```
> restart;
> interface(warnlevel = 0) :            # Maple 12
> with(LinearAlgebra) :
> with(plots) :
```

planes

```
> f1 := z = 0 :    # equation of an  x−y  plane in 3D space
  f2 := x + y + z = 0 :    # equation of a  plane whose normal vector  is (x + y + z)
  f3 := -x − y + 2 z = 0 :  #` `a plane` `containing vector (x + y + z)

> p1 := implicitplot3d(f1, x =-2 ..2, y =-2 ..2, z = 0 ..2, axes = normal,
                        style = patchnogrid,  color = cyan, transparency = 0.6) :
  p2 := implicitplot3d(f2,  x =-2 ..2, y =-2 ..2, z =-2 ..2, axes = normal,
                        style = patchnogrid,  color = green, transparency = 0.6) :
  p3 := implicitplot3d(f3, x =-2 ..2, y =-2 ..2, z =-2 ..2, axes = normal,
                        style = patchnogrid,  color = blue, transparency = 0.6) :
```

The operators/matrices

```
> Rxy := Matrix([[1, 0, 0], [0, 1, 0], [0, 0,-1]]); # a reflection matrix  across the x-y plane
```

$I3 := IdentityMatrix(3):$

$M1 := 2\left(Multiply\left(\frac{\langle 1, 1, 1\rangle}{\sqrt{3}}, Transpose\left(\frac{\langle 1, 1, 1\rangle}{\sqrt{3}}\right)\right)\right) - I3;$ # $2(AA^T)$ - I

$M2 := I3 - 2\left(Multiply\left(\frac{\langle 1, 1, 1\rangle}{\sqrt{3}}, Transpose\left(\frac{\langle 1, 1, 1\rangle}{\sqrt{3}}\right)\right)\right);$ # $\mathrm{I} - 2(AA^T)$

$$Rxy := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$M1 := \begin{bmatrix} -\frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$$

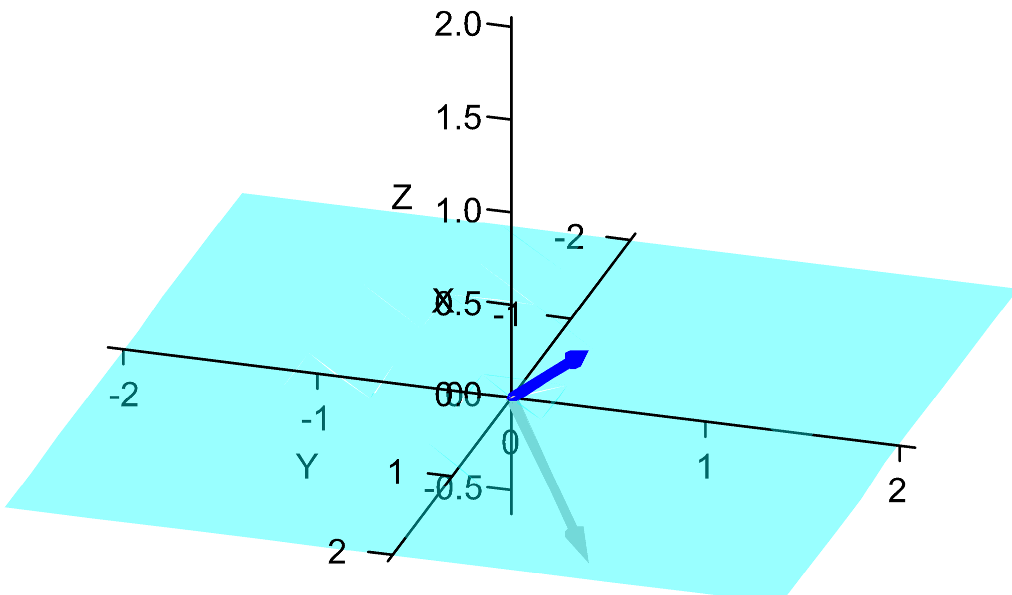
$$M2 := \begin{bmatrix} \frac{1}{3} & -\frac{2}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{bmatrix} \tag{1}$$

Reflection of Vector A = x + y + z across the x-y plane

```
> A := 1/sqrt(3) · ⟨1, 1, 1⟩ :       # vector A
  A1 := Multiply(Rxy, A); # vector A'
  a := arrow(A, color = blue) :
  b := arrow(A1, color = gray) :
  θ := evalf(180/π · cos^(-1)(DotProduct(A, A1))); # angle between A and A1
  display([a, b, p1], axes = normal, tickmarks = [4, 4, 4], scaling = constrained, labels = [X, Y, Z],
          orientation = [17, 66]) ;
```

$$A1 := \begin{bmatrix} \frac{1}{3}\sqrt{3} \\ \frac{1}{3}\sqrt{3} \\ -\frac{1}{3}\sqrt{3} \end{bmatrix}$$

$$\theta := 70.52877934$$

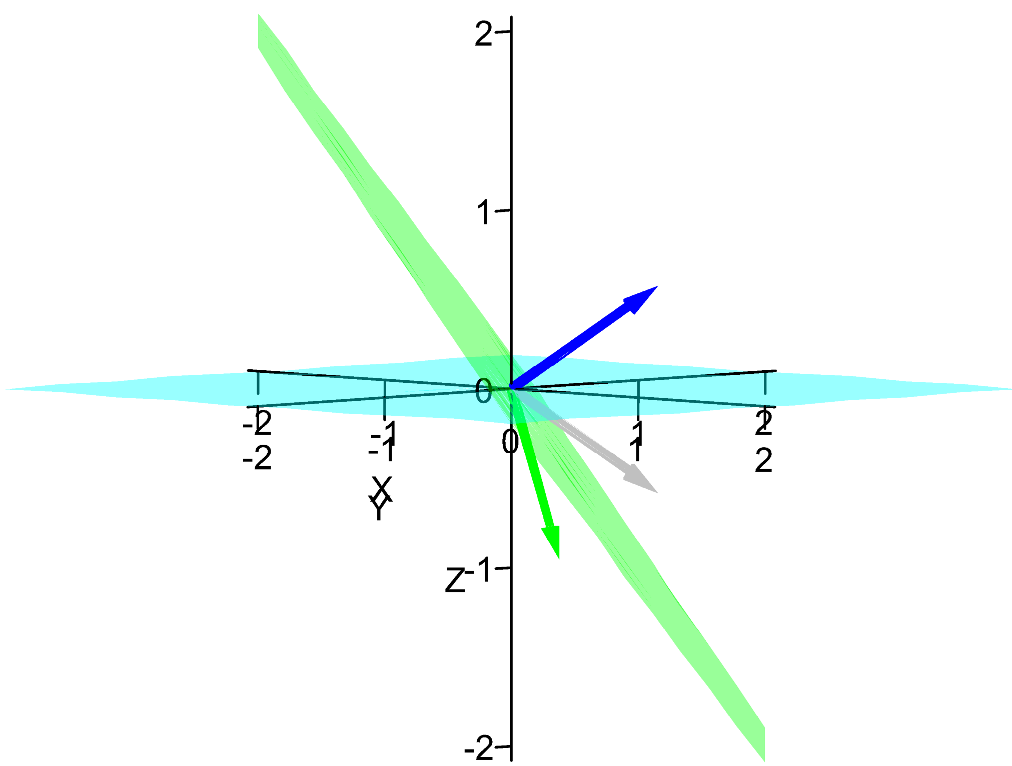


Reflection of Vector $\mathbf{A'} = \mathbf{x} + \mathbf{y} - \mathbf{z}$ by $(\mathbf{I} - 2(\mathbf{A}\mathbf{A}^{\mathbf{T}}))$

> $A2 := factor(Multiply(M2, A1))$; # using $\mathrm{I} - 2(AA^T)$. *Reflection of A1 across plane f2*
$c :=\ arrow(A2, color = green)$:
$\theta := evalf\left(\frac{180}{\pi} \cdot \cos^{-1}(DotProduct(A1, A2))\right)$; # *angle between A1 and A2*
display([a, b, c, p1, p2], axes = normal, tickmarks = [4, 4, 4], scaling = constrained, labels = [X, Y, Z],
orientation = [-45, 86]) ;

$$A2 := \begin{bmatrix} \frac{1}{9}\sqrt{3} \\ \frac{1}{9}\sqrt{3} \\ -\frac{5}{9}\sqrt{3} \end{bmatrix}$$

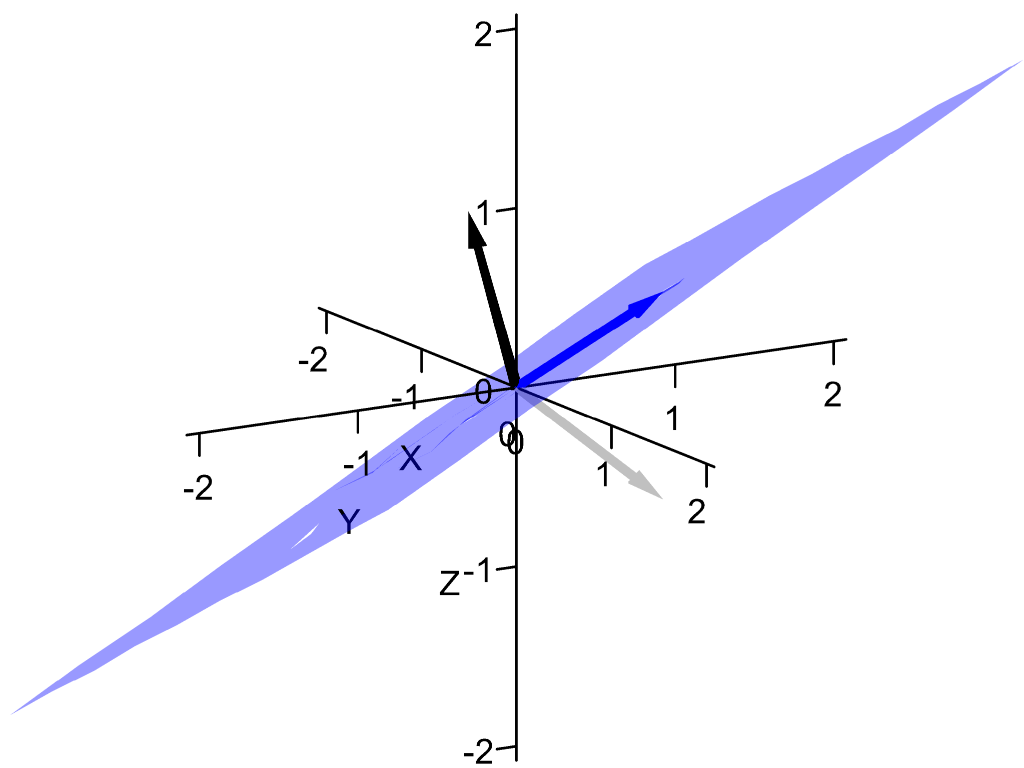
$$\theta := 38.94244126$$



Reflection of Vector A' = x + y - z   by $(2(AA^T) - I)$

> $A3 := Multiply(M1, A1)$;   # using $2(AA^T) - \mathrm{I}$. *Reflection of A1 across vector A; plane f3*
$d := arrow(A3, color = black)$:
display([a, b, d, p3], axes = normal, tickmarks = [4, 4, 4], scaling = constrained, labels = [X, Y, Z], *orientation* = [-31, 76]) ;

$$A3 := \begin{bmatrix} -\frac{1}{9}\sqrt{3} \\ -\frac{1}{9}\sqrt{3} \\ \frac{5}{9}\sqrt{3} \end{bmatrix}$$

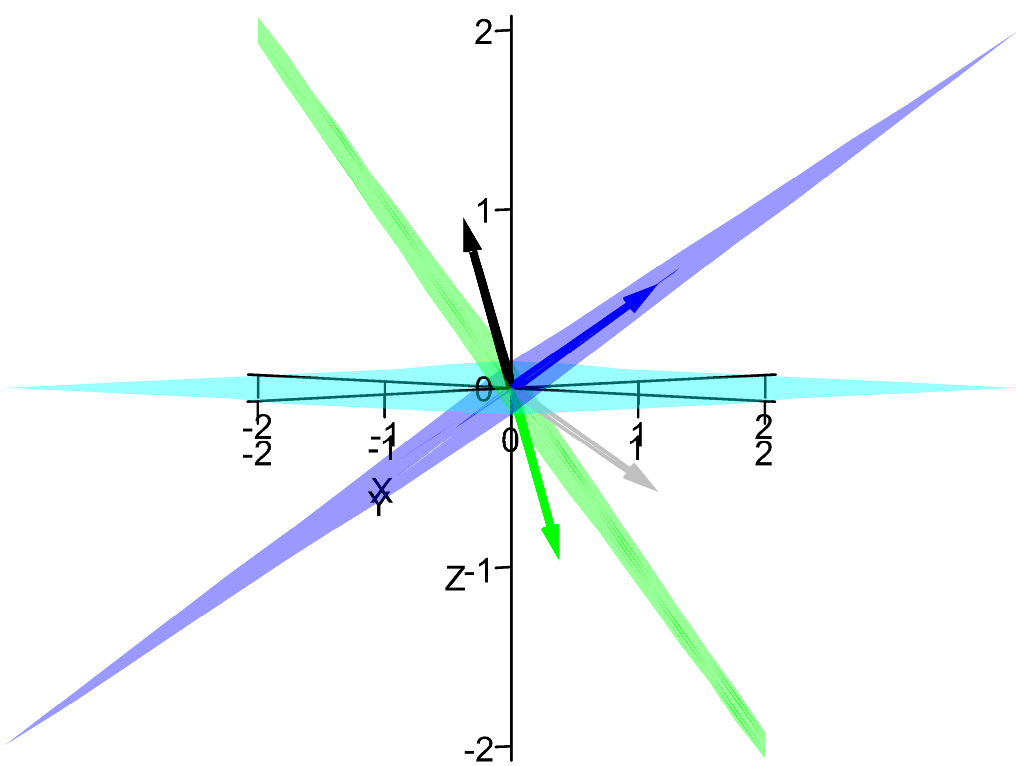


All planes

```
> 'A' = A; 'A3'= A3;
  display( [a, b, c, d, p1, p2, p3 ], axes = normal, tickmarks = [4, 4, 4 ], scaling = constrained, labels
      = [X, Y, Z],                                   orientation = [ -45, 87 ]) ;
```

$$A = \begin{bmatrix} \frac{1}{3}\sqrt{3} \\ \frac{1}{3}\sqrt{3} \\ \frac{1}{3}\sqrt{3} \end{bmatrix}$$

$$A3 = \begin{bmatrix} -\frac{1}{9}\sqrt{3} \\ -\frac{1}{9}\sqrt{3} \\ \frac{5}{9}\sqrt{3} \end{bmatrix}$$

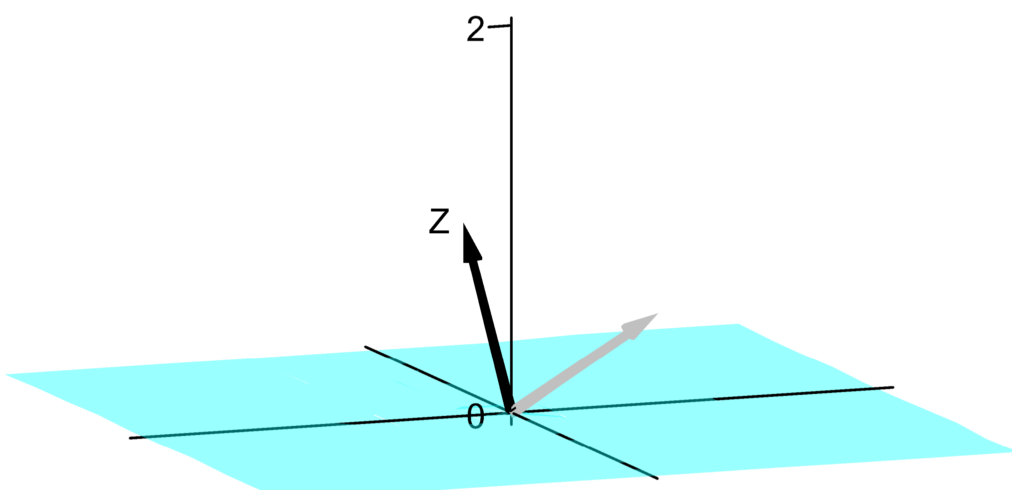


Rotation of vector A by $(2(AA^T) - I) \cdot Rxy$

> $B := Multiply(Multiply(M1, Rxy), A);$
$a := arrow(A, color = gray) : b := arrow(B, color = black) :$
$\theta := evalf\left(\frac{180}{\pi} \cdot \cos^{-1}(DotProduct(A, B))\right);$ # *angle between A and B*
$display([a, b, p1,], \text{axes} = \text{normal}, \text{tickmarks} = [2, 2, 2], \text{scaling} = \text{constrained}, \text{labels} = [" ", " ", Z], orientation = [-21, 80]);$

$$B := \begin{bmatrix} -\frac{1}{9}\sqrt{3} \\ -\frac{1}{9}\sqrt{3} \\ \frac{5}{9}\sqrt{3} \end{bmatrix}$$

$$\theta := 70.52877934$$

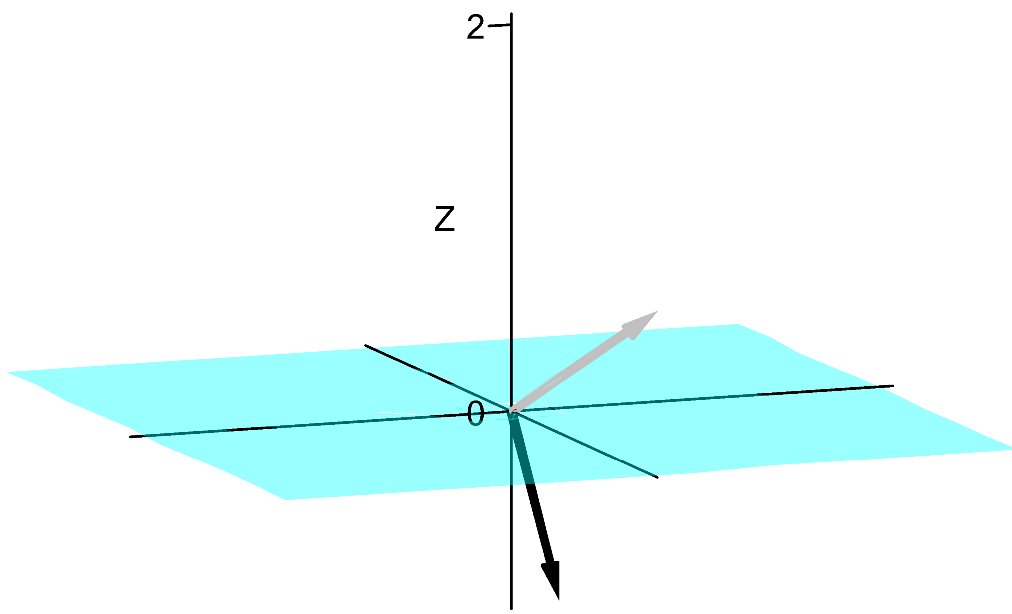


Rotation of vector A by $(I - 2(AA^T)) \cdot Rxy$

> $B := Multiply(Multiply(M2, Rxy), A);$
$a := arrow(A, color = gray) : b := arrow(B, color = \text{black}) :$
$\theta := evalf\left(\frac{180}{\pi} \cdot \cos^{-1}(DotProduct(A, B))\right);$ # *angle between A and B*
$display([\text{a, b, p1, }], \text{axes} = \text{normal, tickmarks} = [2, 2, 2], \text{scaling} = \text{constrained, labels} = [\text{" ", " "}, Z], orientation = [-21, 80]);$

$$B := \begin{bmatrix} \frac{1}{9}\sqrt{3} \\ \frac{1}{9}\sqrt{3} \\ -\frac{5}{9}\sqrt{3} \end{bmatrix}$$

$$\theta := 109.4712207$$



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