```
> restart;
> with(LinearAlgebra) :
```

Reduced Rotation Matrix Elements - a Maple implementation. Maple V12

These two procedures are derived from the C program described by William J. Thompson.
William J. Thompson, Angular Momentum: an illustrated guide
to rotational symmetries for physical systems,
John Wiley and Sons Inc., 1994

The procedure RRM(j,a)

j: half-integrals; i.e., $\frac{1}{2}$, 1, $\frac{3}{2}$, ...

a: angle in radians; i.e., $\frac{\pi}{2}$, π, $\frac{\pi}{4}$, ...

> $djmpm := \textbf{proc}(\,j, mp,\ m, \beta)$

$\quad \textbf{local}\ norm,\ s, minmax,$
$\quad\quad xmin, xmax, x;$

$$norm := \sqrt{(j+mp)!(j-mp)!(j+m)!(j-m)!}\ ;$$

$minmax := [0, mp - m];$
$xmin := \max(minmax);$
$minmax := [j + mp, j - m];$
$xmax := \min(minmax);$

$$s := \sum_{x=xmin}^{xmax} \frac{(-1)^x \cdot \cos(\beta/2)^{(2j+mp-m-2x)} \cdot \sin(\beta/2)^{(2x+m-mp)}}{x!(j+mp-x)!(j-m-x)!(x+m-mp)!};$$

$\textbf{return}\ simplify(norm \cdot s, trig);$
$\textbf{end proc}:$

> $RRM := \textbf{proc}(j, \mathrm{a})$ # *the angle*

$\quad \textbf{local}\ mp, m, r, c, A, B;$

printf ("Angular Momentum\n");
printf ("Reduced Rotation Matrix Elements\n");

if $(j < 0)$ **then**
printf ("\nEnd of Reduced Rotation Matrix Elements: j < 0\n");
return;
end if;

$A := Matrix(2\cdot j + 1);\ B := Matrix(2\cdot j + 1):$

print(*The Matrix elements djm'm(β) are:*);
$r := 0:$
for *mp* **from** *j* **to** $-j$ **by** -1 **do**
$r := r + 1;\ c := 0;$

```
        for m from j to -j by -1 do
            c := c + 1 :
            A[r, c] := djmpm(j, mp, m, β);
            print(d(j, mp, m, β) = A[r, c]);
            B[r, c] := evalf(djmpm(j, mp, m, a));
        end do;
    end do;
    print(`in matrix form;`)
    print(djm'm(β) = Matrix(A));
    print(when β is , a);
    print(`djm'm(β) = Matrix(B));
end proc:
```

Examples:

> $RRM\left(\frac{1}{2}, \frac{\pi}{4}\right);$

Angular Momentum

Reduced Rotation Matrix Elements

The Matrix elements djm'm(β) are:

$$d\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \beta\right) = \cos\left(\frac{1}{2}\beta\right)$$

$$d\left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \beta\right) = -\sin\left(\frac{1}{2}\beta\right)$$

$$d\left(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \beta\right) = \sin\left(\frac{1}{2}\beta\right)$$

$$d\left(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \beta\right) = \cos\left(\frac{1}{2}\beta\right)$$

in matrix form;

$$djm'm(\beta) = \begin{bmatrix} \cos\left(\frac{1}{2}\beta\right) & -\sin\left(\frac{1}{2}\beta\right) \\ \sin\left(\frac{1}{2}\beta\right) & \cos\left(\frac{1}{2}\beta\right) \end{bmatrix}$$

$$when\ \beta\ is,\ \frac{1}{4}\pi$$

$$`djm'm(\beta) = \begin{bmatrix} 0.9238795325 & -0.3826834325 \\ 0.3826834325 & 0.9238795325 \end{bmatrix} \quad \textbf{(1)}$$

$> RRM\left(1, \frac{\pi}{4}\right);$

```
Angular Momentum
Reduced Rotation Matrix Elements
```

The Matrix elements djm'm(β) are:

$$d(1, 1, 1, \beta) = \cos\left(\frac{1}{2}\beta\right)^2$$

$$d(1, 1, 0, \beta) = -\sqrt{2}\cos\left(\frac{1}{2}\beta\right)\sin\left(\frac{1}{2}\beta\right)$$

$$d(1, 1, -1, \beta) = \sin\left(\frac{1}{2}\beta\right)^2$$

$$d(1, 0, 1, \beta) = \sqrt{2}\cos\left(\frac{1}{2}\beta\right)\sin\left(\frac{1}{2}\beta\right)$$

$$d(1, 0, 0, \beta) = 2\cos\left(\frac{1}{2}\beta\right)^2 - 1$$

$$d(1, 0, -1, \beta) = -\sqrt{2}\cos\left(\frac{1}{2}\beta\right)\sin\left(\frac{1}{2}\beta\right)$$

$$d(1, -1, 1, \beta) = \sin\left(\frac{1}{2}\beta\right)^2$$

$$d(1, -1, 0, \beta) = \sqrt{2}\cos\left(\frac{1}{2}\beta\right)\sin\left(\frac{1}{2}\beta\right)$$

$$d(1, -1, -1, \beta) = \cos\left(\frac{1}{2}\beta\right)^2$$

in matrix form;

$$djm'm(\beta) = \begin{bmatrix} \cos\left(\frac{1}{2}\beta\right)^2 & -\sqrt{2}\cos\left(\frac{1}{2}\beta\right)\sin\left(\frac{1}{2}\beta\right) & \sin\left(\frac{1}{2}\beta\right)^2 \\ \sqrt{2}\cos\left(\frac{1}{2}\beta\right)\sin\left(\frac{1}{2}\beta\right) & 2\cos\left(\frac{1}{2}\beta\right)^2 - 1 & -\sqrt{2}\cos\left(\frac{1}{2}\beta\right)\sin\left(\frac{1}{2}\beta\right) \\ \sin\left(\frac{1}{2}\beta\right)^2 & \sqrt{2}\cos\left(\frac{1}{2}\beta\right)\sin\left(\frac{1}{2}\beta\right) & \cos\left(\frac{1}{2}\beta\right)^2 \end{bmatrix}$$

when β *is,* $\frac{1}{4}\pi$

$$`djm'm(\beta) = \begin{bmatrix} 0.8535533905 & -0.5000000000 & 0.1464466095 \\ 0.5000000000 & 0.7071067810 & -0.5000000000 \\ 0.1464466095 & 0.5000000000 & 0.8535533905 \end{bmatrix} \quad \textbf{(2)}$$

> $RRM\left(\frac{3}{2}, \frac{\pi}{4}\right);$

```
Angular Momentum
Reduced Rotation Matrix Elements
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The Matrix elements djm'm(β) are:

$$d\left(\frac{3}{2}, \frac{3}{2}, \frac{3}{2}, \beta\right) = \cos\left(\frac{1}{2}\beta\right)^3$$

$$d\left(\frac{3}{2}, \frac{3}{2}, \frac{1}{2}, \beta\right) = -\sqrt{3}\cos\left(\frac{1}{2}\beta\right)^2\sin\left(\frac{1}{2}\beta\right)$$

$$d\left(\frac{3}{2}, \frac{3}{2}, -\frac{1}{2}, \beta\right) = \sqrt{3}\cos\left(\frac{1}{2}\beta\right)\sin\left(\frac{1}{2}\beta\right)^2$$

$$d\left(\frac{3}{2}, \frac{3}{2}, -\frac{3}{2}, \beta\right) = -\sin\left(\frac{1}{2}\beta\right)^3$$

$$d\left(\frac{3}{2}, \frac{1}{2}, \frac{3}{2}, \beta\right) = \sqrt{3}\cos\left(\frac{1}{2}\beta\right)^2\sin\left(\frac{1}{2}\beta\right)$$

$$d\left(\frac{3}{2}, \frac{1}{2}, \frac{1}{2}, \beta\right) = \left(3\cos\left(\frac{1}{2}\beta\right)^2 - 2\right)\cos\left(\frac{1}{2}\beta\right)$$

$$d\left(\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, \beta\right) = -\left(3\cos\left(\frac{1}{2}\beta\right)^2 - 1\right)\sin\left(\frac{1}{2}\beta\right)$$

$$d\left(\frac{3}{2}, \frac{1}{2}, -\frac{3}{2}, \beta\right) = \sqrt{3}\cos\left(\frac{1}{2}\beta\right)\sin\left(\frac{1}{2}\beta\right)^2$$

$$d\left(\frac{3}{2}, -\frac{1}{2}, \frac{3}{2}, \beta\right) = \sqrt{3}\cos\left(\frac{1}{2}\beta\right)\sin\left(\frac{1}{2}\beta\right)^2$$

$$d\left(\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \beta\right) = \left(3\cos\left(\frac{1}{2}\beta\right)^2 - 1\right)\sin\left(\frac{1}{2}\beta\right)$$

$$d\left(\frac{3}{2}, -\frac{1}{2}, -\frac{1}{2}, \beta\right) = \left(3\cos\left(\frac{1}{2}\beta\right)^2 - 2\right)\cos\left(\frac{1}{2}\beta\right)$$

$$d\left(\frac{3}{2}, -\frac{1}{2}, -\frac{3}{2}, \beta\right) = -\sqrt{3}\cos\left(\frac{1}{2}\beta\right)^2\sin\left(\frac{1}{2}\beta\right)$$

$$d\left(\frac{3}{2}, -\frac{3}{2}, \frac{3}{2}, \beta\right) = \sin\left(\frac{1}{2}\beta\right)^3$$

$$d\left(\frac{3}{2}, -\frac{3}{2}, \frac{1}{2}, \beta\right) = \sqrt{3}\cos\left(\frac{1}{2}\beta\right)\sin\left(\frac{1}{2}\beta\right)^2$$

$$d\left(\frac{3}{2}, -\frac{3}{2}, -\frac{1}{2}, \beta\right) = \sqrt{3}\cos\left(\frac{1}{2}\beta\right)^2\sin\left(\frac{1}{2}\beta\right)$$

$$d\left(\frac{3}{2}, -\frac{3}{2}, -\frac{3}{2}, \beta\right) = \cos\left(\frac{1}{2}\beta\right)^3$$

in matrix form;

$$djm'm(\beta) = \begin{bmatrix} \cos\left(\frac{1}{2}\beta\right)^3 & -\sqrt{3}\cos\left(\frac{1}{2}\beta\right)^2\sin\left(\frac{1}{2}\beta\right) & \sqrt{3}\cos\left(\frac{1}{2}\beta\right)\sin\left(\frac{1}{2}\beta\right)^2 & -\sin\left(\frac{1}{2}\beta\right)^3 \\ \sqrt{3}\cos\left(\frac{1}{2}\beta\right)^2\sin\left(\frac{1}{2}\beta\right) & \left(3\cos\left(\frac{1}{2}\beta\right)^2-2\right)\cos\left(\frac{1}{2}\beta\right) & -\left(3\cos\left(\frac{1}{2}\beta\right)^2-1\right)\sin\left(\frac{1}{2}\beta\right) & \sqrt{3}\cos\left(\frac{1}{2}\beta\right)\sin\left(\frac{1}{2}\beta\right)^2 \\ \sqrt{3}\cos\left(\frac{1}{2}\beta\right)\sin\left(\frac{1}{2}\beta\right)^2 & \left(3\cos\left(\frac{1}{2}\beta\right)^2-1\right)\sin\left(\frac{1}{2}\beta\right) & \left(3\cos\left(\frac{1}{2}\beta\right)^2-2\right)\cos\left(\frac{1}{2}\beta\right) & -\sqrt{3}\cos\left(\frac{1}{2}\beta\right)^2\sin\left(\frac{1}{2}\beta\right) \\ \sin\left(\frac{1}{2}\beta\right)^3 & \sqrt{3}\cos\left(\frac{1}{2}\beta\right)\sin\left(\frac{1}{2}\beta\right)^2 & \sqrt{3}\cos\left(\frac{1}{2}\beta\right)^2\sin\left(\frac{1}{2}\beta\right) & \cos\left(\frac{1}{2}\beta\right)^3 \end{bmatrix}$$

when β *is,* $\frac{1}{4}\pi$

$$`djm'm(\beta) = \begin{bmatrix} 0.7885805075 & -0.5657583600 & 0.2343447858 & -0.0560426913 \\ 0.5657583600 & 0.5179824572 & -0.5972387915 & 0.2343447858 \\ 0.2343447858 & 0.5972387915 & 0.5179824572 & -0.5657583600 \\ 0.0560426913 & 0.2343447858 & 0.5657583600 & 0.7885805075 \end{bmatrix} \qquad (3$$

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