000\rangle + c_1|011\rangle + c_2|010\rangle + c_3|001\rangle + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + c_7|111\rangle \quad (14)$$

> T11 := RowOperation( $\mathbb{B}$ , [2, 6]); # swaps rows 2 and 6  
 $S := \text{Multiply}(T11, \text{Transpose}(\text{States})) : |\psi 0\rangle := |\psi 0\rangle; |\psi 1\rangle := \text{Multiply}(\text{CoEff}, S)[1, 1];$

$$T11 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$|\psi 0\rangle := c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + c_3|011\rangle + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + c_7|111\rangle$$

$$|\psi 1\rangle := c_0|000\rangle + c_1|101\rangle + c_2|010\rangle + c_3|011\rangle + c_4|100\rangle + c_5|001\rangle + c_6|110\rangle + c_7|111\rangle \quad (15)$$

> T12 := RowOperation( $\mathbb{B}$ , [3, 7]); # swaps rows 3 and 7

$S := \text{Multiply}(T12, \text{Transpose}(\text{States})) :$

$|\psi 0\rangle := |\psi 0\rangle;$

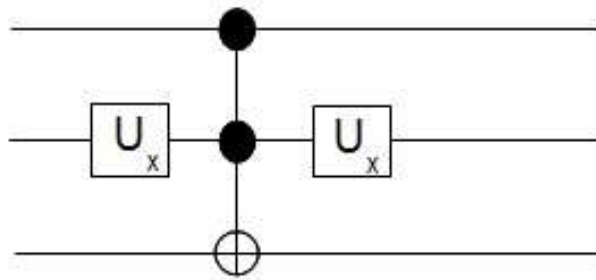
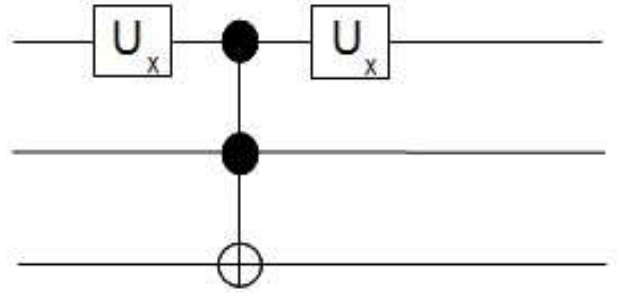
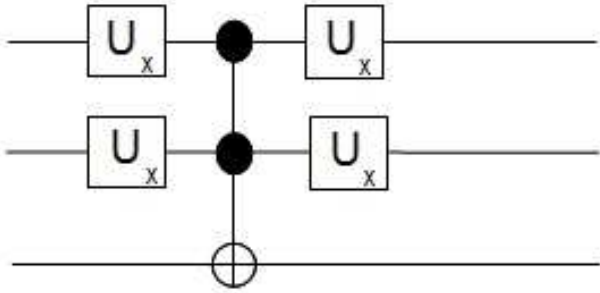
$|\psi 1\rangle := \text{Multiply}(\text{CoEff}, S)[1, 1];$

$$T12 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$|\psi 0\rangle := c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + c_3|011\rangle + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + c_7|111\rangle$$

$$|\psi 1\rangle := c_0|000\rangle + c_1|001\rangle + c_2|110\rangle + c_3|011\rangle + c_4|100\rangle + c_5|101\rangle + c_6|010\rangle + c_7|111\rangle \quad (16)$$

## Other versions of the Toffoli gate



>  $M1 := TP(Ux, TP(Ux, \mathbb{D})) :$  #` Compare with T4  
 $G1 := Multiply(M1, Multiply(T1, M1)) ;$   
 $S := Multiply(G1, Transpose(States)) :$   
 $|\psi0\rangle := |\psi0\rangle ;$   
 $|\psi1\rangle := Multiply(CoEff, S)[1, 1] ;$

$$G1 := \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$|\psi0\rangle := c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + c_3|011\rangle + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + c_7|111\rangle$$

$$|\psi1\rangle := c_0|001\rangle + c_1|000\rangle + c_2|010\rangle + c_3|011\rangle + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + c_7|111\rangle$$

(17)

>  $M2 := TP(Ux, TP(\mathbb{D}, \mathbb{D})) :$  # **Compare with T8**  
 $G2 := Multiply(M2, Multiply(T1, M2)) ;$   
 $S := Multiply(G2, Transpose(States)) :$   
 $|\psi0\rangle := |\psi0\rangle ;$   
 $|\psi1\rangle := Multiply(CoEff, S)[1, 1];$

$$G2 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$|\psi0\rangle := c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + c_3|011\rangle + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + c_7|111\rangle$$

$$|\psi1\rangle := c_0|000\rangle + c_1|001\rangle + c_2|011\rangle + c_3|010\rangle + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + c_7|111\rangle \quad (18)$$

>  $M3 := TP(\mathbb{D}, TP(Ux, \mathbb{D})) :$  # **Compare with T7**  
 $G3 := Multiply(M3, Multiply(T1, M3)) ;$   
 $S := Multiply(G3, Transpose(States)) :$   
 $|\psi0\rangle := |\psi0\rangle ;$   
 $|\psi1\rangle := Multiply(CoEff, S)[1, 1];$

$$G3 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$|\psi0\rangle := c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + c_3|011\rangle + c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + c_7|111\rangle$$

$$|\psi1\rangle := c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + c_3|011\rangle + c_4|101\rangle + c_5|100\rangle + c_6|110\rangle + c_7|111\rangle \quad (19)$$