> *restart* :

> *interface*(*warnlevel* = 0) :   # **Maple 12**

> *with*(*plots*) :

> *with*(*LinearAlgebra*) :

Reflection operators: 1) I - 2(nn^t) vs 2) 2(nn^t) - I

> q := 2 :

A unit sphere, an x-y plane and a z-x plane - as a references

> $sphere := implicitplot3d\left(x^2 + z^2 + y^2 = 1, x = -1 ..1, z = -1 ..1, y = -1 ..1, axes = normal, style = patchnogrid, transparency = .7, color = gray\right) :$

$XYpl := implicitplot3d(z = 0, x = -1 ..1, y = -1 ..1, z = -1 ..1, axes = normal, color = blue, transparency = 0.6, style = surface) :$

$ZXpl := implicitplot3d(y = 0, x = -1 ..1, y = -1 ..1, z = -1 ..1, axes = normal, color = red, transparency = 0.4, style = surface) :$

Vector V_0 ; interesting set of angles $\alpha = \left\{ \frac{1.047}{2}, \frac{0.7227}{2}, \frac{0.3554}{2}, \frac{0.1769}{2}, \frac{0.1251}{2}, \frac{0.06248}{2} \right\}$

$$\mathbf{V_0 = x + \cos(\alpha)\, y + \sin(\alpha)\, z}$$

> $V_0 := \frac{1}{\sqrt{1 + \cos^2(\alpha) + \sin^2(\alpha)}}\ Vector([1, \cos(\alpha), \sin(\alpha)]);$

$$V_0 := \begin{bmatrix} \frac{1}{\sqrt{1 + \cos(\alpha)^2 + \sin(\alpha)^2}} \\ \frac{\cos(\alpha)}{\sqrt{1 + \cos(\alpha)^2 + \sin(\alpha)^2}} \\ \frac{\sin(\alpha)}{\sqrt{1 + \cos(\alpha)^2 + \sin(\alpha)^2}} \end{bmatrix} \qquad (1)$$

This is the first reflection operator

> U := *Matrix*([[1, 0, 0], [0, 1, 0], [0, 0, -1]]);

$$U := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad (2)$$

Second reflection operator: $2V_0V_0^T - I$

```
> I3 := IdentityMatrix(3) :
  if q = 2 then
      r := 2·Multiply(V0, Transpose(V0)) - I3;
   else
      r := I3 - 2·Multiply(V0, Transpose(V0)) ;
  end if;
  α := 0.3554/2 :
```

$$r := \begin{bmatrix} \frac{2}{1+\cos(\alpha)^2+\sin(\alpha)^2}-1 & \frac{2\cos(\alpha)}{1+\cos(\alpha)^2+\sin(\alpha)^2} & \frac{2\sin(\alpha)}{1+\cos(\alpha)^2+\sin(\alpha)^2} \\ \frac{2\cos(\alpha)}{1+\cos(\alpha)^2+\sin(\alpha)^2} & \frac{2\cos(\alpha)^2}{1+\cos(\alpha)^2+\sin(\alpha)^2}-1 & \frac{2\cos(\alpha)\sin(\alpha)}{1+\cos(\alpha)^2+\sin(\alpha)^2} \\ \frac{2\sin(\alpha)}{1+\cos(\alpha)^2+\sin(\alpha)^2} & \frac{2\cos(\alpha)\sin(\alpha)}{1+\cos(\alpha)^2+\sin(\alpha)^2} & \frac{2\sin(\alpha)^2}{1+\cos(\alpha)^2+\sin(\alpha)^2}-1 \end{bmatrix} \tag{3}$$

Vector M is the reflection of Vector V_0 about the x-y plane

$M = x + \cos(\alpha)y - \sin(\alpha)z$

```
> 'V0' = V0 ;
  M := simplify(Multiply(U, V0)) :
' U· V0 '= M;
```

$$V_0 = \begin{bmatrix} 0.7071067814 \\ 0.6959718706 \\ 0.1249926222 \end{bmatrix}$$

$$U\ V_0 = \begin{bmatrix} 0.7071067814 \\ 0.6959718706 \\ -0.1249926222 \end{bmatrix} \tag{4}$$

Vector N is the reflection of Vector M across the plane of vector $\mathbf{V_0}$

$$\mathbf{M} = x + \cos(\alpha)\,y - \sin(\alpha)\,\mathbf{z}$$

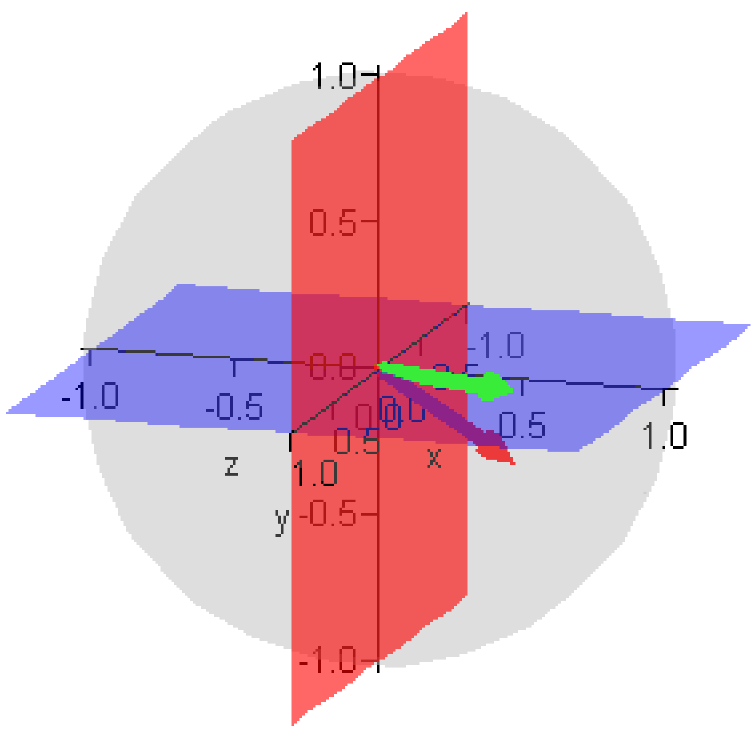
```
> N := simplify(Multiply(r, M)) :
'N ' = N;
if q = 2 then
   p2 := implicitplot3d(-sin(α)·y + cos(α)·z, x = -1..1, y = -1..1, z = -1..1, axes = normal, color
     = green, transparency = 0.6, style = surface) :  # the plane of vector V0
                                                       # x-y plane rotated by α about the x-axis
 else
   p2 := implicitplot3d(x + cos(α)·y + sin(α)·z, x = -1..1, y = -1..1, z = -1..1, axes = normal, color
     = green, transparency = 0.6, style = surface) :  # plane perpendiculer to vector V0
 end if;
```

$$N = \begin{bmatrix} 0.6629178244 \\ 0.6524787633 \\ 0.3671667498 \end{bmatrix}$$

$$p2 := PLOT3D(...) \tag{5}$$

Display the initial vector $\mathbf{V_0}$ (green) and its reflection; vector M(red).
This is the reflextion of $\mathbf{V_0}$ across the x-y plane.

```
> a1 := arrow(V0, color = green) : b1 := arrow(M, color = red) :
display([sphere, XYpl, ZXpl, a1, b1], axes = normal, scaling = constrained, tickmarks = [4, 4, 4],
    orientation = [17, 77]) ;
```



Display the vector(blue) resulting from the reflection of vector M(red) across the plane of vector V_0(green plane); the mirror plane.

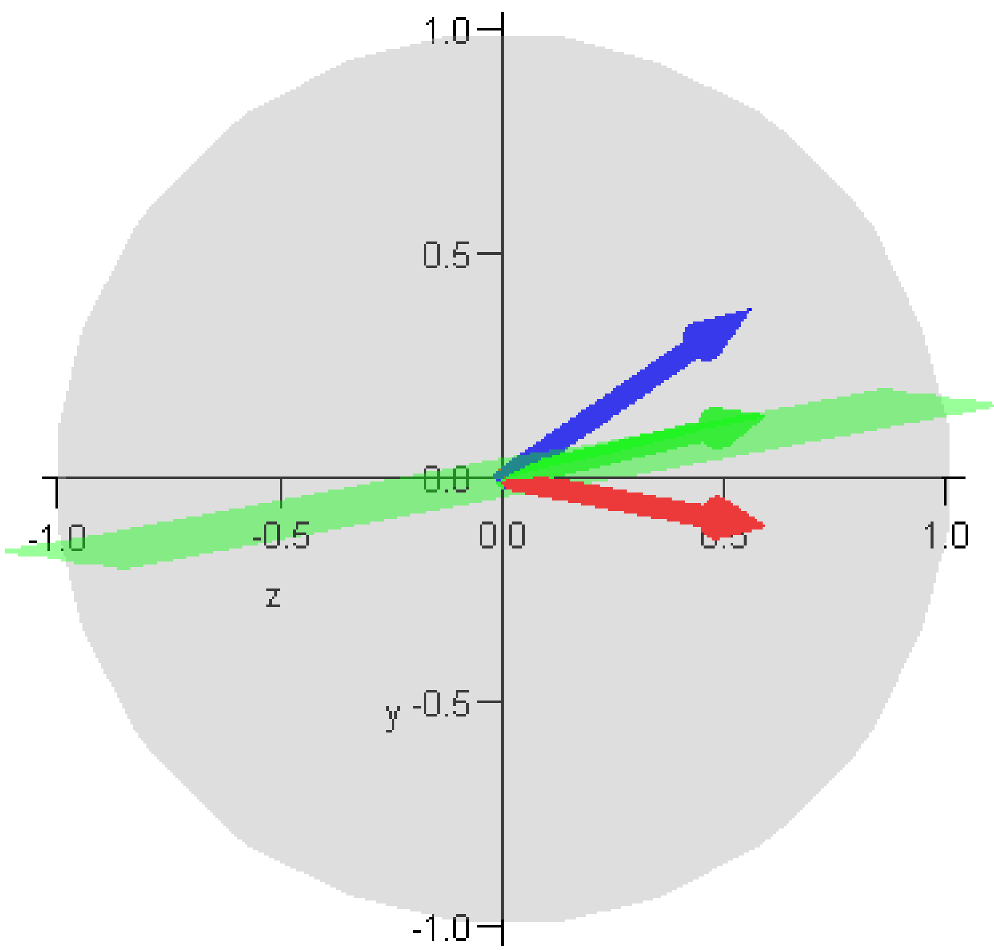
```
> b1 := arrow(M, color = red) :
  c1 :=  arrow(N, color = blue) :
  k := 1;
  P := N[3]^2;   # square of the coefficient of the z -component
  display([sphere, p2, a1, b1, c1], axes = normal, scaling = constrained, tickmarks = [4, 4, 4], orientation
      = [8, 91]);
```

$$\theta := evalf\left(\left(\frac{180}{\pi}\right)\cdot\cos^{-1}\left(DotProduct\left(N, Vector\left(\frac{[1, \cos(\alpha), \sin(\alpha)]}{\sqrt{1+\cos^2(\alpha)+\sin^2(\alpha)}}\right)\right)\right)\right);$$

\# *angle between N and* V_0\`\`

$$k := 1$$

$$P := 0.1348114222$$



$$\theta := 14.36065930 \tag{6}$$

Repeat the preceding reflection steps until the square of the coefficient of the z-component reaches a maximum. This is equivalent to a series of CCW rotations of the initial vector V_0.

```
> do
    k := k + 1;  Pold := N[3]^2;   # square of the coefficient of the z -component
    M := simplify(Multiply(U, N)) :  N := simplify(Multiply(r, M)) :  Pnew := N[3]^2;
    if  Pnew <  Pold then printf("\nThe Maximum Value is %f, after %d iterations\n", Pold, k
  - 1); quit(0); end if;
    a2 := arrow(M,  color = red) : a3 := arrow(N, color = blue) : # display the vectors
    display([sphere, p2, ZXpl, a1, a2, a3], axes = normal, scaling = constrained,  tickmarks = [4, 4,
  4],                              orientation = [13, 75]);
```

$$\theta := evalf\left(\left(\frac{180}{\pi}\right)\cdot\cos^{-1}\left(DotProduct\left(N, Vector\left(\frac{[1, \cos(\alpha), \sin(\alpha)]}{\sqrt{1+\cos^2(\alpha)+\sin^2(\alpha)}}\right)\right)\right)\right);$$

```
  print( ); print( );
end do;
```

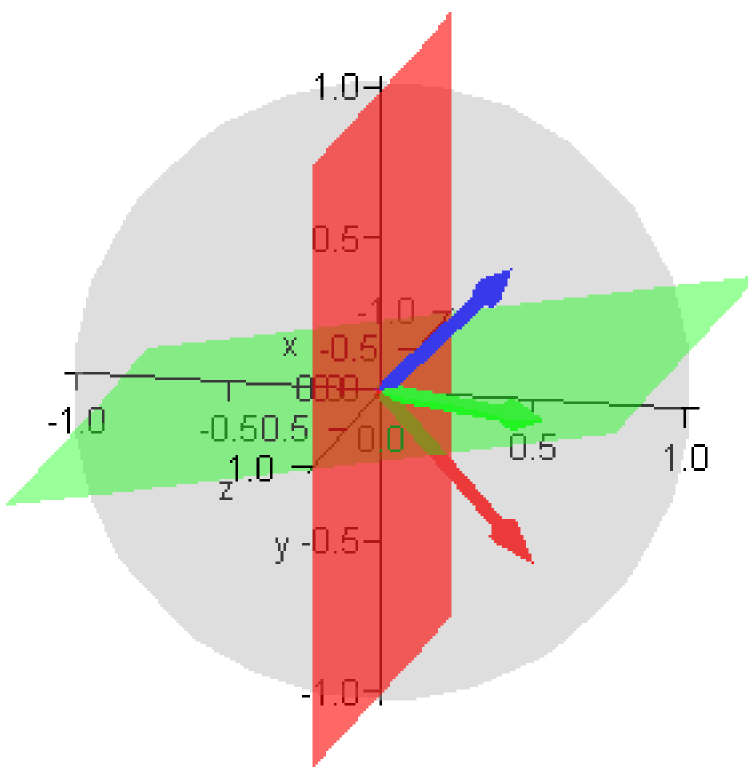
$$Pold := 0.1348114222$$

$$k := 2$$

$$M := \begin{bmatrix} 0.6629178244 \\ 0.6524787633 \\ -0.3671667498 \end{bmatrix}$$

$$N := \begin{bmatrix} 0.5773013940 \\ 0.5682105472 \\ 0.5863956643 \end{bmatrix}$$

$$Pnew := 0.3438598751$$



$$\theta := 28.72131880$$

$$Pold := 0.3438598751$$

$$k := 3$$

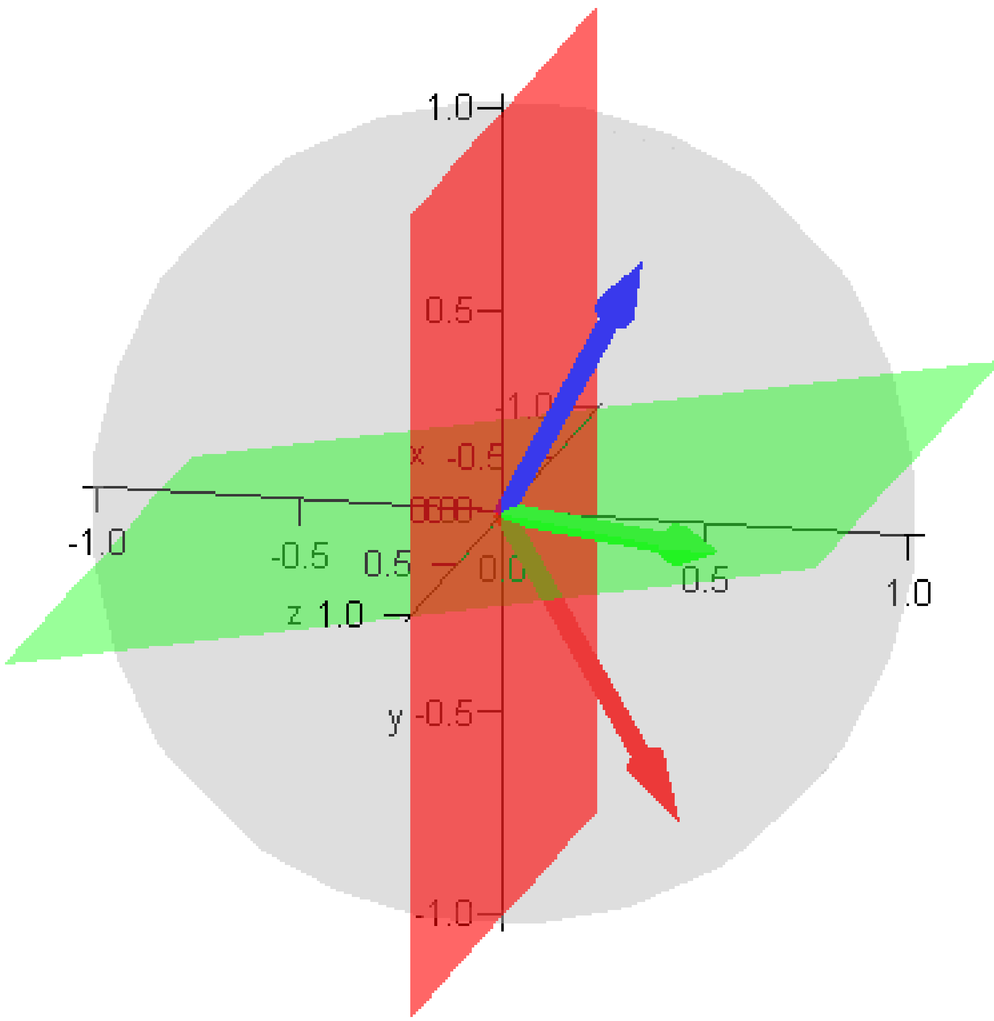
$$M := \begin{bmatrix} 0.5773013940 \\ 0.5682105472 \\ -0.5863956643 \end{bmatrix}$$

$$N := \begin{bmatrix} 0.4556078858 \\ 0.4484333637 \\ 0.7689791760 \end{bmatrix}$$

$$Pnew := 0.5913289731$$

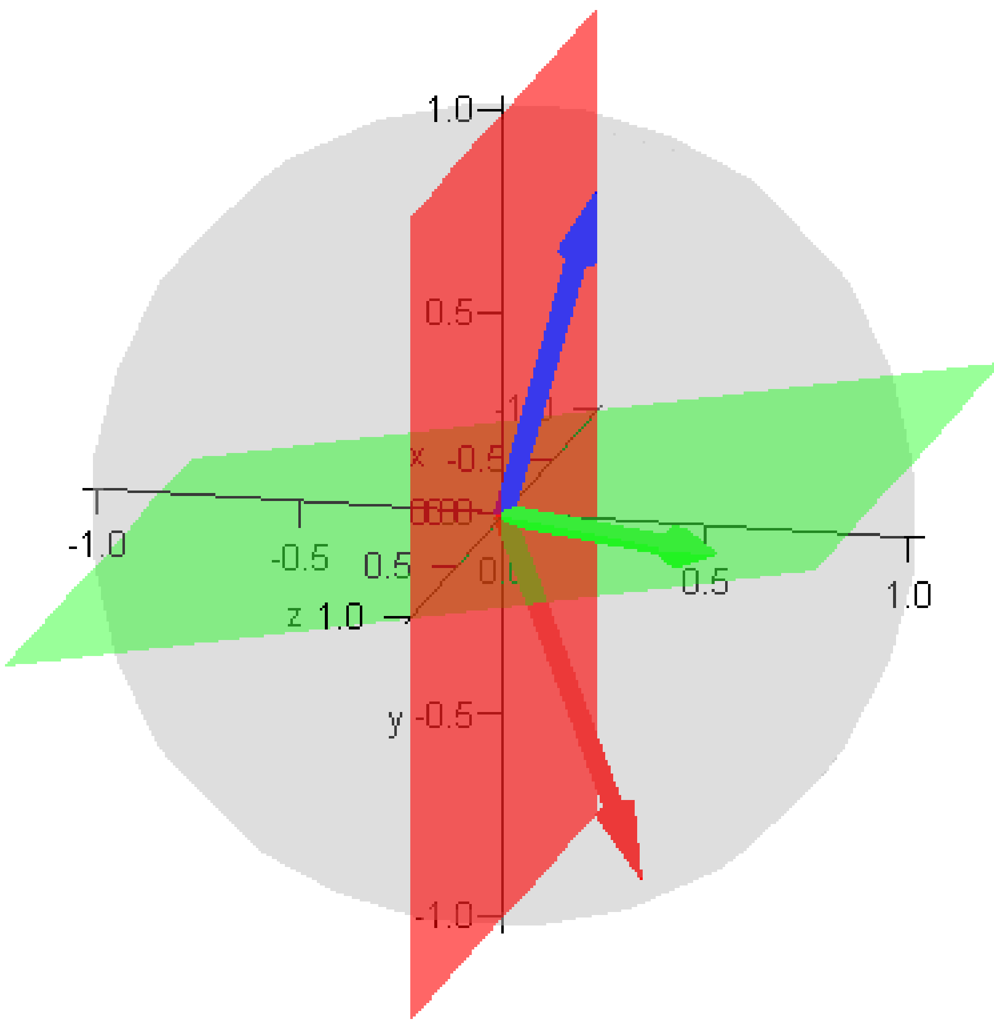
$$a2 := PLOT3D(\ldots)$$

$$a3 := PLOT3D(\ldots)$$



$$\theta := 43.08197828$$

$$Pold := 0.5913289731$$
$$k := 4$$
$$M := \begin{bmatrix} 0.4556078858 \\ 0.4484333637 \\ -0.7689791760 \end{bmatrix}$$
$$N := \begin{bmatrix} 0.3054422459 \\ 0.3006324037 \\ 0.9035071625 \end{bmatrix}$$
$$Pnew := 0.8163251927$$
$$a2 := PLOT3D(\ldots)$$
$$a3 := PLOT3D(\ldots)$$



$$\theta := 57.44263768$$

$$Pold := 0.8163251927$$

$$k := 5$$

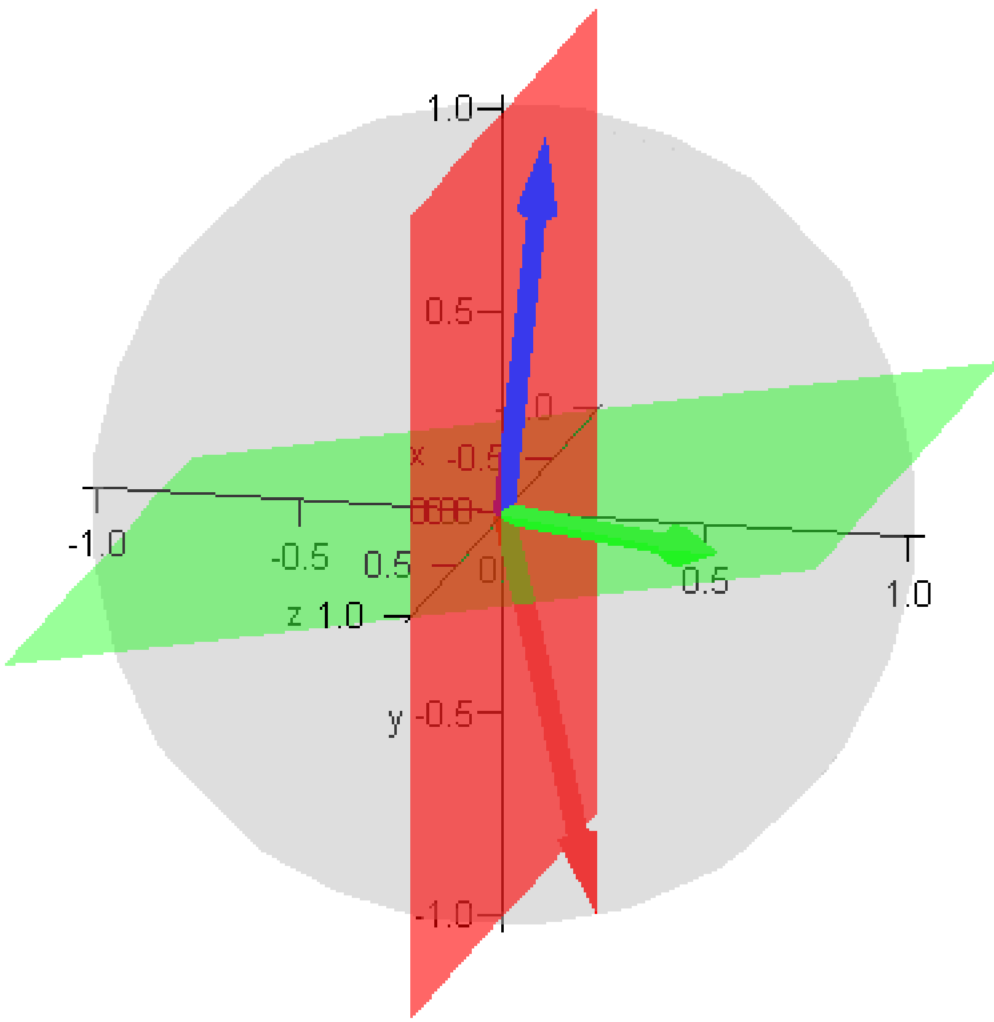
$$M := \begin{bmatrix} 0.3054422459 \\ 0.3006324037 \\ -0.9035071625 \end{bmatrix}$$

$$N := \begin{bmatrix} 0.1361887193 \\ 0.1340441361 \\ 0.9815726171 \end{bmatrix}$$

$$Pnew := 0.9634848026$$

$$a2 := PLOT3D(\ldots)$$

$$a3 := PLOT3D(\ldots)$$



$$\theta := 71.80329712$$

$$Pold := 0.9634848026$$

$$k := 6$$

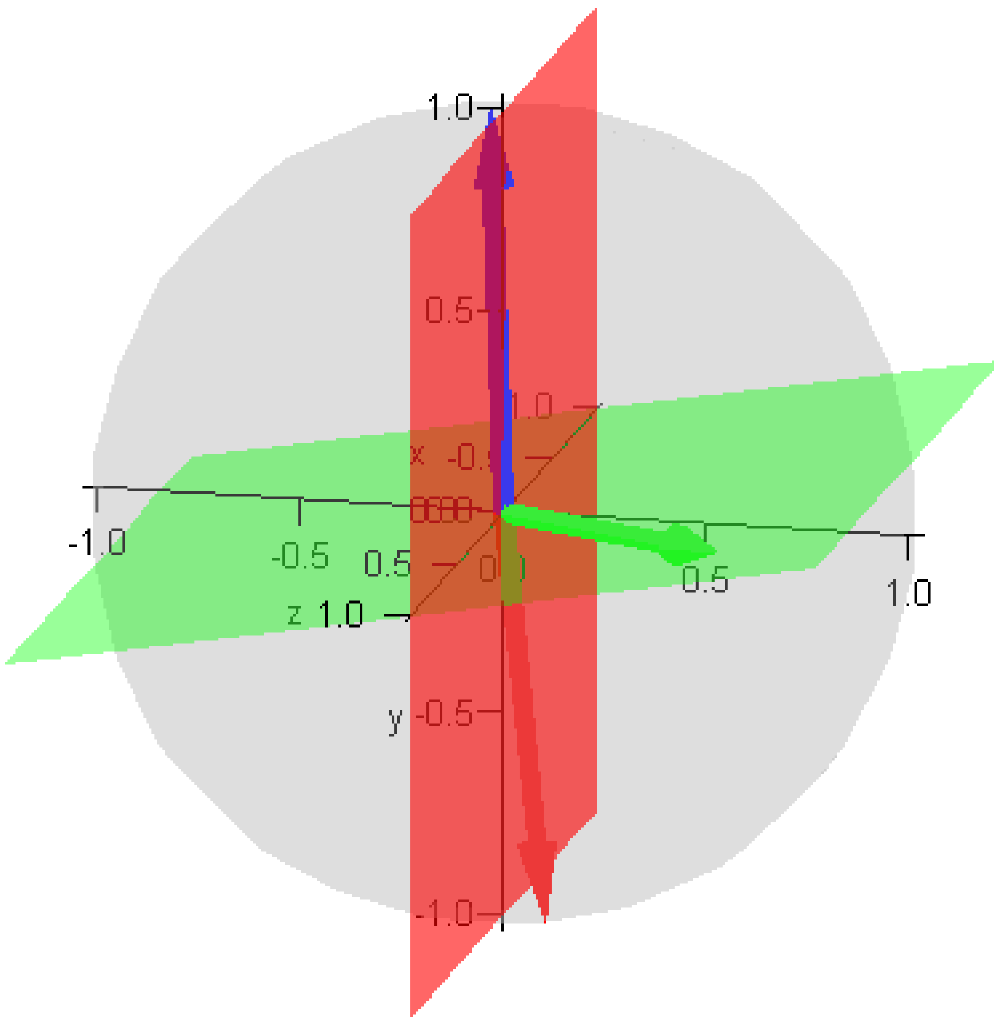
$$M := \begin{bmatrix} 0.1361887193 \\ 0.1340441361 \\ -0.9815726171 \end{bmatrix}$$

$$N := \begin{bmatrix} -0.04157559770 \\ -0.04092090080 \\ 0.9982970248 \end{bmatrix}$$

$$Pnew := 0.9965969497$$

$$a2 := PLOT3D(\ldots)$$

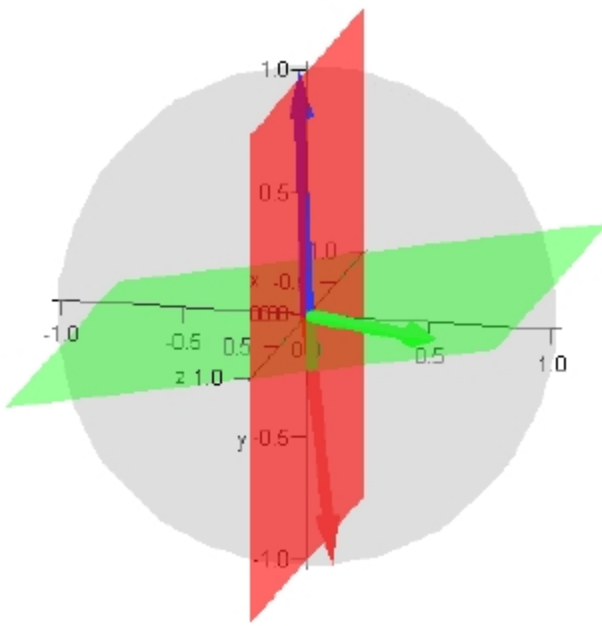
$$a3 := PLOT3D(\ldots)$$



$$\theta := 86.16395662$$

$$Pold := 0.9965969497$$
$$k := 7$$
$$M := \begin{bmatrix} -0.04157559770 \\ -0.04092090080 \\ -0.9982970248 \end{bmatrix}$$
$$N := \begin{bmatrix} -0.2167417464 \\ -0.2133286835 \\ 0.9526352336 \end{bmatrix}$$
$$Pnew := 0.9075138883$$

```
The Maximum Value is 0.996597, after 6 iterations
```



$\theta := 86.16395662$