```
> restart; interface(displayprecision = 3) :
> with(plots) :
```

Using Maple's Associated Laguerre Polynomial Function

```
> MALa := proc(n, a, x) simplify( (n + a)! LaguerreL(n, a, x) ); end proc:
```

Hydrogen Atom Radial Function

```
> R := (n, l) → sqrt( 4·(n − l − 1)! / ( n^4 · ((n + l)!)^3 ) ) · e^(−ρ/n) · ( (2/n)·ρ )^l · MALa( (n − l − 1), (2·l + 1), ( (2/n)·ρ ) ) :
```

Hydrogen Atom Radial Probability

```
> P := (n, l) → ρ^2 · (abs(R(n, l)))^2 :
```

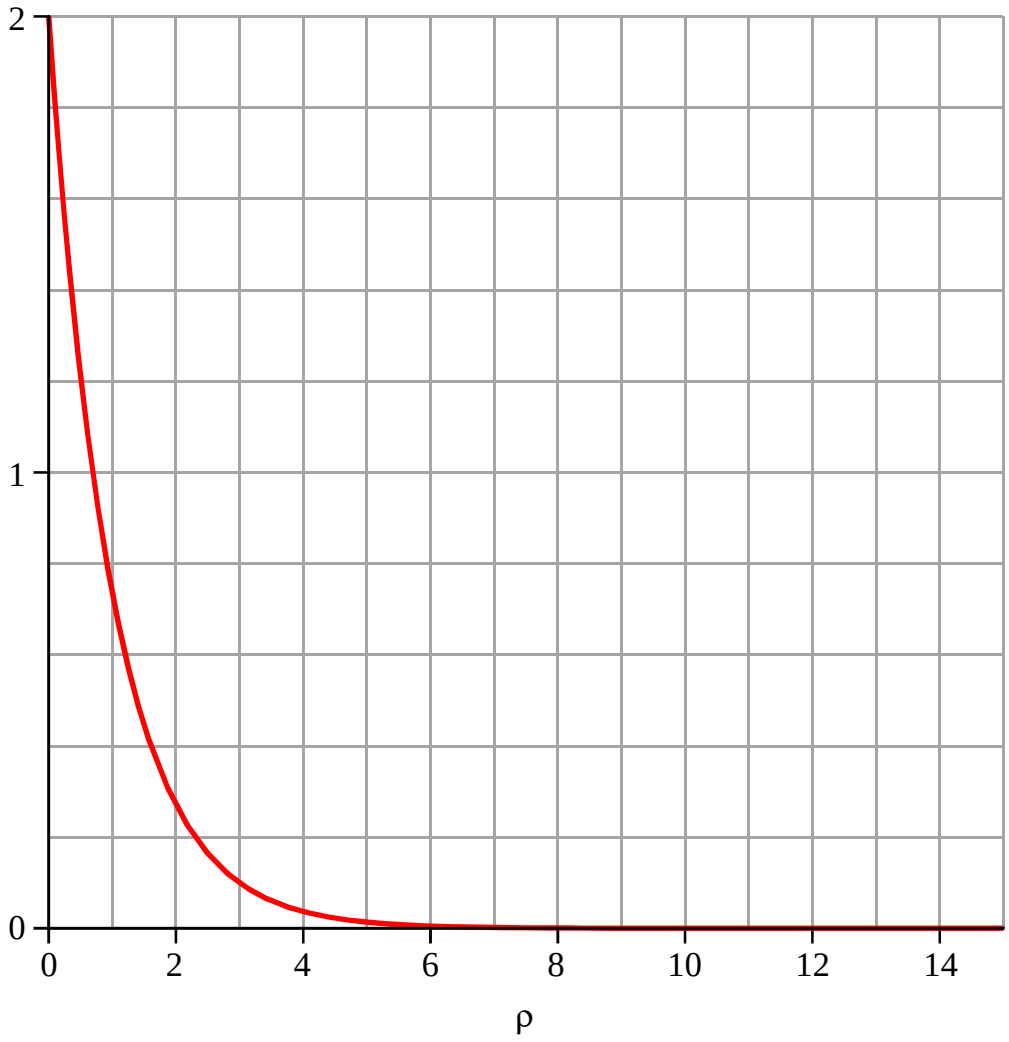
Graph of the Radial Function $R(n, l)$

```
> graph := proc(n, l)
    local g, N, v, h;
    N := n·15; v := 1; h := 1;
    if n = 3 then N := 35 end if ;
    if n = 2 then v := 12/7; h := 7/6 end if;
    if n ≥ 3 then v := 2 end if;
    g := plot( R(n, l), ρ = 0 ..N,
        color = red, axis[1] = [gridlines = [14·v, linestyle = solid]],
        thickness = 2, axis[2] = [gridlines = [12·h, linestyle = solid]],
        tickmarks = [10, 3]) :
    display([g]);
  end proc:
```

Graph of the Radial Probability $R^2(n, l)\ \rho^2$

```
> graphP := proc(n, l)
    local g, N, h, v;
    N := n·15; v := 1; h := 1;
    if n = 3 then N := 35 end if ;
    if n = 2 then v := 12/7; h := 7/6 end if;
    if n ≥ 3 then v := 2 end if;
    g := plot( P(n, l), ρ = 0 ..N,
        color = red, axis[1] = [gridlines = [14·v, linestyle = solid]],
        thickness = 2, axis[2] = [gridlines = [12·h, linestyle = solid]],
        tickmarks = [10, 3]) :
  display([g]);
  end proc:
```

```
> graph(1, 0);
'R[1, 0](ρ)' = R(1, 0);
```

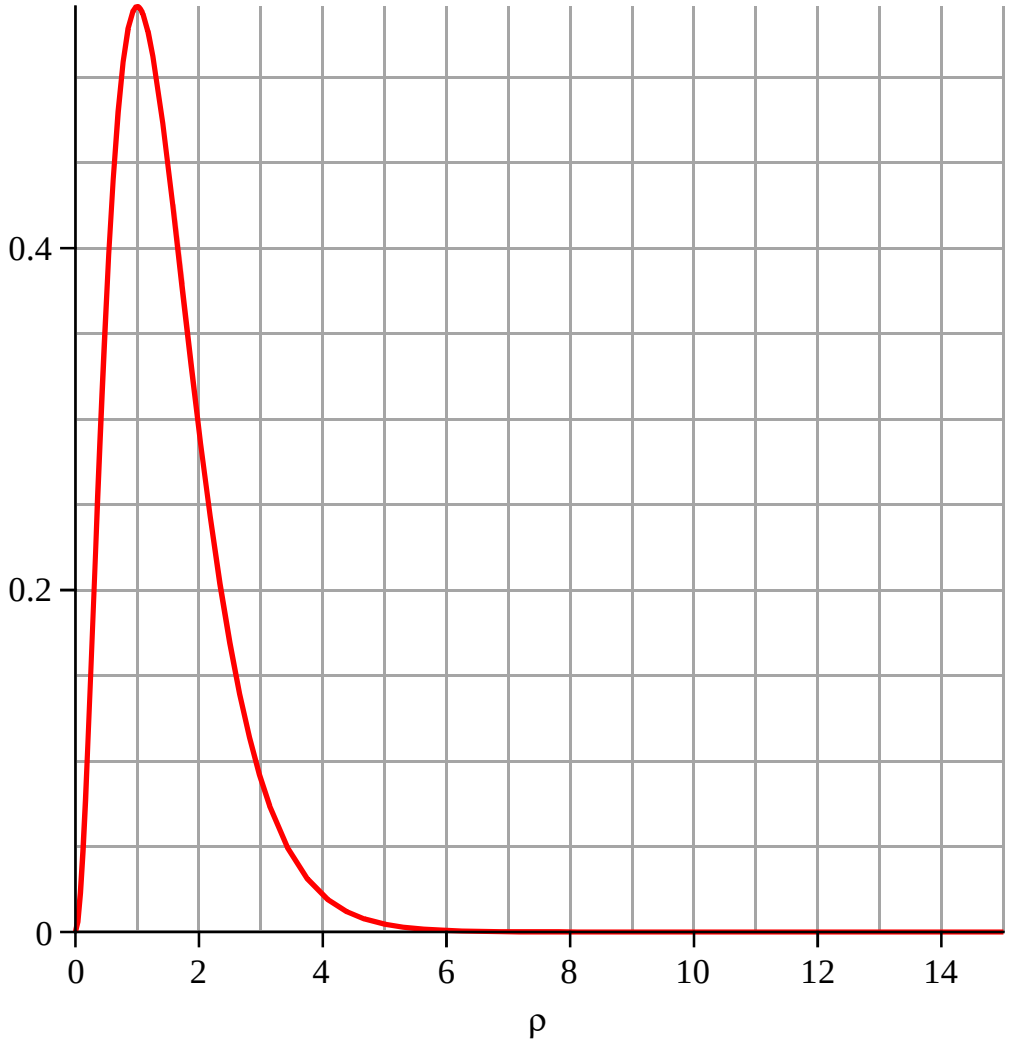


$$R_{1,\,0}(\rho) = 2\,e^{-\rho} \qquad \textbf{(1)}$$

> *graphP*(1, 0);

'$\rho^2 \cdot (\mathrm{abs}(R[1,0](\rho)))^2$';

'$\int_0^\infty \rho^2 |R(1,0)|^2 \, d\rho$ '= $\int_0^\infty \rho^2 |R(1,0)|^2 \, d\rho$;



$$\rho^2 \left| R_{1,0}(\rho) \right|^2$$

$$\int_0^\infty \rho^2 |R(1,0)|^2 \, d\rho = 1 \qquad \textbf{(2)}$$

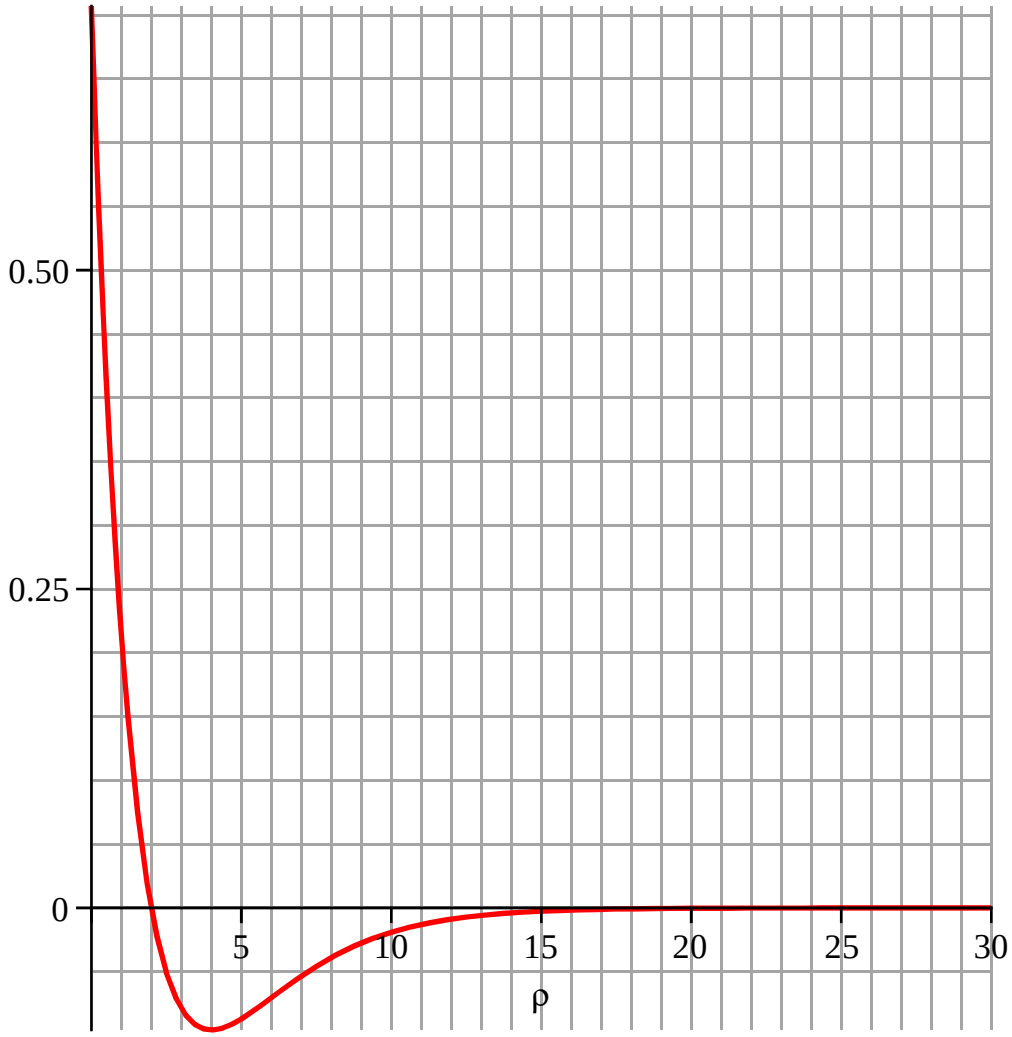
> $\rho[\mathrm{max}] := \max\left(solve\left(\frac{d}{d\rho} \left(\rho^2 (R(1,0))^2 \right) = 0 \right) \right)$;

$r[\mathrm{max}] = \mathrm{evalf}(0.529 \cdot \rho[\mathrm{max}])$; $r[\mathrm{max}] := $'$r[\mathrm{max}]$': $\rho[\mathrm{max}] := $'$\rho[\mathrm{max}]$':

$$\rho_{max} := 1$$

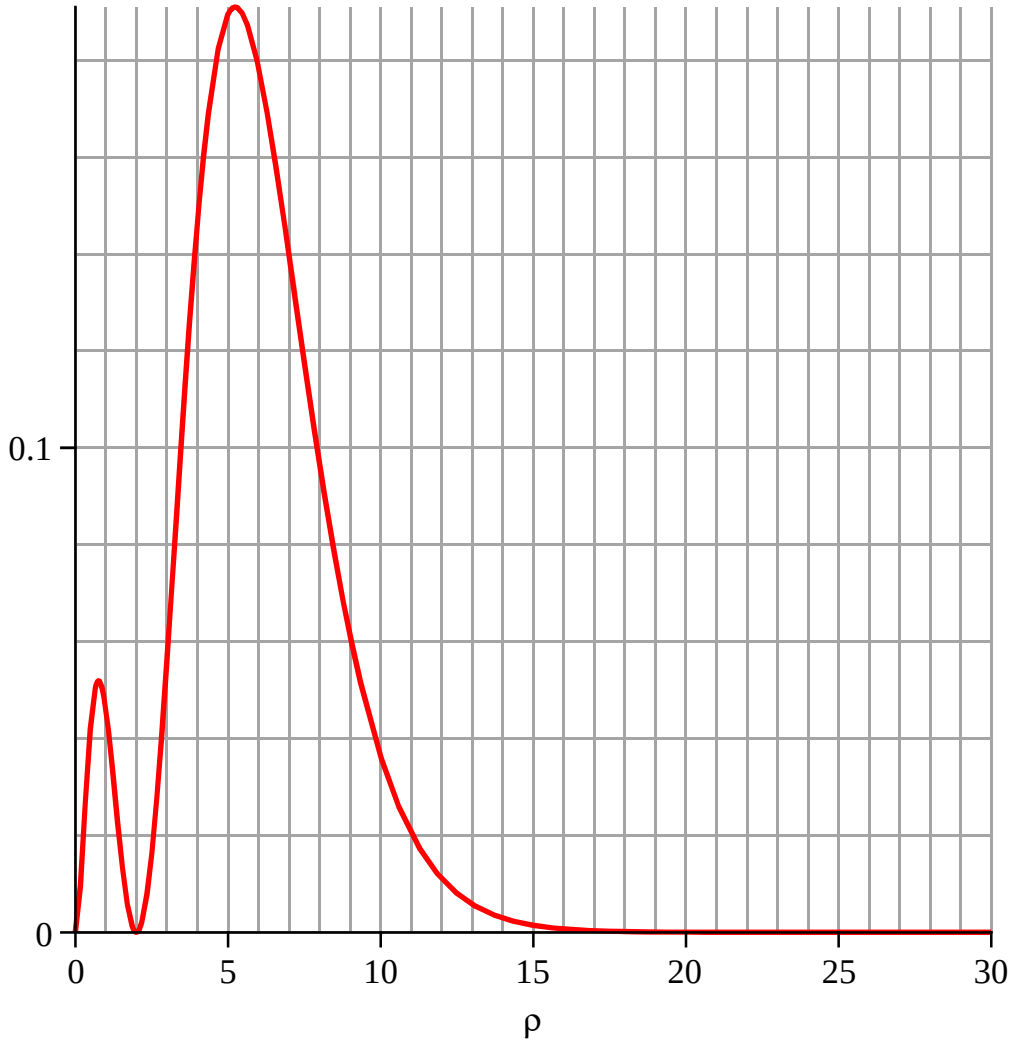
$$r_{max} = 0.529 \qquad \textbf{(3)}$$

```
> graph(2, 0);
'R[2, 0](ρ)' = combine(R(2, 0));
```



$$R_{2,\,0}(\rho) = \left(\frac{1}{2} - \frac{1}{4}\,\rho\right)\sqrt{2}\;e^{-\frac{1}{2}\rho} \qquad \textbf{(4)}$$

```
> graphP(2, 0);
  'ρ^2·(abs(R[2, 0](ρ)))^2';
  '∫_0^∞ ρ^2 |R(2, 0)|^2 dρ '= ∫_0^∞ ρ^2 |R(2, 0)|^2 dρ;
```



$$\rho^2 \left| R_{2,\,0}(\rho) \right|^2$$

$$\int_0^\infty \rho^2 |R(2, 0)|^2 \, d\rho = 1 \qquad \textbf{(5)}$$

```
> ρ[max][1] := fsolve( d/dρ (ρ^2 (R(2, 0))^2) = 0, ρ, 0..1.5 ); r[max][1] := 0.529·ρ[max][1]; ρ :='ρ':
  ρ[max][2] := fsolve( d/dρ (ρ^2 (R(2, 0))^2) = 0, ρ, 1.5..20 ); r[max][2] := 0.529·ρ[max][2]; ρ :='ρ':
```

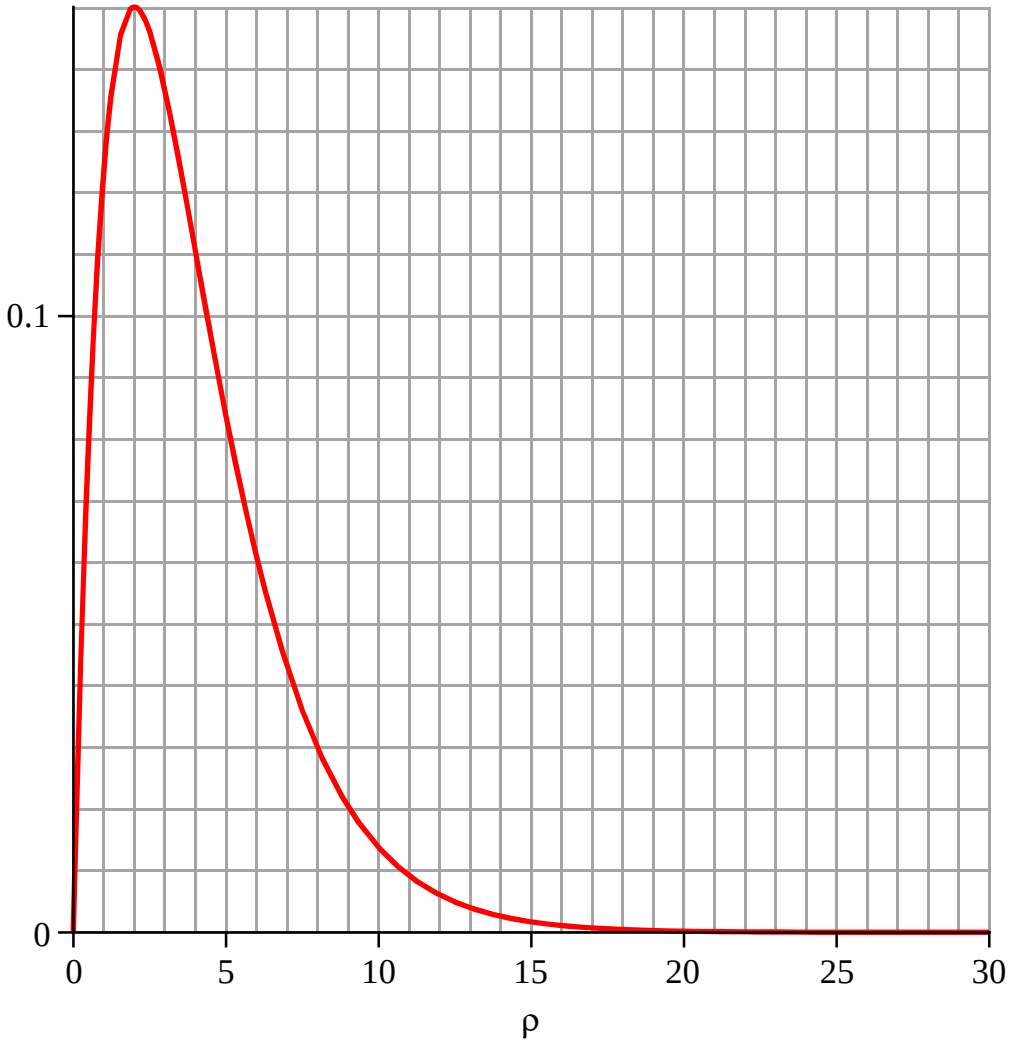
$$\rho_{max_1} := 0.764$$

$$r_{max_1} := 0.404$$

$$\rho_{max_2} := 5.236$$

$$r_{max_2} := 2.770 \qquad \textbf{(6)}$$

```
> graph(2, 1);
'R[2, 1](ρ)' = combine(R(2, 1));
```

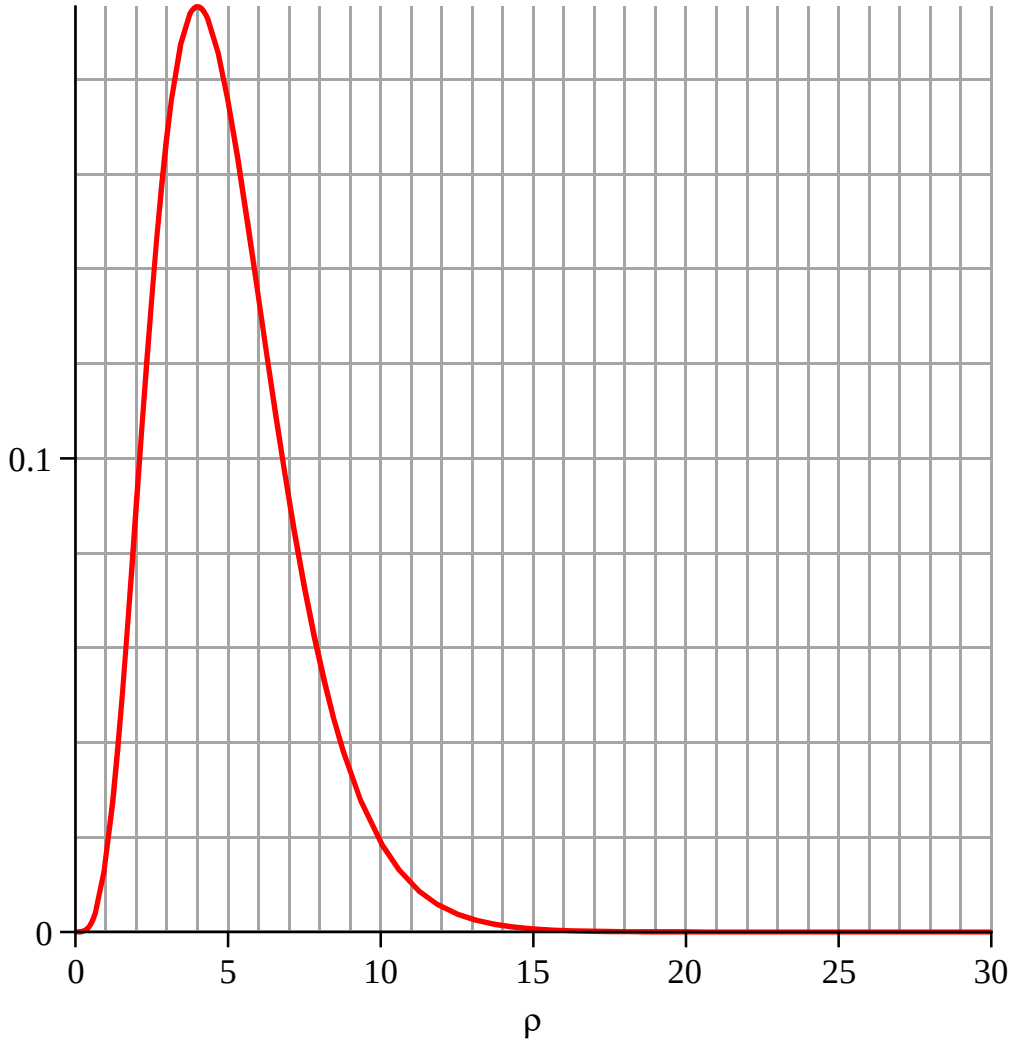


$$R_{2,1}(\rho) = \frac{1}{12}\sqrt{6}\, e^{-\frac{1}{2}\rho}\, \rho \qquad (7)$$

> *graphP*(2, 1);

'$\rho^2 \cdot (\mathrm{abs}(R[2,1](\rho)))^2$';

'$\int_0^\infty \rho^2 |R(2,1)|^2 \, d\rho$ '= $\int_0^\infty \rho^2 |R(2,1)|^2 \, d\rho$;



$$\rho^2 \left| R_{2,1}(\rho) \right|^2$$

$$\int_0^\infty \rho^2 |R(2,1)|^2 \, d\rho = 1 \qquad \textbf{(8)}$$

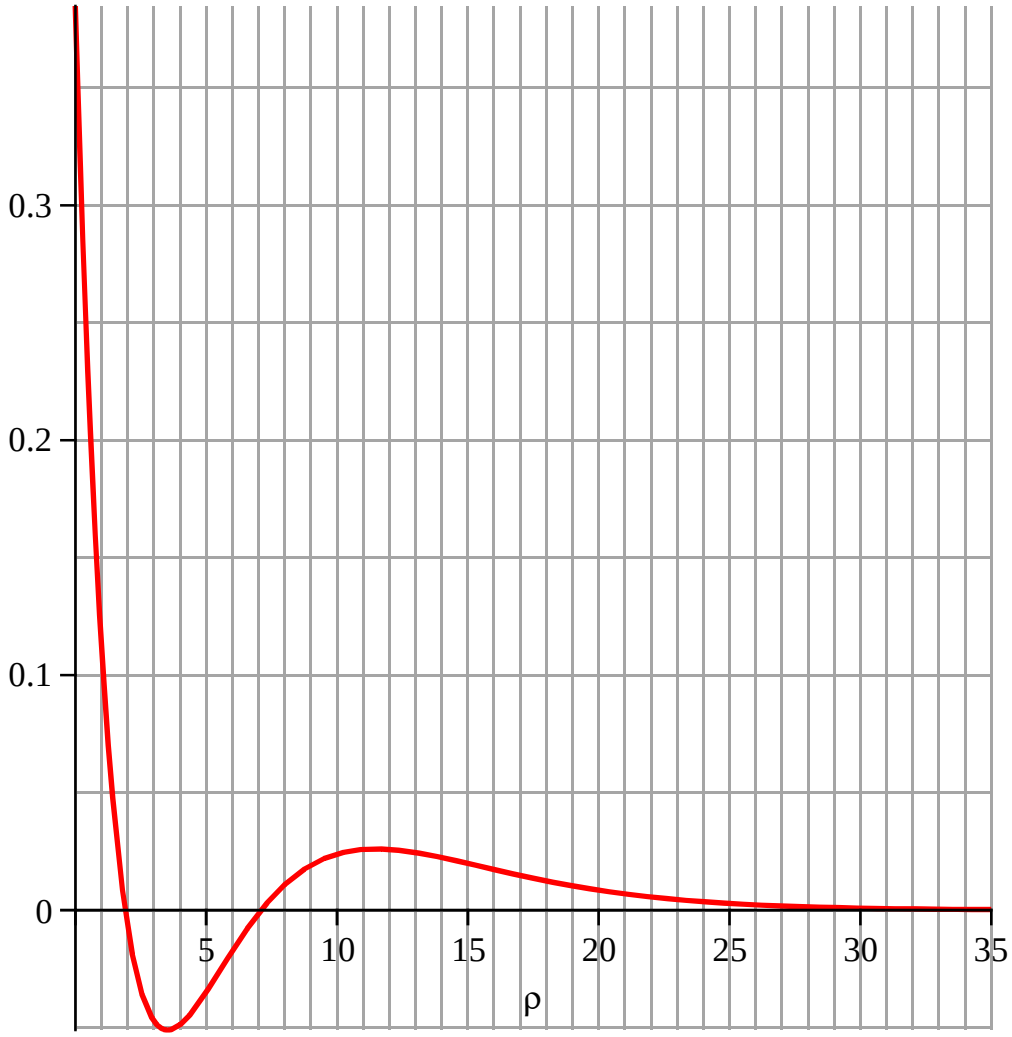
> $\rho[\mathrm{max}] := \max\left(solve\left(\frac{d}{d\rho} \left(\rho^2 (R(2,1))^2 \right) = 0 \right) \right)$;

r[max] = evalf(0.529·ρ[max]); r[max] :='r[max]': ρ[max] :='ρ[max]':

$$\rho_{max} := 4$$

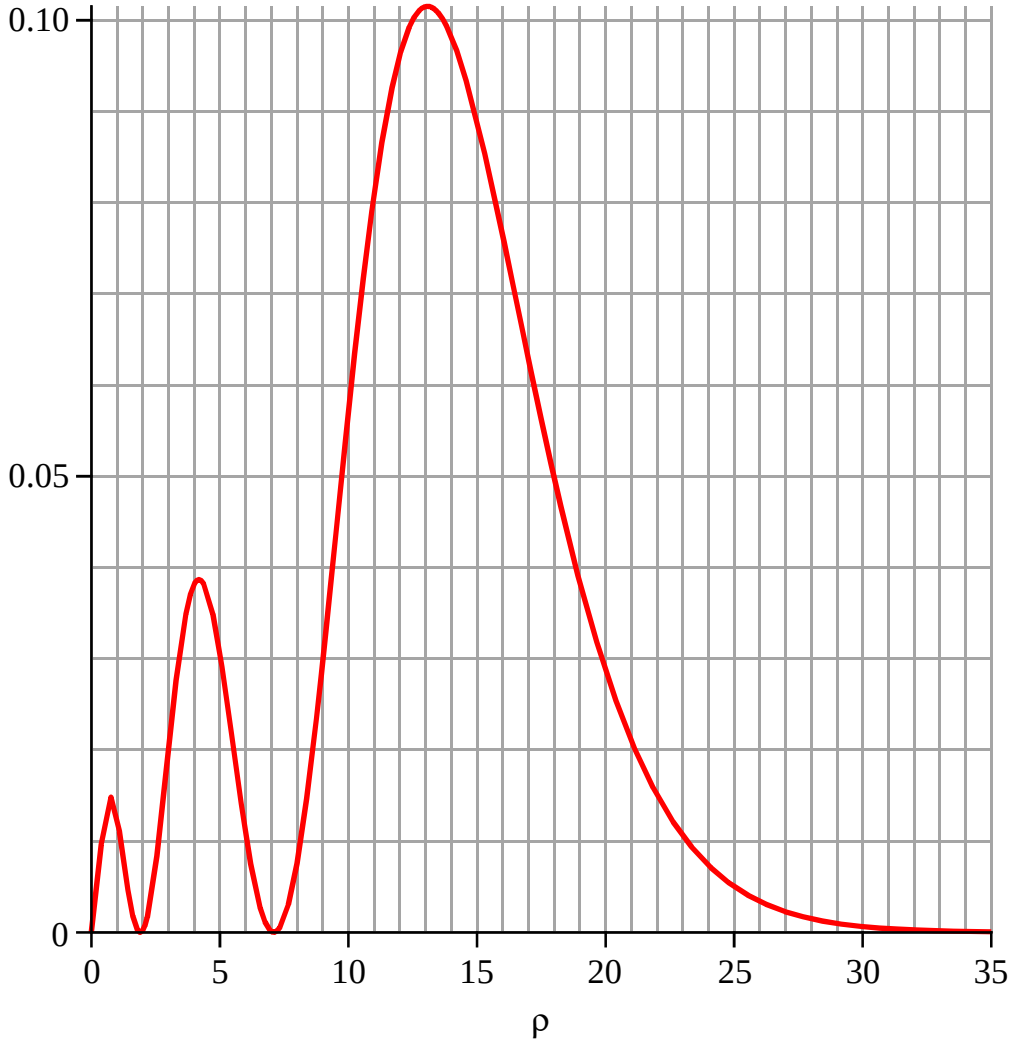
$$r_{max} = 2.116 \qquad \textbf{(9)}$$

```
> graph(3, 0);
'R[3, 0](ρ)' = combine(R(3, 0));
```



$$R_{3,\,0}(\rho) = \left(\frac{2}{9} - \frac{4}{27}\,\rho + \frac{4}{243}\,\rho^2\right)\sqrt{3}\;e^{-\frac{1}{3}\rho} \qquad \textbf{(10)}$$

```
> graphP(3, 0);
  'ρ^2·(abs(R[3, 0](ρ)))^2';
  '∫_0^∞ ρ^2 |R(3, 0)|^2 dρ '= ∫_0^∞ ρ^2 |R(3, 0)|^2 dρ;
```



$$\rho^2 \left| R_{3,\,0}(\rho) \right|^2$$

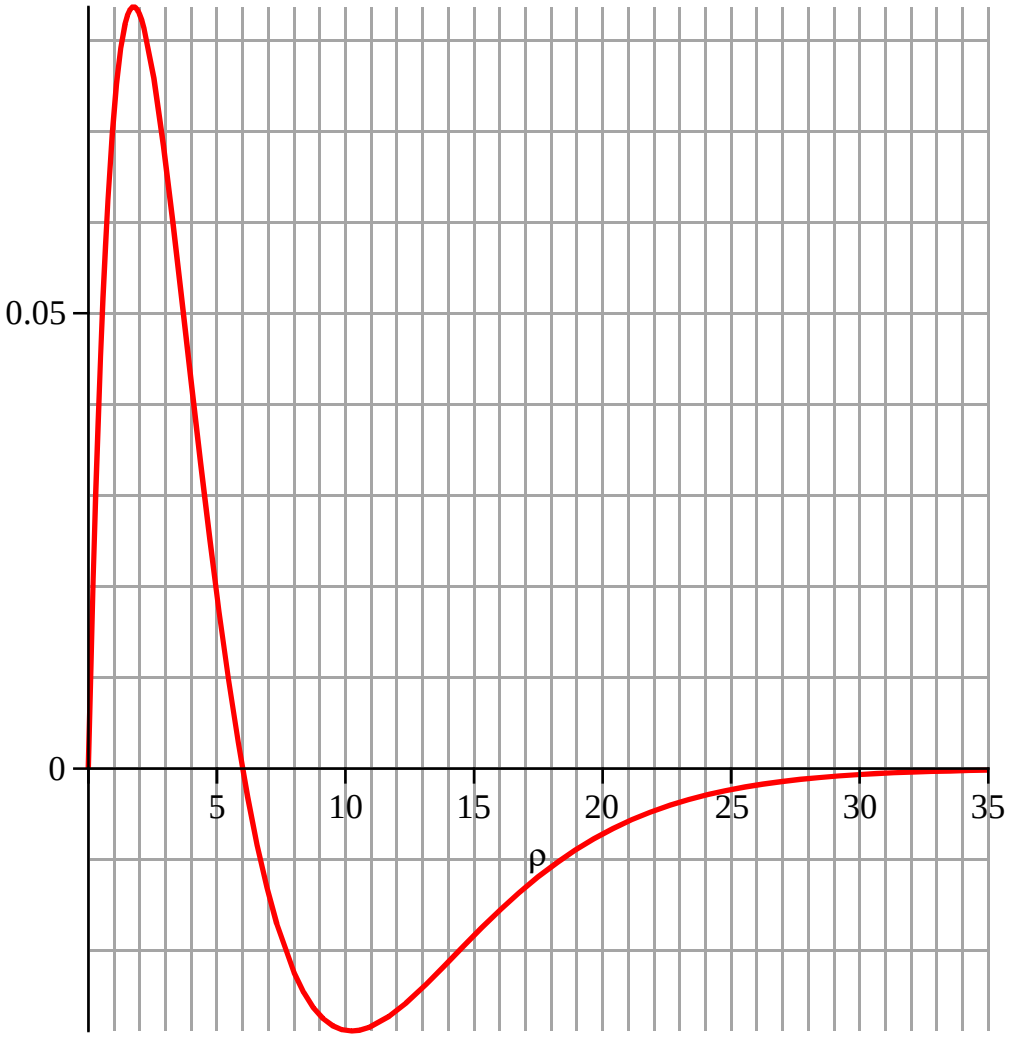
$$\int_0^\infty \rho^2 |R(3, 0)|^2 \, d\rho = 1 \qquad \textbf{(11)}$$

```
> v[1] := fsolve(d/dρ (ρ^2 (R(3, 0))^2) = 0, ρ, 0..2) : u[1] := 0.529·v[1] : ρ := 'ρ':
  v[2] := fsolve(d/dρ (ρ^2 (R(3, 0))^2) = 0, ρ, 2..7) : u[2] := 0.529·v[2] : ρ := 'ρ':
  v[3] := fsolve(d/dρ (ρ^2 (R(3, 0))^2) = 0, ρ, 7..35) : u[3] := 0.529·v[3] : ρ := 'ρ':
  ρ[max] := [v[1], v[2], v[3]]; ρ[max] := 'ρ[max]':
  r[max] := [u[1], u[2], u[3]]; r[max] := 'r[max]':
```

$$\rho_{max} := [0.740, 4.186, 13.074]$$

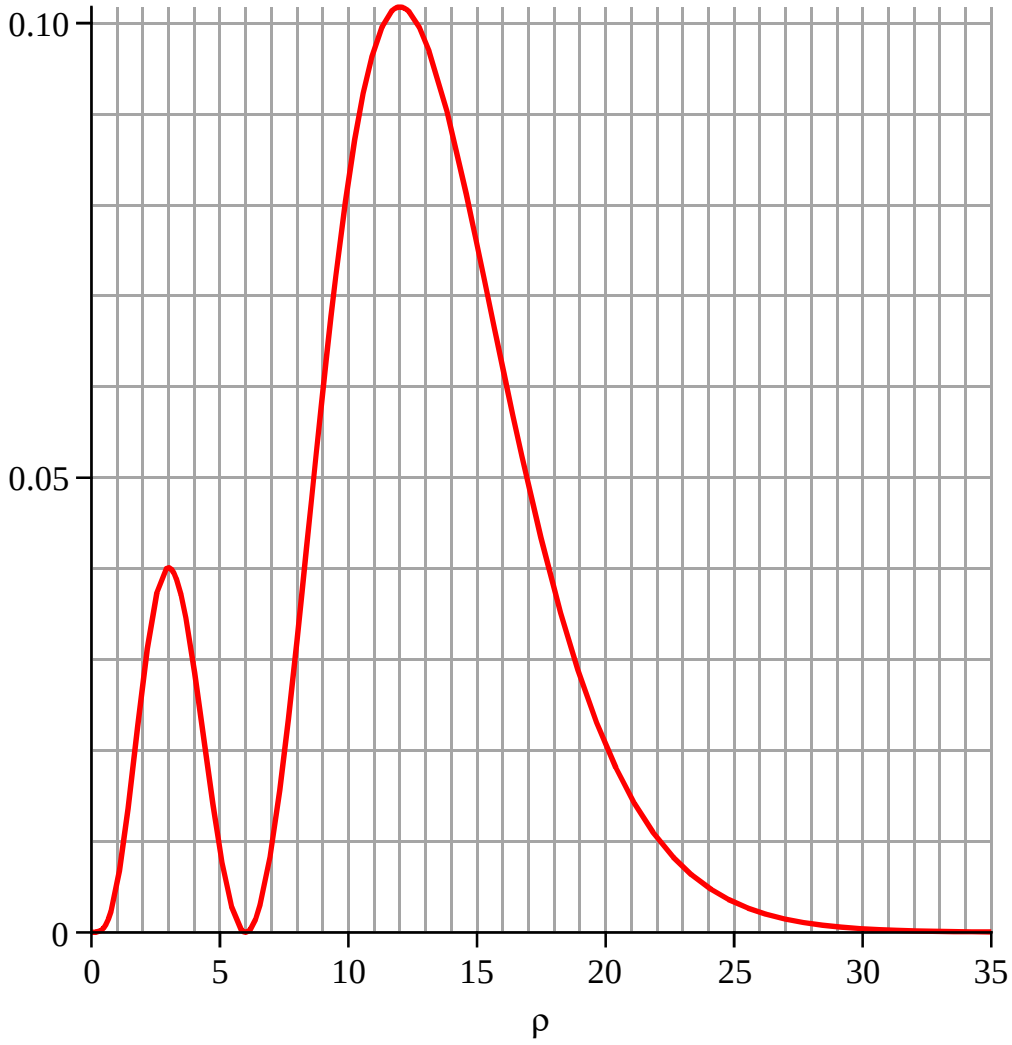
$$r_{max} := [0.391, 2.214, 6.916] \qquad \textbf{(12)}$$

```
> graph(3, 1);
'R[3, 1](ρ)' = combine(R(3, 1));
```



$$R_{3,1}(\rho) = \left(\frac{4}{81} - \frac{2}{243}\,\rho\right)\rho\sqrt{6}\,e^{-\frac{1}{3}\rho} \qquad \textbf{(13)}$$

```
> graphP(3, 1);
  'ρ^2·(abs(R[3, 1](ρ)))^2';
  '∫_0^∞ ρ^2 |R(3, 1)|^2 dρ '= ∫_0^∞ ρ^2 |R(3, 1)|^2 dρ;
```



$$\rho^2 \left| R_{3,\,1}(\rho) \right|^2$$

$$\int_0^{\infty} \rho^2 |R(3, 1)|^2 \, d\rho = 1 \qquad \textbf{(14)}$$

```
> ρ[max][1] := fsolve( d/dρ (ρ^2 (R(3, 1))^2) = 0, ρ, 0..6 ); r[max][1] := 0.529·ρ[max][1]; ρ :='ρ':
  ρ[max][2] := fsolve( d/dρ (ρ^2 (R(3, 1))^2) = 0, ρ, 6..35 ); r[max][2] := 0.529·ρ[max][2]; ρ :='ρ':
```

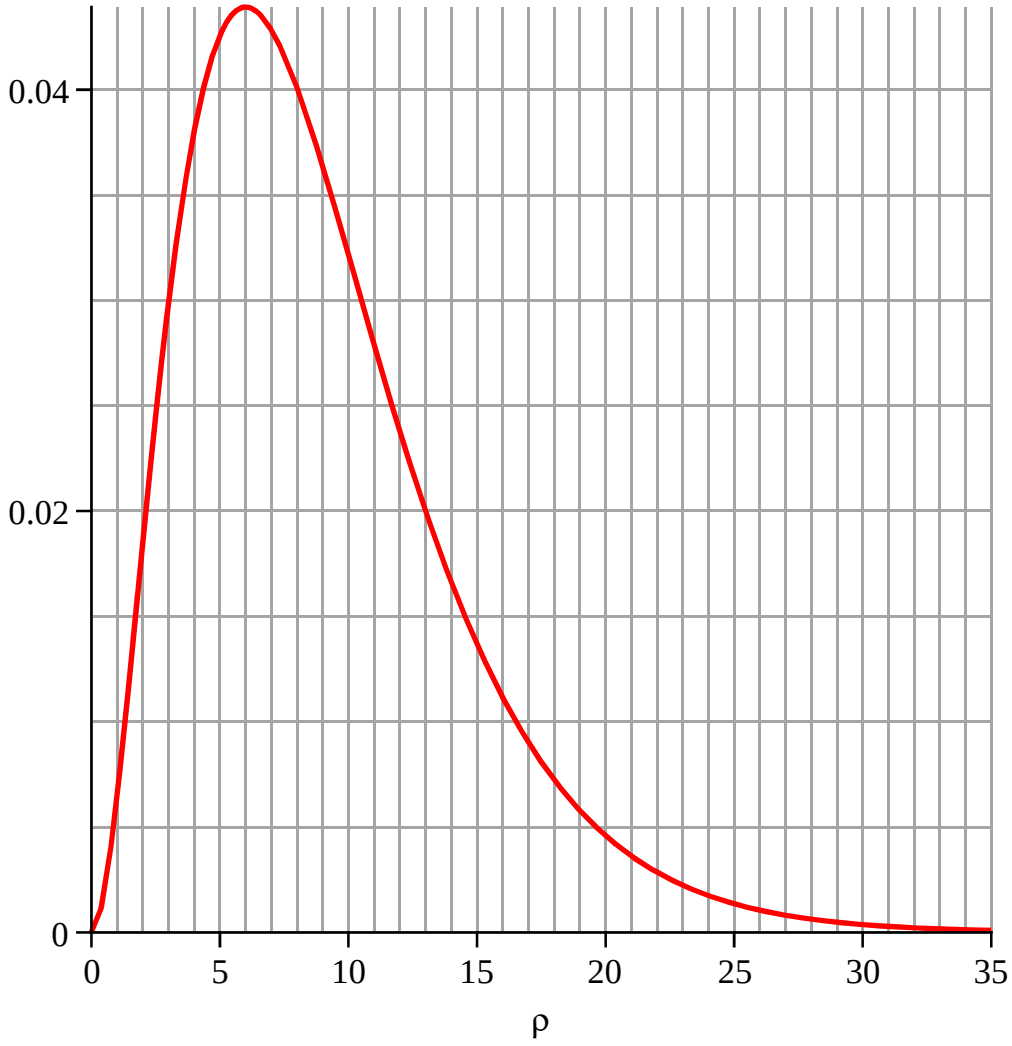
$$\rho_{max_1} := 3.000$$

$$r_{max_1} := 1.587$$

$$\rho_{max_2} := 12.000$$

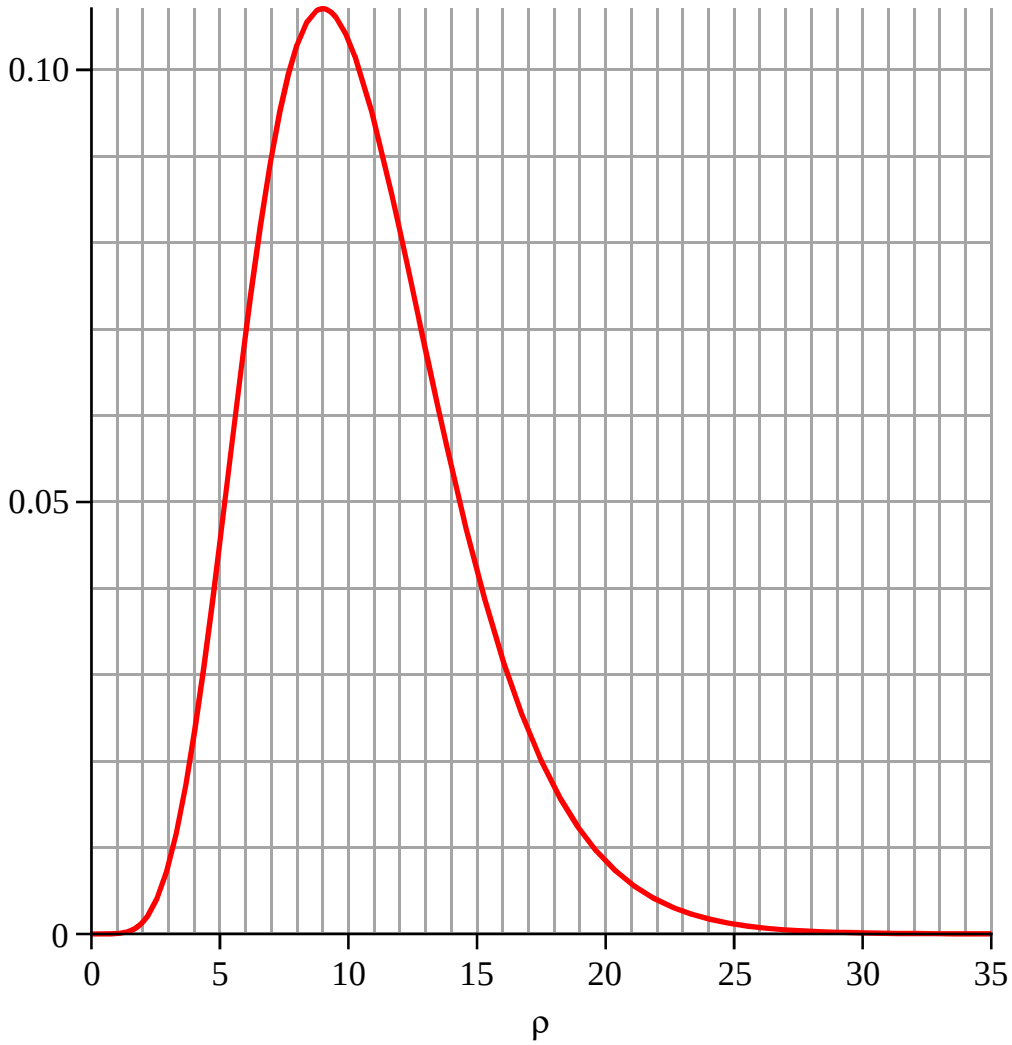
$$r_{max_2} := 6.348 \qquad \textbf{(15)}$$

```
> graph(3, 2);
'R[3, 2](ρ)' = combine(R(3, 2));
```



$$R_{3,2}(\rho) = \frac{2}{1215}\sqrt{30}\, e^{-\frac{1}{3}\rho}\, \rho^2 \qquad \textbf{(16)}$$

```
> graphP(3, 2);
'ρ^2·(abs(R[3, 2](ρ)))^2';
'∫_0^∞ ρ^2 |R(3, 2)|^2 dρ '= ∫_0^∞ ρ^2 |R(3, 2)|^2 dρ;
```



$$\rho^2 \left| R_{3,\,2}(\rho) \right|^2$$

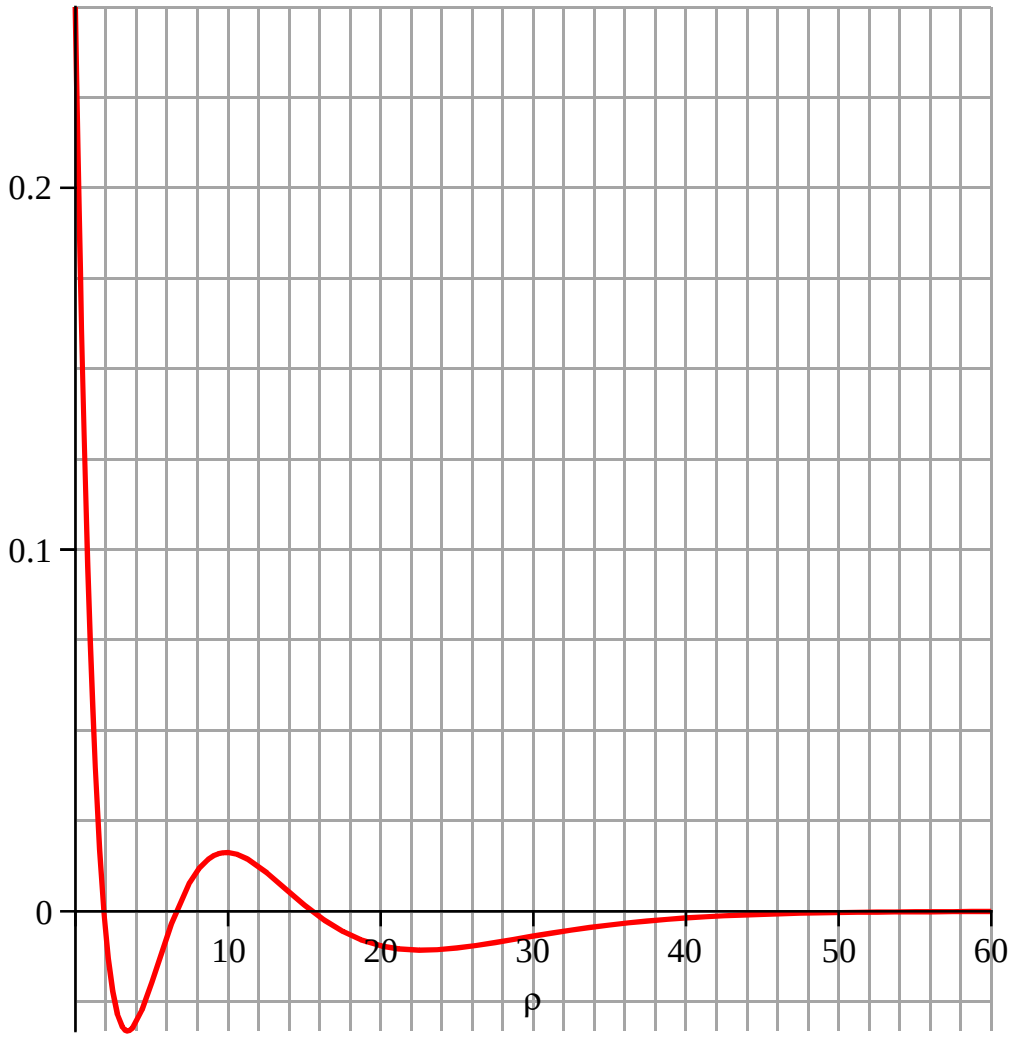
$$\int_0^{\infty} \rho^2 |R(3,2)|^2 \, d\rho = 1 \qquad \textbf{(17)}$$

```
> ρ[max] := max(solve(d/dρ (ρ^2 (R(3, 2))^2) = 0));
r[max] = evalf(0.529·ρ[max]);
```

$$\rho_{max} := 9$$

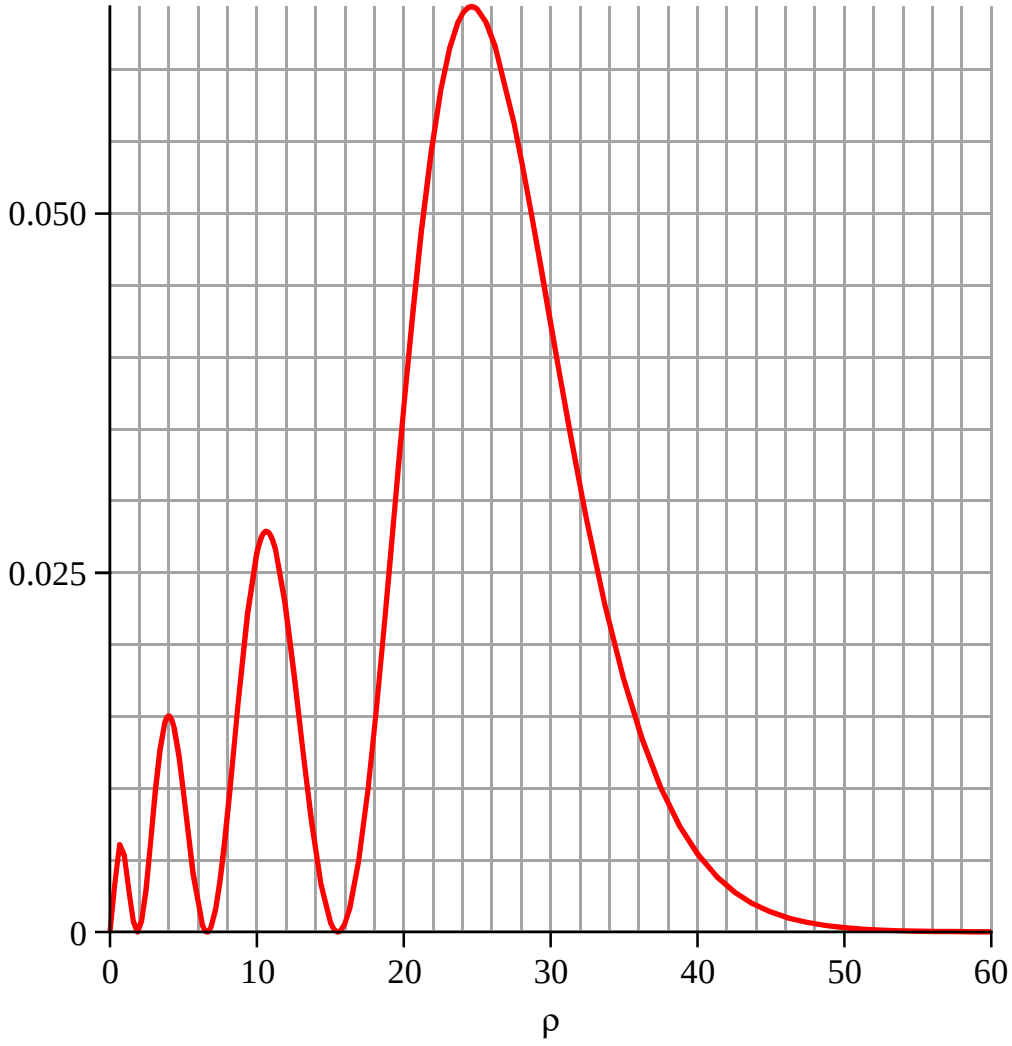
$$r_{max} = 4.761 \qquad \textbf{(18)}$$

```
> graph(4, 0);
'R[4, 0](ρ)' = combine(R(4, 0));
```



$$R_{4,0}(\rho) = \left(\frac{1}{4} - \frac{3}{16}\rho + \frac{1}{32}\rho^2 - \frac{1}{768}\rho^3\right) e^{-\frac{1}{4}\rho} \qquad \textbf{(19)}$$

> *graphP*(4, 0); 'ρ^2·(abs(R[4, 0](ρ)))^2'; '∫_0^∞ ρ^2 |R(4, 0)|^2 dρ '= ∫_0^∞ ρ^2 |R(4, 0)|^2 dρ;



$$\rho^2 \left| R_{4,\,0}(\rho) \right|^2$$

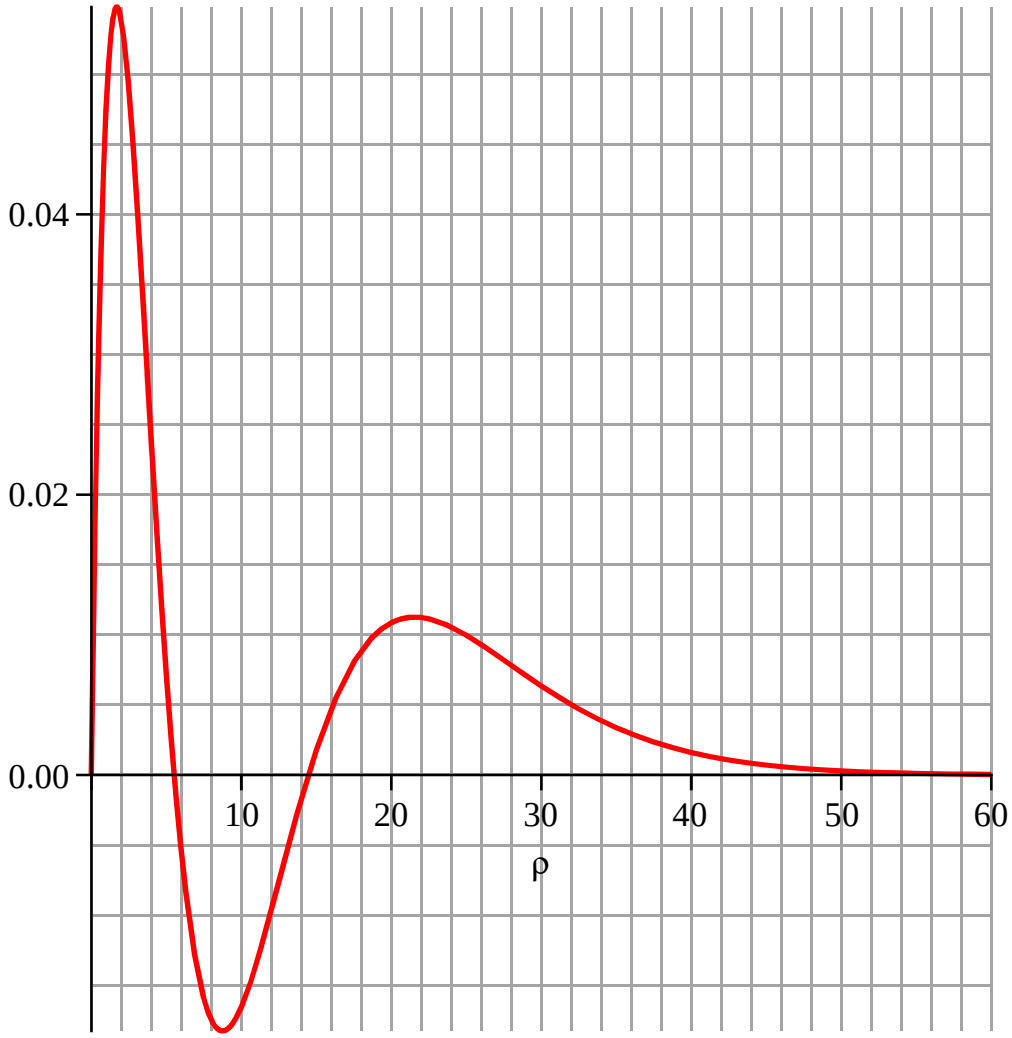
$$\int_0^\infty \rho^2 |R(4, 0)|^2 \, d\rho = 1 \qquad \textbf{(20)}$$

> v[1] := *fsolve*(d/dρ (ρ^2 (R(4, 0))^2) = 0, ρ, 0..2): u[1] := 0.529·v[1]: ρ :='ρ':
v[2] := *fsolve*(d/dρ (ρ^2 (R(4, 0))^2) = 0, ρ, 2..6): u[2] := 0.529·v[2]: ρ :='ρ':
v[3] := *fsolve*(d/dρ (ρ^2 (R(4, 0))^2) = 0, ρ, 6..16): u[3] := 0.529·v[3]: ρ :='ρ':
v[4] := *fsolve*(d/dρ (ρ^2 (R(4, 0))^2) = 0, ρ, 16..35): u[4] := 0.529·v[4]: ρ :='ρ':
ρ[max] := [v[1], v[2], v[3], *v*[4]]; ρ[max] :='ρ[max]':
r[max] := [u[1], u[2], u[3], *u*[4]]; r[max] :='r[max]':

$$\rho_{max} := [0.732, 4.000, 10.650, 24.618]$$

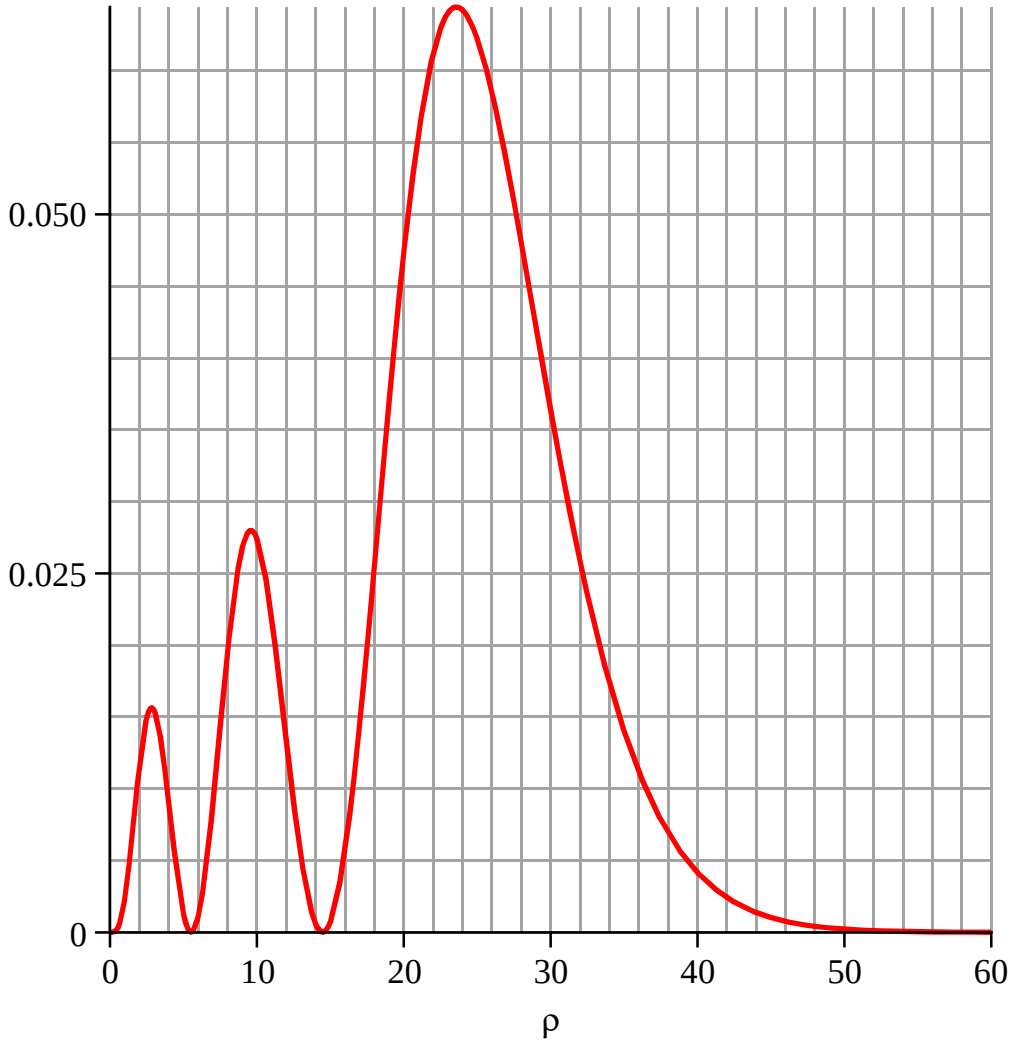
$$r_{max} := [0.387, 2.116, 5.634, 13.023] \qquad \textbf{(21)}$$

```
> graph(4, 1);
'R[4, 1](ρ)' = combine(R(4, 1));
```



$$R_{4,\,1}(\rho) = \left(\frac{1}{48} - \frac{1}{192}\,\rho + \frac{1}{3840}\,\rho^2\right)\rho\sqrt{15}\;e^{-\frac{1}{4}\rho} \qquad \textbf{(22)}$$

```
> graphP(4, 1);
'ρ^2·(abs(R[4, 1](ρ)))^2';
'∫_0^∞ ρ^2 |R(4, 1)|^2 dρ '= ∫_0^∞ ρ^2 |R(4, 1)|^2 dρ;
```



$$\rho^2 \left| R_{4,\,1}(\rho) \right|^2$$

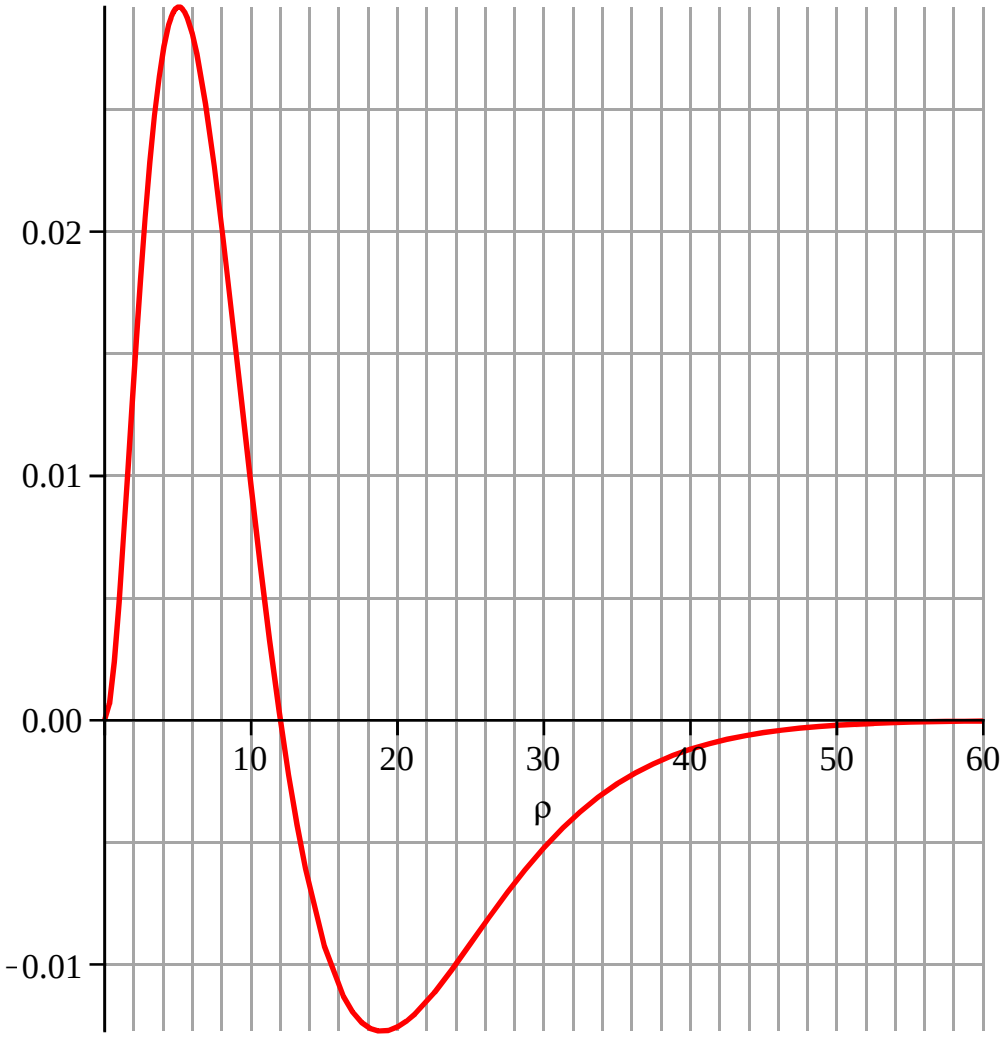
$$\int_0^\infty \rho^2 |R(4, 1)|^2 \, d\rho = 1 \qquad \textbf{(23)}$$

```
> v[1] := fsolve(d/dρ (ρ^2 (R(4, 1))^2) = 0, ρ, 0..5) : u[1] := 0.529·v[1] : ρ :='ρ':
  v[2] := fsolve(d/dρ (ρ^2 (R(4, 1))^2) = 0, ρ, 5..14) : u[2] := 0.529·v[2] : ρ :='ρ':
  v[3] := fsolve(d/dρ (ρ^2 (R(4, 1))^2) = 0, ρ, 14..50) : u[3] := 0.529·v[3] : ρ :='ρ':
  ρ[max] := [v[1], v[2], v[3]]; ρ[max] :='ρ[max]':
  r[max] := [u[1], u[2], u[3]]; r[max] :='r[max]':
```

$$\rho_{max} := [2.830, 9.589, 23.580]$$

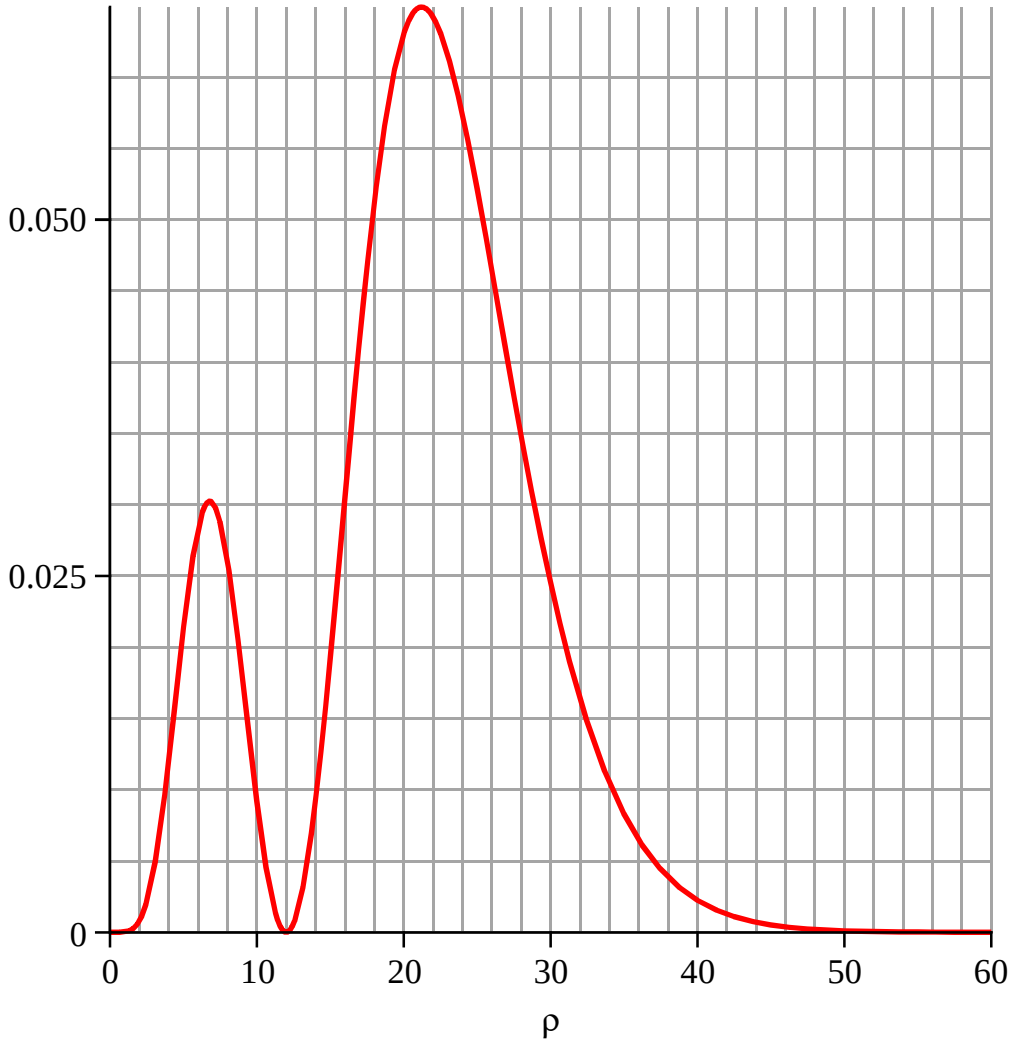
$$r_{max} := [1.497, 5.073, 12.474] \qquad \textbf{(24)}$$

```
> graph(4, 2);
'R[4, 2](ρ)' = combine(R(4, 2));
```



$$R_{4,\,2}(\rho) = \left(\frac{1}{320} - \frac{1}{3840}\,\rho\right)\rho^2\sqrt{5}\,e^{-\frac{1}{4}\rho} \qquad \textbf{(25)}$$

```
> graphP(4, 2);
  'ρ^2·(abs(R[4, 2](ρ)))^2';
  '∫_0^∞ ρ^2 |R(4, 2)|^2 dρ '= ∫_0^∞ ρ^2 |R(4, 2)|^2 dρ;
```



$$\rho^2 \left| R_{4,\,2}(\rho) \right|^2$$

$$\int_0^\infty \rho^2 \, |R(4, 2)|^2 \, d\rho = 1 \qquad \textbf{(26)}$$

```
> ρ[max][1] := fsolve( d/dρ (ρ^2 (R(4, 2))^2) = 0, ρ, 0..12 ); r[max][1] := 0.529·ρ[max][1]; ρ :='ρ':
  ρ[max][2] := fsolve( d/dρ (ρ^2 (R(4, 2))^2) = 0, ρ, 12..50 ); r[max][2] := 0.529·ρ[max][2]; ρ :='ρ':
```

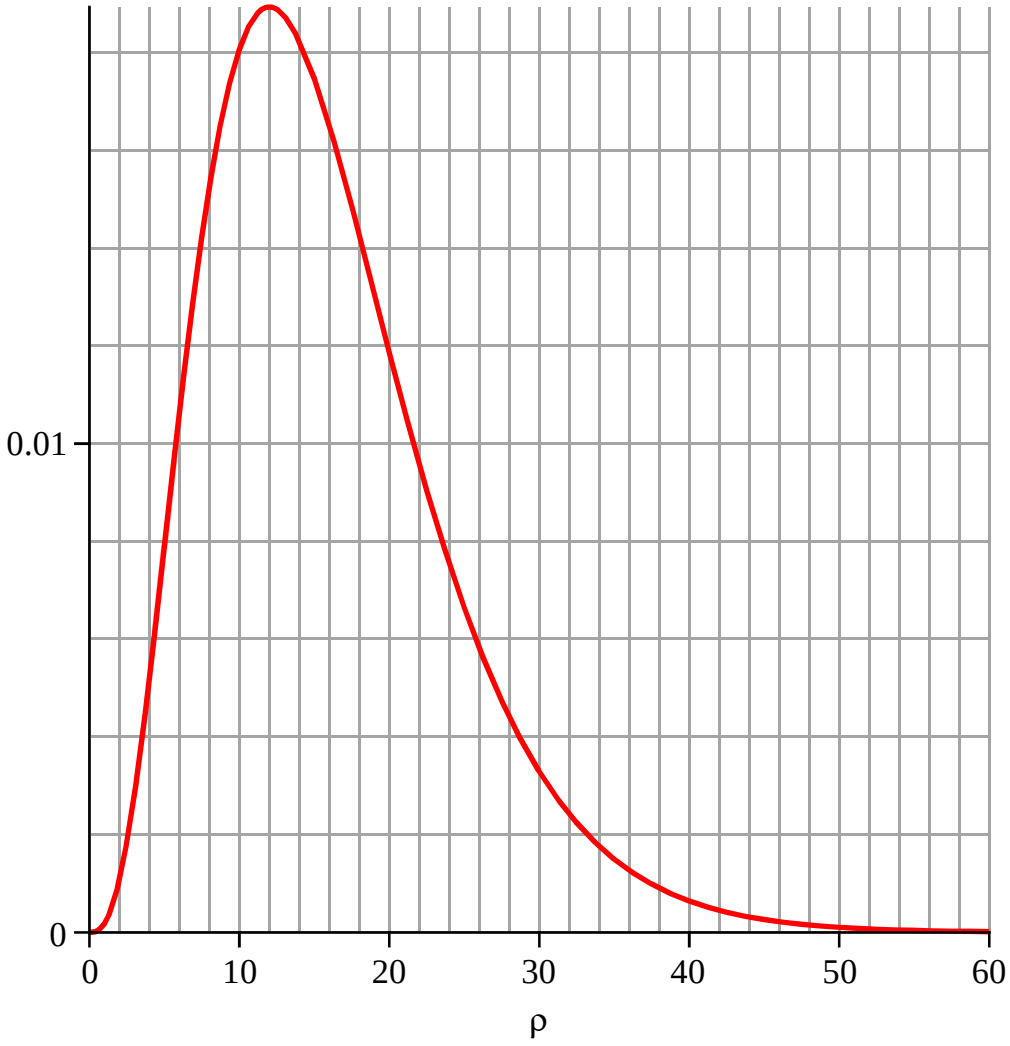
$$\rho_{max_1} := 6.789$$

$$r_{max_1} := 3.591$$

$$\rho_{max_2} := 21.211$$

$$r_{max_2} := 11.221 \qquad \textbf{(27)}$$

```
> graph(4, 3);
'R[4, 3](ρ)' = combine(R(4, 3));
```

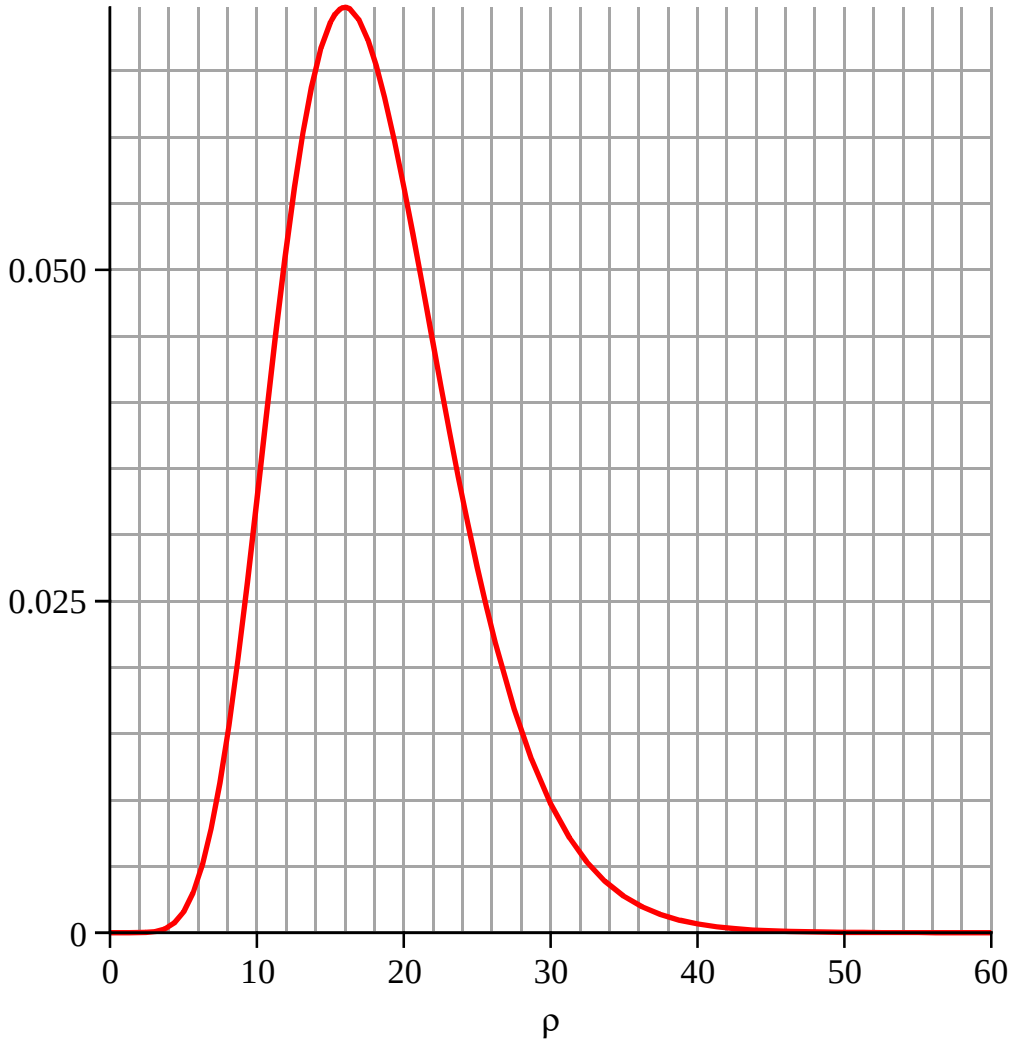


$$R_{4,3}(\rho) = \frac{1}{26880}\sqrt{35}\, e^{-\frac{1}{4}\rho}\, \rho^{3} \qquad \textbf{(28)}$$

> *graphP*(4, 3);

'$\rho^2 \cdot (\mathrm{abs}(R[4,3](\rho)))^2$';

'$\int_0^\infty \rho^2 |R(4,3)|^2 \, d\rho$ '= $\int_0^\infty \rho^2 |R(4,3)|^2 \, d\rho$;



$$\rho^2 \left| R_{4,3}(\rho) \right|^2$$

$$\int_0^\infty \rho^2 |R(4,3)|^2 \, d\rho = 1 \qquad \textbf{(29)}$$

> $\rho[\max] := \max\left(solve\left(\frac{d}{d\rho} \left(\rho^2 (R(4,3))^2 \right) = 0 \right) \right)$;

$r[\max] = \mathrm{evalf}(0.529 \cdot \rho[\max])$;

$$\rho_{max} := 16$$

$$r_{max} = 8.464 \qquad \textbf{(30)}$$

>