```
> restart;
>
```

Wigner 6j coefficients

A Maple implementation. Maple V12

These procedures are derived from the C program described by William J. Thompson.
William J. Thompson, Angular Momentum: An Illustrated Guide
to Rotational Symmetries for Physical Systems,
John Wiley and Sons Inc., 1994

The Racah W coefficient : $\langle \mathbf{j1, j2, j3, j', J, M} | \mathbf{j1, j2, j3, j'', JM} \rangle = \sqrt{(2\,\mathbf{j'}+1)(2\,\mathbf{j''}+1)}\,\mathbf{W}(\mathbf{j1, j2, J, j3, j', j''})$

The Wigner 6j symbol: $\begin{bmatrix} j1 & j2 & j' \\ j3 & \mathrm{J} & j'' \end{bmatrix} = (-1)^{\mathrm{j1}+\mathrm{j2}+\mathrm{j3}+\mathrm{J}}\,\mathbf{W}(\mathbf{j1, j2, J, j3, j', j''})$

where $(-1)^{\mathrm{j1}+\mathrm{j2}+\mathrm{j3}+\mathrm{J}}\,\mathbf{W}(\mathbf{j1, j2, J, j3, j', j''})$ **is implemented in procedure E6j**( )

```
> TriangleBroken := proc( x, y, z)
                      local sum;
                      sum := trunc(2·( x + y + z)); # print(sum) ;
                      if ((type(sum, odd)) or (y < abs(x − z)) or (y > x + z)) then
                         printf("\n!! (%a,%a,%a) breaks triangle rule: try again\n", x, y, z);
                        return 1;
                     else return 0 end if;
                   end proc:
```

```
> Δ := proc(a, b, c)

        return ( sqrt( (a + b − c)!(a + c − b)!(b + c − a)! / (a + b + c + 1)! ) );

     end proc:
```

```
> E6j := proc(a1, a2, a3, a4, a5, a6)
          local maxl, minl, n, s,
                k, kmax, kmin, p,
                t0, t1, t2, t3, t4, t5, t6;

   #  Normalization

       n := Δ(a1, a2, a5)·Δ(a1, a3, a6)·Δ(a2, a4, a6)·Δ(a3, a4, a5);   #print (Norm = n);

   #  Minimum summation index kmin

       minl := [ 0, a1 + a4 − a5 − a6, a2 + a3 − a5 − a6];      # print("minl = ", minl);
      kmin := max(minl);              #   print("kmin ", kmin);
```

```
#   Maximun summation index kmax

    maxl := [a1 + a2 + a3 + a4 + 1, a1 + a2 − a5, a3 + a4 − a5, a1 + a3 − a6, a2 + a4 − a6];
  #  print("maxl = ",  maxl);
   kmax := min(maxl[ ]);            #       print("kmax ", kmax);

# The sum

    p := a1 + a2 + a3 + a4;
    t0 := a1 + a2 + a3 + a4 + 1;
    t1 := a5 + a6 − a1 − a4;
    t2 := a5 + a6 − a2 − a3;
    t3 := a1 + a2 − a5;
    t4 := a3 + a4 − a5;
    t5 := a1 + a3 − a6;
    t6 := a2 + a4 − a6;
```

$$s := \sum_{k=kmin}^{kmax} \frac{(-1)^{p+k} \cdot (\mathrm{t0} - \mathrm{k})!}{k!\,(t1+k)! \cdot (t2+k)! \cdot (t3-k)! \cdot (\mathrm{t4}-k)! \cdot (t5-k)!\,(\mathrm{t6}-k)!};$$

```
  return  simplify(n·s);        #    W(j1, j2, J, j3, j', j")

end proc:

> W6j :=proc(j1, j2, j3, j4, j5, j6)
            local  Coeff, S6j;

            printf("\nAngular Momentum: Wigner 6j coefficient \n");

           if (j1 < 0) then
              printf("\nEnd 6j coupling coefficients\n");
              return 0;
           end if;

           if ((TriangleBroken( j1, j2, j5) > 0) or (TriangleBroken( j1, j3, j6) > 0) or
              (TriangleBroken( j2, j4, j6) > 0) or (TriangleBroken( j3, j4, j5) > 0 )) then
                printf("\n"); ;
           else
                Coeff := (E6j( j1, j2, j3, j4, j5, j6));
                S6j := Matrix([[ j1, j2, j5], [ j4, j3, j6]]);
                 print('6 j  coefficient ',  S6j = Coeff);
                 printf("                                              ");
                 printf("                           = %6f \n\n", simplify(Coeff) );
           end if;
       end proc:
```

Examples using the following coupling scheme:

j1 + j2 = j12   j2 + j3 = j23
j12 + j3 = J   j1 + j23 = J

where j1 = j2 = j3 = 1/2

W6j(j1,j2,J,j3,j12,j23) = $W6j\left(\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}, 1, 1\right)$

> $W6j\left(\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}, 1, 1\right);$

```
Angular Momentum: Wigner 6j coefficient
```

$$6\,j\ coefficient, \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ \frac{1}{2} & \frac{3}{2} & 1 \end{bmatrix} = -\frac{1}{3}$$

```
= -0.333333
```

where j1 = j2 = 1/2, and j3 = 1

> $W6j\left(\frac{1}{2}, \frac{1}{2}, 2, 1, 1, \frac{3}{2}\right);$

```
Angular Momentum: Wigner 6j coefficient
```

$$6\,j\ coefficient, \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ 1 & 2 & \frac{3}{2} \end{bmatrix} = \frac{1}{6}\sqrt{3}$$

```
= 0.288675
```

>