```
> restart;
> interface(warnlevel = 0) :          #
```

Maple 12

```
> with(plots) :
> with(LinearAlgebra) :
```

Define the basis vectors:

basis set 1 { x, y, z } ; the standard basis

basis set 2 { v1, v2, v3 }

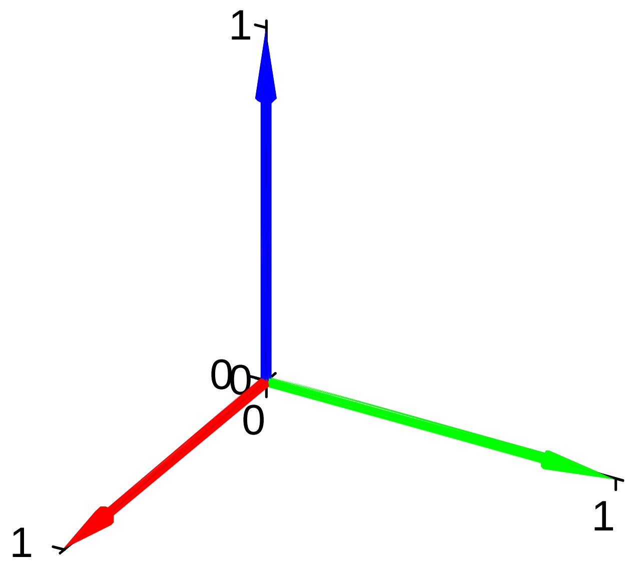
Notice that these are orthogonal bases: x • y = x · z = y · z = 0

v1 · v2 = v1 · v3 = v2 · v3 =0

```
> e1 := Vector([1, 0, 0]) :
  e2 := Vector([0, 1, 0]) :
  e3 := Vector([0, 0, 1]) :
  v1 := Vector([1, −1, 1]) :
  v2 := Vector([−1, 1, 2]) :
  v3 := Vector([1, 1, 0]) :
```

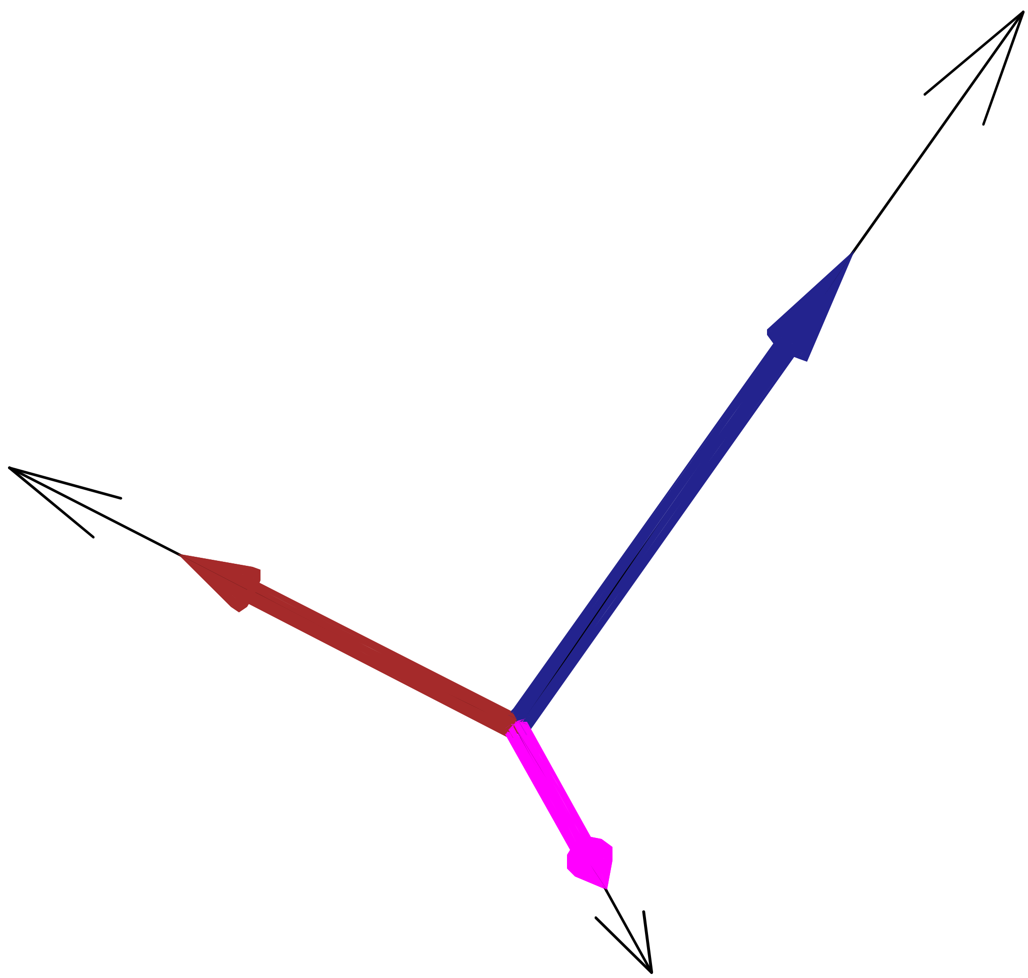
Plotting bases set 1

```
> x := arrow(e1,  color = red, width = 0.025) :
  y := arrow(e2, color = green, width = 0.025) :
  z := arrow(e3, color = blue, width = 0.025) :
  display([x, y, z], axes = normal, scaling = constrained,  tickmarks = [2, 2, 2], orientation = [30, 61]);
```



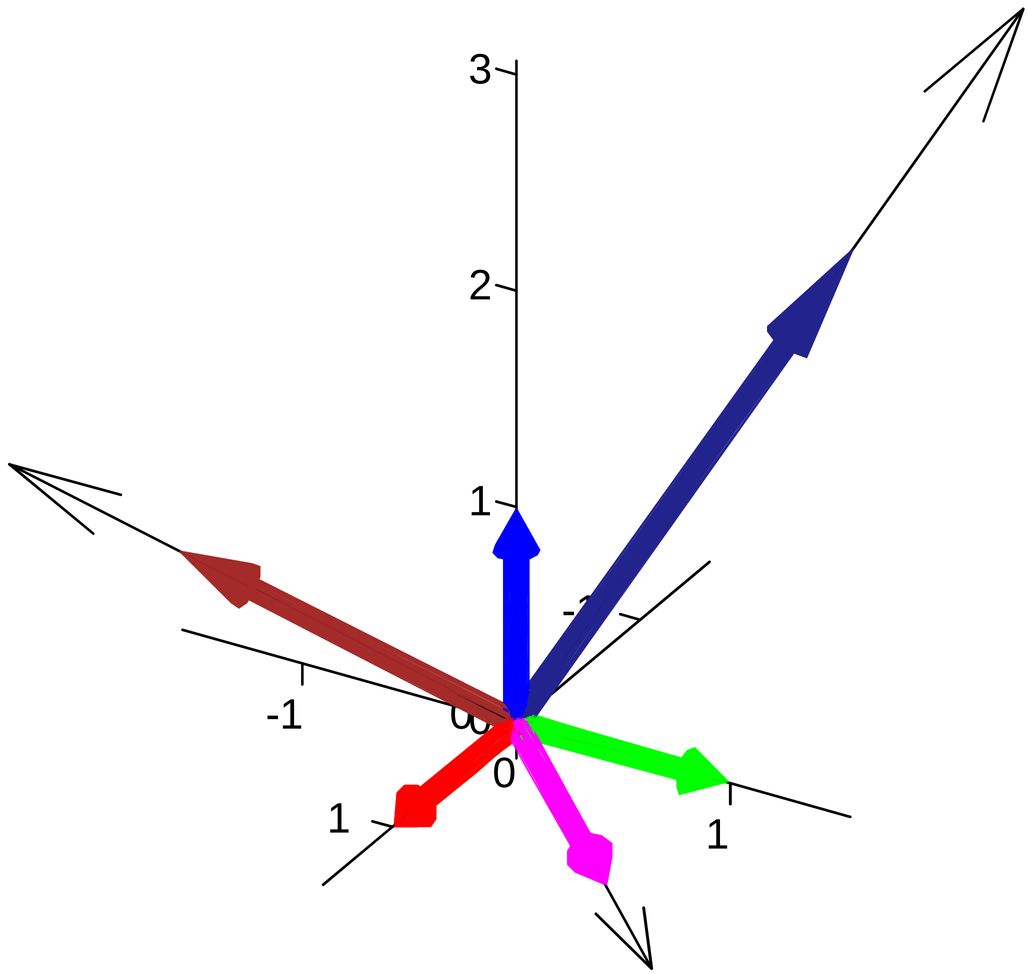
Plotting basis set 2

```
> u1 := arrow(v1, color = brown, width = 0.1) :
  u2 := arrow(v2, color = navy, width = 0.1) :
  u3 := arrow(v3, color = magenta,  width = 0.1) :
  a := arrow(1.5 · v1, color = black, width = 0.05, shape = arrow, head_length = 0.5, head_width = 0.2) :
  b := arrow(1.5 · v2, color = black, width = 0.05, shape = arrow, head_length = 0.5, head_width = 0.3) :
  c := arrow(1.5 · v3, color = black,  width = 0.05, shape = arrow, head_length = 0.5, head_width = 0.2) :
  display([u1, u2, u3, a, b, c], axes = none, scaling = constrained,  orientation = [30, 61]);
```



Plotting both bases

```
> x := arrow(e1,  color = red, width = 0.1) :
  y := arrow(e2, color = green, width = 0.1) :
  z := arrow(e3, color = blue, width = 0.1) :
  display([x, y, z, u1, u2, u3, a, b, c], axes = normal,  scaling = constrained,  tickmarks = [3, 3, 3],
                                         orientation = [30, 61]);
```



Defining vector $R(t)$. $R(t)$ represents a vector in 3D space in the standard basis

$$R(t) = X(t)\mathbf{x} + Y(t)\mathbf{y} + \mathbf{z}$$
$$X(t) = \cos(t)$$
$$Y(t) = \sin(t)$$

Writing the matrix form of vector R(t) in the standard basis

```
> R := t → Vector([cos(t), sin(t), 1]) : 'R(t)'= R(t);
```

$$R(t) = \begin{bmatrix} \cos(t) \\ \sin(t) \\ 1 \end{bmatrix} \tag{1}$$

Writing the transformation matrix from the basis set 2 to the standard basis

> $\mathbb{S} := Matrix([\mathrm{v1}, \mathrm{v2}, \mathrm{v3}]);$

$$\mathbb{S} := \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} \tag{2}$$

Define the matrix S as the inverse of matrix $\mathbb{S}$. This is the transformation matrix from the standard basis to basis set 2

> $S := MatrixInverse(\mathbb{S});$

$$S := \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \\ -\frac{1}{6} & \frac{1}{6} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} \tag{3}$$

The relationship between matrix S and its inverse is the identity matrix

> $'\mathbb{S} \cdot S' = \mathrm{Multiply}(\mathbb{S}, S);$

$$\mathbb{S}\, S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

The position vector in the basis set 2 is given by

> $\mathcal{R} := t \rightarrow Multiply(S, R(t)) :$
> $'\mathcal{R}(t)' = \mathcal{R}(t);$

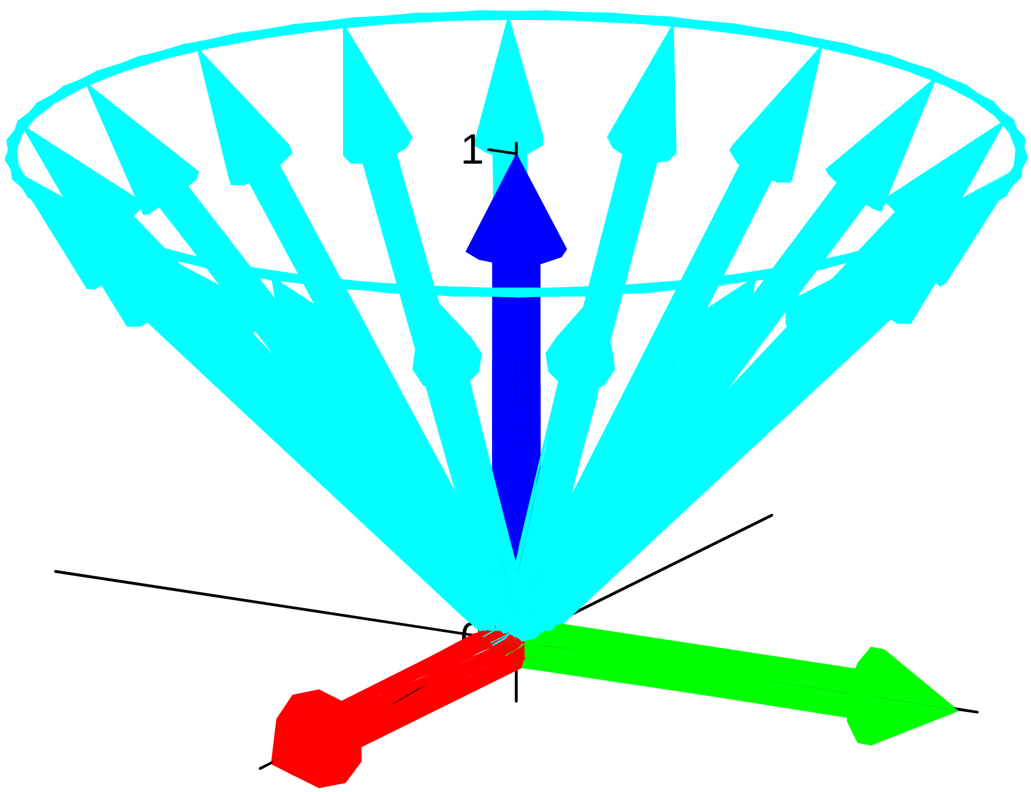
$$\mathcal{R}(t) = \begin{bmatrix} \frac{1}{3}\cos(t) - \frac{1}{3}\sin(t) + \frac{1}{3} \\ -\frac{1}{6}\cos(t) + \frac{1}{6}\sin(t) + \frac{1}{3} \\ \frac{1}{2}\cos(t) + \frac{1}{2}\sin(t) \end{bmatrix} \tag{5}$$

> $'R(t)' = Multiply(\mathbb{S}, \mathcal{R}(t));$

$$R(t) = \begin{bmatrix} \cos(t) \\ \sin(t) \\ 1 \end{bmatrix} \tag{6}$$

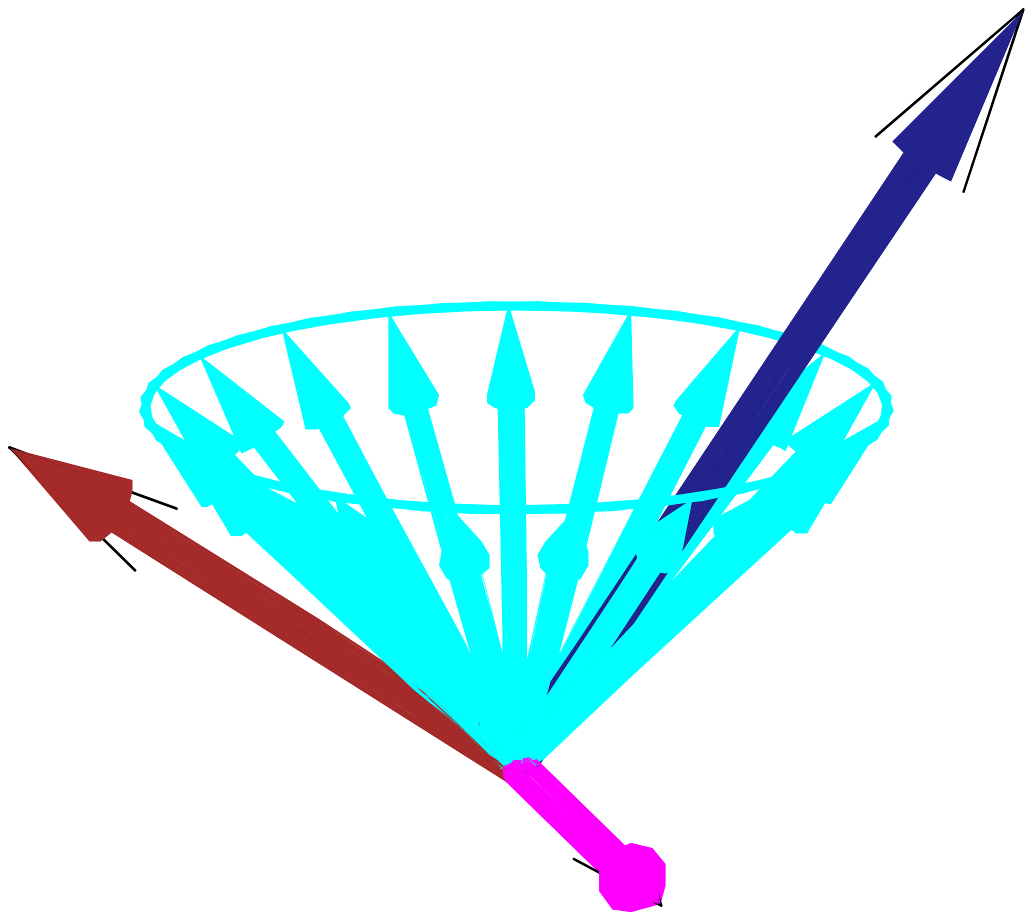
Plotting the standard basis

```
> lv := [seq( R(t), t = 0 ..6, 1/3 )] :
  ar := arrow(lv, color = cyan) :
  r := spacecurve(R(t),  color = cyan, thickness = 2, t = 0 ..4π) :
  display([x, y, z, r, ar], axes = normal, scaling = constrained, tickmarks = [2, 2, 2],
                                              orientation = [29, 74]);
```



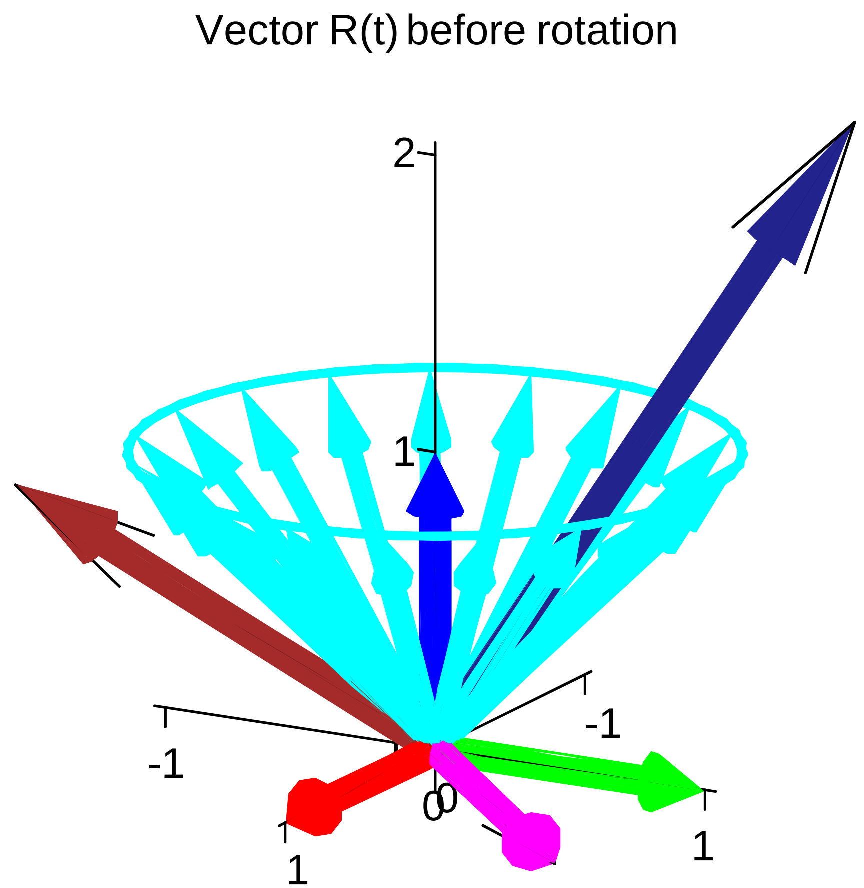
Plotting vector R(t) and basis set 2

```
> a := arrow(v1, color = black, width = 0.05, shape = arrow, head_length = 0.5, head_width = 0.2) :
  b := arrow(v2, color = black, width = 0.05, shape = arrow, head_length = 0.5, head_width = 0.3) :
  c := arrow(v3, color = black,  width = 0.05, shape = arrow, head_length = 0.5, head_width = 0.2) :
  display([u1, u2, u3, a, b, c, r, ar], axes = none, scaling = constrained, tickmarks = [0, 0, 0],
      orientation = [29, 74]);
```



Plotting both bases

> *display*([u1, u2, u3, *a*, *b*, *c*, x, y, z, *r*, *ar*], axes = *normal*, scaling = constrained, *tickmarks* = [3, 3, 3], *orientation* = [29, 74], *title* = "Vector R(t) before rotation");



The Euler Rotation matrix is given as:

$$\mathbf{R(\alpha,\beta,\gamma) = Rz(\alpha)Ry(\beta)Rz(\gamma)}$$

let $\alpha = \gamma = \frac{\pi}{2}$ **and** $\beta = \pi$

> $Rz := \theta \rightarrow Matrix([[\cos(\theta), -\sin(\theta), 0], [\sin(\theta), \cos(\theta), 0], [0, 0, 1]]) :$
$Rx := \theta \rightarrow Matrix([[1, 0, 0], [0, \cos(\theta), -\sin(\theta)], [0, \sin(\theta), \cos(\theta)],]) :$
$Ry := \theta \rightarrow Matrix([[\cos(\theta), 0, \sin(\theta)], [0, 1, 0], [-\sin(\theta), 0, \cos(\theta)]]) :$

> $\alpha := \frac{\pi}{2} : \beta := \pi : \gamma1 := \frac{\pi}{2} :$
$Erot := Multiply(Rz(\alpha), Multiply(Ry(\beta), Rz(\gamma1)));$

$$Erot := \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad (7)$$

Define vector R1(t) = Erot · R(t). Vector R1(t) is written in the standard basis

> $R1 := t \rightarrow Multiply(Erot, R(t)) :$
$'R1(t)' = R1(t);$

$$R1(t) = \begin{bmatrix} -\cos(t) \\ \sin(t) \\ -1 \end{bmatrix} \quad (8)$$

$$\mathbf{R1(t) = -\cos(t)x + \sin(t)y - z}$$

Plotting vectors R(t) and R1(t) in the standard basis

```
> lv1 := [seq( R1(t), t = 0..6, 1/3 )]:
  ar1 := arrow(lv1, color = gray) :
  r1 := spacecurve(R1(t), color = gray, thickness = 2, t = 0..4π) :
  display([x, y, z, r1, ar1], axes = normal, scaling = constrained, tickmarks = [3, 3, 3],
                                    orientation = [30, 61]);
```