> $restart;$

# Laguerre Polynomials

> $L[0](x) = simplify\left(e^{x} \cdot \frac{d^{0}}{d x^{0}} \left(x^{0} \cdot e^{-x}\right)\right);$

$$L_0(x) = 1 \tag{1}$$

> $L[1](x) = simplify\left(e^{x} \cdot \frac{d^{1}}{d x^{1}} \left(x^{1} \cdot e^{-x}\right)\right);$

$$L_1(x) = 1 - x \tag{2}$$

> $L[2](x) = simplify\left(e^{x} \cdot \frac{d^{2}}{d x^{2}} \left(x^{2} \cdot e^{-x}\right)\right);$

$$L_2(x) = 2 - 4x + x^2 \tag{3}$$

> $L[3](x) = simplify\left(e^{x} \cdot \frac{d^{3}}{d x^{3}} \left(x^{3} \cdot e^{-x}\right)\right);$

$$L_3(x) = 6 - 18x + 9x^2 - x^3 \tag{4}$$

> $L[4](x) = simplify\left(e^{x} \cdot \frac{d^{4}}{d x^{4}} \left(x^{4} \cdot e^{-x}\right)\right);$

$$L_4(x) = 24 - 96x + 72x^2 - 16x^3 + x^4 \tag{5}$$

## This Procedure Generates Laguerre Polynomials

> $La :=$ **proc**$(n)$
**local** $l$;
**if** $n = 0$ **then** $l := simplify\left(e^{r} \left(e^{-r} \cdot r^{n}\right)\right);$
**else**
$l := simplify\left(e^{r} \cdot \frac{d^{n}}{d r^{n}} \left(e^{-r} \cdot r^{n}\right)\right);$
**end if**;
**end proc**:

## Laguerre Polynomials from $L_0(r)$ to $L_8(r)$

```
> for i from 0 to 8 do
    L[i](r) = La(i);
  end do;
```

$$L_0(r) = 1$$
$$L_1(r) = -r + 1$$
$$L_2(r) = r^2 - 4r + 2$$
$$L_3(r) = -r^3 + 9r^2 - 18r + 6$$
$$L_4(r) = r^4 - 16r^3 + 72r^2 - 96r + 24$$
$$L_5(r) = -r^5 + 25r^4 - 200r^3 + 600r^2 - 600r + 120$$
$$L_6(r) = r^6 - 36r^5 + 450r^4 - 2400r^3 + 5400r^2 - 4320r + 720$$
$$L_7(r) = -r^7 + 49r^6 - 882r^5 + 7350r^4 - 29400r^3 + 52920r^2 - 35280r + 5040$$
$$L_8(r) = r^8 - 64r^7 + 1568r^6 - 18816r^5 + 117600r^4 - 376320r^3 + 564480r^2 - 322560r + 40320 \quad \textbf{(6)}$$

## Maple's Laguerre Polynomial Generator

```
> for n from 0 to 8 do
    L[n](r) = simplify(n! LaguerreL(n, r));
  end do;
```

$$L_0(r) = 1$$
$$L_1(r) = -r + 1$$
$$L_2(r) = r^2 - 4r + 2$$
$$L_3(r) = -r^3 + 9r^2 - 18r + 6$$
$$L_4(r) = r^4 - 16r^3 + 72r^2 - 96r + 24$$
$$L_5(r) = -r^5 + 25r^4 - 200r^3 + 600r^2 - 600r + 120$$
$$L_6(r) = r^6 - 36r^5 + 450r^4 - 2400r^3 + 5400r^2 - 4320r + 720$$
$$L_7(r) = 5040 - 35280r + 52920r^2 - 29400r^3 + 7350r^4 - 882r^5 + 49r^6 - r^7$$
$$L_8(r) = 40320 - 322560r + 564480r^2 - 376320r^3 + 117600r^4 - 18816r^5 + 1568r^6 - 64r^7 + r^8 \quad \textbf{(7)}$$

# Associated Laguerre Polynomials

$$> L[0](0)(x) = simplify\left((-1)^0 \cdot \frac{d^0}{dx^0}\left(e^x \cdot \frac{d^0}{dx^0}\left(x^0 \cdot e^{-x}\right)\right)\right);$$

$$L_0(0)(x) = 1 \tag{8}$$

$$> L[0](1)(x) = simplify\left((-1)^1 \cdot \frac{d^1}{dx^1}\left(e^x \cdot \frac{d^1}{dx^1}\left(x^1 \cdot e^{-x}\right)\right)\right);$$

$$L_0(1)(x) = 1 \tag{9}$$

$$> L[0](2)(x) = simplify\left((-1)^2 \cdot \frac{d^2}{dx^2}\left(e^x \cdot \frac{d^2}{dx^2}\left(x^2 \cdot e^{-x}\right)\right)\right);$$

$$L_0(2)(x) = 2 \tag{10}$$

$$> L[0](4)(x) = simplify\left((-1)^4 \cdot \frac{d^4}{dx^4}\left(e^x \cdot \frac{d^4}{dx^4}\left(x^4 \cdot e^{-x}\right)\right)\right);$$

$$L_0(4)(x) = 24 \tag{11}$$

$$> L[1](2)(x) = simplify\left((-1)^2 \cdot \frac{d^2}{dx^2}\left(e^x \cdot \frac{d^3}{dx^3}\left(x^3 \cdot e^{-x}\right)\right)\right);$$

$$L_1(2)(x) = 18 - 6x \tag{12}$$

$$> L[1](3)(x) = simplify\left((-1)^3 \cdot \frac{d^3}{dx^3}\left(e^x \cdot \frac{d^4}{dx^4}\left(x^4 \cdot e^{-x}\right)\right)\right);$$

$$L_1(3)(x) = 96 - 24x \tag{13}$$

## This Procedure Generates Associated Laguerre Polynomials

> *ALa* := **proc**($a$, $b$)
**local** *al*, *s*;
$s := a + b$;
**if** $b = 0$ **then** $al := simplify\left((-1)^b\, La(s)\right)$
**else**
$$al := simplify\left((-1)^b \cdot \frac{d^b}{dr^b}\, La(s)\right);$$
**end if**;
**end proc**:

## Associated Laguerre Polynomials from $L_0(0)(r)$ to $L_5(5)(r)$

```
> for i from 0 to 5 do
    for j from 0 to 5 do
        print(L[i](j)(x) = ALa(i, j));
    end do;
  end do;
```

$$L_0(0)(x) = 1$$

$$L_0(1)(x) = 1$$

$$L_0(2)(x) = 2$$

$$L_0(3)(x) = 6$$

$$L_0(4)(x) = 24$$

$$L_0(5)(x) = 120$$

$$L_1(0)(x) = -r + 1$$

$$L_1(1)(x) = -2r + 4$$

$$L_1(2)(x) = -6r + 18$$

$$L_1(3)(x) = -24r + 96$$

$$L_1(4)(x) = -120r + 600$$

$$L_1(5)(x) = -720r + 4320$$

$$L_2(0)(x) = r^2 - 4r + 2$$

$$L_2(1)(x) = 3r^2 - 18r + 18$$

$$L_2(2)(x) = 12r^2 - 96r + 144$$

$$L_2(3)(x) = 60r^2 - 600r + 1200$$

$$L_2(4)(x) = 360r^2 - 4320r + 10800$$

$$L_2(5)(x) = 2520r^2 - 35280r + 105840$$

$$L_3(0)(x) = -r^3 + 9r^2 - 18r + 6$$

$$L_3(1)(x) = -4r^3 + 48r^2 - 144r + 96$$

$$L_3(2)(x) = -20r^3 + 300r^2 - 1200r + 1200$$

$$L_3(3)(x) = -120r^3 + 2160r^2 - 10800r + 14400$$

$$L_3(4)(x) = -840r^3 + 17640r^2 - 105840r + 176400$$

$$L_3(5)(x) = -6720r^3 + 161280r^2 - 1128960r + 2257920$$

$$L_4(0)(x) = r^4 - 16\,r^3 + 72\,r^2 - 96\,r + 24$$
$$L_4(1)(x) = 5\,r^4 - 100\,r^3 + 600\,r^2 - 1200\,r + 600$$
$$L_4(2)(x) = 30\,r^4 - 720\,r^3 + 5400\,r^2 - 14400\,r + 10800$$
$$L_4(3)(x) = 210\,r^4 - 5880\,r^3 + 52920\,r^2 - 176400\,r + 176400$$
$$L_4(4)(x) = 1680\,r^4 - 53760\,r^3 + 564480\,r^2 - 2257920\,r + 2822400$$
$$L_4(5)(x) = 15120\,r^4 - 544320\,r^3 + 6531840\,r^2 - 30481920\,r + 45722880$$
$$L_5(0)(x) = -r^5 + 25\,r^4 - 200\,r^3 + 600\,r^2 - 600\,r + 120$$
$$L_5(1)(x) = -6\,r^5 + 180\,r^4 - 1800\,r^3 + 7200\,r^2 - 10800\,r + 4320$$
$$L_5(2)(x) = -42\,r^5 + 1470\,r^4 - 17640\,r^3 + 88200\,r^2 - 176400\,r + 105840$$
$$L_5(3)(x) = -336\,r^5 + 13440\,r^4 - 188160\,r^3 + 1128960\,r^2 - 2822400\,r + 2257920$$
$$L_5(4)(x) = -3024\,r^5 + 136080\,r^4 - 2177280\,r^3 + 15240960\,r^2 - 45722880\,r + 45722880$$
$$L_5(5)(x) = -30240\,r^5 + 1512000\,r^4 - 27216000\,r^3 + 217728000\,r^2 - 762048000\,r + 914457600 \quad \textbf{(14)}$$

## Maple's Associated Laguerre Polynomial Generator

```
> MALa := proc(n, a, x) simplify((n + a)! LaguerreL(n, a, x)); end proc:
> for i from 0 to 5 do
   for j from 0 to 5 do
      print(L[i](j)(r) = MALa(i, j, r));
   end do;
  end do;
```

$$L_0(0)(r) = 1$$
$$L_0(1)(r) = 1$$
$$L_0(2)(r) = 2$$
$$L_0(3)(r) = 6$$
$$L_0(4)(r) = 24$$
$$L_0(5)(r) = 120$$
$$L_1(0)(r) = 1 - r$$
$$L_1(1)(r) = 4 - 2\,r$$
$$L_1(2)(r) = 18 - 6\,r$$
$$L_1(3)(r) = 96 - 24\,r$$
$$L_1(4)(r) = 600 - 120\,r$$
$$L_1(5)(r) = 4320 - 720\,r$$

$$L_2(0)(r) = 2 - 4r + r^2$$
$$L_2(1)(r) = 18 - 18r + 3r^2$$
$$L_2(2)(r) = 144 - 96r + 12r^2$$
$$L_2(3)(r) = 1200 - 600r + 60r^2$$
$$L_2(4)(r) = 10800 - 4320r + 360r^2$$
$$L_2(5)(r) = 2520r^2 - 35280r + 105840$$
$$L_3(0)(r) = -r^3 + 9r^2 - 18r + 6$$
$$L_3(1)(r) = -4r^3 + 48r^2 - 144r + 96$$
$$L_3(2)(r) = -20r^3 + 300r^2 - 1200r + 1200$$
$$L_3(3)(r) = -120r^3 + 2160r^2 - 10800r + 14400$$
$$L_3(4)(r) = -840r^3 + 17640r^2 - 105840r + 176400$$
$$L_3(5)(r) = -6720r^3 + 161280r^2 - 1128960r + 2257920$$
$$L_4(0)(r) = r^4 - 16r^3 + 72r^2 - 96r + 24$$
$$L_4(1)(r) = 5r^4 - 100r^3 + 600r^2 - 1200r + 600$$
$$L_4(2)(r) = 30r^4 - 720r^3 + 5400r^2 - 14400r + 10800$$
$$L_4(3)(r) = 210r^4 - 5880r^3 + 52920r^2 - 176400r + 176400$$
$$L_4(4)(r) = 1680r^4 - 53760r^3 + 564480r^2 - 2257920r + 2822400$$
$$L_4(5)(r) = 15120r^4 - 544320r^3 + 6531840r^2 - 30481920r + 45722880$$
$$L_5(0)(r) = -r^5 + 25r^4 - 200r^3 + 600r^2 - 600r + 120$$
$$L_5(1)(r) = -6r^5 + 180r^4 - 1800r^3 + 7200r^2 - 10800r + 4320$$
$$L_5(2)(r) = -42r^5 + 1470r^4 - 17640r^3 + 88200r^2 - 176400r + 105840$$
$$L_5(3)(r) = -336r^5 + 13440r^4 - 188160r^3 + 1128960r^2 - 2822400r + 2257920$$
$$L_5(4)(r) = -3024r^5 + 136080r^4 - 2177280r^3 + 15240960r^2 - 45722880r + 45722880$$
$$L_5(5)(r) = -30240r^5 + 1512000r^4 - 27216000r^3 + 217728000r^2 - 762048000r + 914457600 \quad \textbf{(15)}$$

## This Procedure Generates Associated Laguerre Polynomials Using Maple's Laguerre Polynomial Function

```
> MALa2 :=proc(a, b)
          local al, s;
           s := a + b ;
           if b = 0 then al := simplify(s!· LaguerreL(s, r))
             else
```

$$al := (-1)^b \cdot \frac{d^b}{d r^b}(simplify(s! \cdot \mathrm{LaguerreL}(s, r)));$$

```
           end if;
        end proc:
```

## Associated Laguerre Polynomials from $L_0^0(r)$ to $L_5^5(r)$

```
> for i from 0 to 5 do
    for j from 0 to 5 do
         print(L[i](j)(r) = MALa2(i, j));
    end do;
  end do;
```

$$L_0(0)(r) = 1$$
$$L_0(1)(r) = 1$$
$$L_0(2)(r) = 2$$
$$L_0(3)(r) = 6$$
$$L_0(4)(r) = 24$$
$$L_0(5)(r) = 120$$
$$L_1(0)(r) = 1 - r$$
$$L_1(1)(r) = 4 - 2r$$
$$L_1(2)(r) = 18 - 6r$$
$$L_1(3)(r) = 96 - 24r$$
$$L_1(4)(r) = 600 - 120r$$
$$L_1(5)(r) = 4320 - 720r$$
$$L_2(0)(r) = 2 - 4r + r^2$$
$$L_2(1)(r) = 18 - 18r + 3r^2$$
$$L_2(2)(r) = 144 - 96r + 12r^2$$
$$L_2(3)(r) = 1200 - 600r + 60r^2$$
$$L_2(4)(r) = 10800 - 4320r + 360r^2$$
$$L_2(5)(r) = 105840 - 35280r + 2520r^2$$

$$L_3(0)(r) = 6 - 18\,r + 9\,r^2 - r^3$$
$$L_3(1)(r) = 96 - 144\,r + 48\,r^2 - 4\,r^3$$
$$L_3(2)(r) = 1200 - 1200\,r + 300\,r^2 - 20\,r^3$$
$$L_3(3)(r) = 14400 - 10800\,r + 2160\,r^2 - 120\,r^3$$
$$L_3(4)(r) = 176400 - 105840\,r + 17640\,r^2 - 840\,r^3$$
$$L_3(5)(r) = 2257920 - 1128960\,r + 161280\,r^2 - 6720\,r^3$$
$$L_4(0)(r) = 24 - 96\,r + 72\,r^2 - 16\,r^3 + r^4$$
$$L_4(1)(r) = 600 - 1200\,r + 600\,r^2 - 100\,r^3 + 5\,r^4$$
$$L_4(2)(r) = 10800 - 14400\,r + 5400\,r^2 - 720\,r^3 + 30\,r^4$$
$$L_4(3)(r) = 176400 - 176400\,r + 52920\,r^2 - 5880\,r^3 + 210\,r^4$$
$$L_4(4)(r) = 2822400 - 2257920\,r + 564480\,r^2 - 53760\,r^3 + 1680\,r^4$$
$$L_4(5)(r) = 45722880 - 30481920\,r + 6531840\,r^2 - 544320\,r^3 + 15120\,r^4$$
$$L_5(0)(r) = 120 - 600\,r + 600\,r^2 - 200\,r^3 + 25\,r^4 - r^5$$
$$L_5(1)(r) = 4320 - 10800\,r + 7200\,r^2 - 1800\,r^3 + 180\,r^4 - 6\,r^5$$
$$L_5(2)(r) = 105840 - 176400\,r + 88200\,r^2 - 17640\,r^3 + 1470\,r^4 - 42\,r^5$$
$$L_5(3)(r) = 2257920 - 2822400\,r + 1128960\,r^2 - 188160\,r^3 + 13440\,r^4 - 336\,r^5$$
$$L_5(4)(r) = 45722880 - 45722880\,r + 15240960\,r^2 - 2177280\,r^3 + 136080\,r^4 - 3024\,r^5$$
$$L_5(5)(r) = 914457600 - 762048000\,r + 217728000\,r^2 - 27216000\,r^3 + 1512000\,r^4 - 30240\,r^5 \tag{16}$$