```
> restart;
>
```

Clebsch-Gordan coefficients

A Maple implementation. Maple V12

These procedures are derived from the C program described by William J. Thompson.
William J. Thompson, Angular Momentum: An Illustrated Guide
to Rotational Symmetries for Physical Systems,
John Wiley and Sons Inc., 1994

The 3j Wigner coefficients.

$$\begin{bmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{bmatrix} = (-1)^{j_1 - j_2 - m_3} \cdot \mathrm{N} \cdot S$$

$$\mathrm{N} = \sqrt{\frac{(j3+j1-j2)!\,(j3-j1+j2)!\,(j1+j2-j3)!\,(j3-m3)!\,(j3+m3)!}{(j1+j2+j3+1)!\,(j1-m1)!\,(j1+m1)!\,(j2-m2)!\,(j2+m2)!}}$$

$$\mathrm{S} = \sum_{kmin}^{kmax} \frac{(-1)^{(kmin+j2+m2)}\,(j2+j3+m1-k)!\,(j1-m1+k)!}{(k)!\,(j3-j1+j2-k)!\,(j3-m3-k)!\,(k+j1-j2+m3)!}$$

where
kmin = $maximun(0, j2-j1-m3)$, **and** kmax = $minimum(j3-j1+j2, j3-m3)$

The Clebsch-Gordan Coefficients from Wigner 3j Coefficients

$$\langle j_1,j_2;m_1,m_2|j_1,j_2;j_3,m_3\rangle = (-1)^{j_1 - j_2 + m_3}\sqrt{2j_3+1}\begin{bmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & -m_3 \end{bmatrix}$$

******* Note: We are using the following convention *******

$$[\pm \mathrm{N}] \equiv \pm\sqrt{N}$$

```
> TriangleBroken := proc( j1, j2, j3)
      local sum;
      sum := 2·( j1 + j2 + j3);
      if ((type(sum, odd)) or (j2 < abs(j1 - j3)) or (j2 > j1 + j3)) then
        return 1;
      else return 0 end if;
    end proc:
```

```
> EvenOrOdd := proc( j, m)
        local sum;
        sum := 2·( j + m);   # print(sum);
        if type( sum, even) then return 0;
        else return 1 end if;
      end proc:
```

This procedure returns a signed squared coefficient.

$$\pm a\sqrt{n} = \pm a^2 n = \pm \mathbf{N}$$

```
> SQ := proc(n)
      local sign, c;
      if (evalf(n) < 0) then sign := -1 else sign := 1 end if;
      c := simplify( (n·n) ·sign);
      return (c);
   end proc:
```

```
> CG := proc(j1, j2, j3, m1, m2, M)
      local k, kmin, kmax, m3, N, S, WC, CG;
      m3 := -1·M;
     if (j1 < 0) then
         printf("\nEnd 3j - CG coefficient\n");
         return 0;
    end if;
    if (m1 + m2 + m3 ≠ 0) then
       printf("\n!! m1+m2+m3 not zero; try again\n\n");
    else
         if (TriangleBroken(j1, j2, j3) ≠ 0) then
           printf("\n!! (%a,%a,%a) breaks triangle rule: try again\n\n", j1, j2, j3);
         else
             if (abs(m1) > j1 or abs(m2) > j2 or abs(m3) > j3) then
              printf("\n!! An m values is too big; try again\n\n");
             else
                 if (EvenOrOdd(j1, m1) + EvenOrOdd(j2, m2) + EvenOrOdd(j3, m3) > 0) then
                   printf("\n!! An m does not match a j; try again\n\n");
                 else
                     kmin := max(0, j2 - j1 - m3);  kmax := min( j3 - j1 + j2, j3 - m3);
```

$$N := \sqrt{\frac{(j3+j1-j2)!(j3-j1+j2)!(j1+j2-j3)!(j3-m3)!(j3+m3)!}{(j1+j2+j3+1)!(j1-m1)!(j1+m1)!(j2-m2)!(j2+m2)!}};$$

$$S := \sum_{k=kmin}^{kmax} \frac{(-1)^{(k+j2+m2)}\cdot(j2+j3+m1-k)!(j1-m1+k)!}{(k)!(j3-j1+j2-k)!(j3-m3-k)!(k+j1-j2+m3)!};$$

$$WC := (-1)^{j1-j2-m3}\cdot N\cdot S;$$

$$CG := (-1)^{j1-j2-m3}\cdot\sqrt{2\cdot j3+1}\cdot WC;$$

```
                     printf("The Clebsh-Gordan coefficient <%a,%a;%a,%a|%a,%a;%a,%a> = [%a] =
    %lf \n\n", j1, j2, m1, m2, j1, j2, j3, -m3, SQ(CG), CG );
                 end if;
             end if;
         end if;
     end if;
  end proc:
```

Examples:

CG(j1,j2,j3,m1,m2,m3) or CG(j1,j2,J,m1,m2,M)

```
> CG(1, 1, 2, 1, 1, 2);
The Clebsh-Gordan coefficient <1,1;1,1|1,1;2,2> = [1] = 1.000000
```

```
> CG(1, 1, 2, 0, 1, 1);
The Clebsh-Gordan coefficient <1,1;0,1|1,1;2,1> = [1/2] = 0.707107
```

```
> CG(1, 1, 2, 1, 0, 1);
The Clebsh-Gordan coefficient <1,1;1,0|1,1;2,1> = [1/2] = 0.707107
```

```
> CG(1, 1, 2, -1, 1, 0);
The Clebsh-Gordan coefficient <1,1;-1,1|1,1;2,0> = [1/6] = 0.408248
```

```
> CG(1, 1, 2, 0, 0, 0);
The Clebsh-Gordan coefficient <1,1;0,0|1,1;2,0> = [2/3] = 0.816497
```

```
> CG(1, 1, 2, 1, -1, 0);
The Clebsh-Gordan coefficient <1,1;1,-1|1,1;2,0> = [1/6] = 0.408248
```

```
> CG(1, 1, 2, -1, 0, -1);
The Clebsh-Gordan coefficient <1,1;-1,0|1,1;2,-1> = [1/2] = 0.707107
```

```
> CG(1, 1, 2, 0, -1, -1);
The Clebsh-Gordan coefficient <1,1;0,-1|1,1;2,-1> = [1/2] = 0.707107
```

```
> CG(1, 1, 2, -1, -1, -2);
The Clebsh-Gordan coefficient <1,1;-1,-1|1,1;2,-2> = [1] = 1.000000
```

```
> CG(1, 1, 1, 0, 1, 1);
The Clebsh-Gordan coefficient <1,1;0,1|1,1;1,1> = [-1/2] = -0.707107
```

```
> CG(1, 1, 1, 1, 0, 1);
The Clebsh-Gordan coefficient <1,1;1,0|1,1;1,1> = [1/2] = 0.707107
```

```
> CG(1, 1, 1, 0, 0, 0);
The Clebsh-Gordan coefficient <1,1;0,0|1,1;1,0> = [0] = 0.000000
```

```
> CG(1, 1, 1, 1, -1, 0);
The Clebsh-Gordan coefficient <1,1;1,-1|1,1;1,0> = [1/2] = 0.707107
```

```
> CG(1, 1, 1, -1, 1, 0);
The Clebsh-Gordan coefficient <1,1;-1,1|1,1;1,0> = [-1/2] = -0.707107
```

```
> CG(1, 1, 1, 0, -1, -1);
The Clebsh-Gordan coefficient <1,1;0,-1|1,1;1,-1> = [1/2] = 0.707107
```

> $CG(1, 1, 1, -1, 0, -1);$

```
The Clebsh-Gordan coefficient <1,1;-1,0|1,1;1,-1> = [-1/2] = -0.707107
```

> $CG(1, 1, 0, 1, -1, 0);$

```
The Clebsh-Gordan coefficient <1,1;1,-1|1,1;0,0> = [1/3] = 0.577350
```

> $CG(1, 1, 0, -1, 1, 0);$

```
The Clebsh-Gordan coefficient <1,1;-1,1|1,1;0,0> = [1/3] = 0.577350
```

> $CG(1, 1, 0, 0, 0, 0);$

```
The Clebsh-Gordan coefficient <1,1;0,0|1,1;0,0> = [-1/3] = -0.577350
```

> $CG\left(\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}, \frac{1}{2}, 1\right);$

```
The Clebsh-Gordan coefficient <1/2,1/2;1/2,1/2|1/2,1/2;1,1> = [1] = 1.000000
```

> $CG\left(\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}, -\frac{1}{2}, 0\right);$

```
The Clebsh-Gordan coefficient <1/2,1/2;1/2,-1/2|1/2,1/2;1,0> = [1/2] = 0.707107
```

> $CG\left(\frac{1}{2}, \frac{1}{2}, 1, -\frac{1}{2}, \frac{1}{2}, 0\right);$

```
The Clebsh-Gordan coefficient <1/2,1/2;-1/2,1/2|1/2,1/2;1,0> = [1/2] = 0.707107
```

> $CG\left(\frac{1}{2}, \frac{1}{2}, 1, -\frac{1}{2}, -\frac{1}{2}, -1\right);$

```
The Clebsh-Gordan coefficient <1/2,1/2;-1/2,-1/2|1/2,1/2;1,-1> = [1] = 1.000000
```

> $CG\left(2, \frac{3}{2}, \frac{5}{2}, -2, -\frac{1}{2}, -\frac{5}{2}\right);$

```
The Clebsh-Gordan coefficient <2,3/2;-2,-1/2|2,3/2;5/2,-5/2> = [-4/7] = -0.755929
```

>