> $restart:$

> $interface(warnlevel = 0):$ # **Maple 12**

> $with(plots):$

> $with(LinearAlgebra):$

A unit circle - as a reference

> $circle := implicitplot(x^2 + y^2 = 1, \ x = -1..1, y = -1..1, \ axes = \ normal, \ color = \ black):$

Vector V_0 . An interesting set of angles $\alpha = \left\{ \frac{1.047}{2}, \frac{0.7227}{2}, \frac{0.3554}{2}, \frac{0.1769}{2}, \frac{0.1251}{2}, \frac{0.06248}{2} \right\}$

$$\mathbf{V_0 = \cos(\alpha)x + \sin(\alpha)y}$$

> $V_0 := Vector([\cos(\alpha), \ \sin(\alpha)]);$
> $\alpha := \frac{0.1769}{2};$

$$V_0 := \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \end{bmatrix}$$

$$\alpha := 0.08845000000 \tag{1}$$

This is the first reflection operator

$$\mathrm{U} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

> $\mathrm{U} := \theta \rightarrow Matrix([[1, 0], [0, \cos(\theta)]]):$

This is the second reflection operator

$$\mathrm{R} = \begin{bmatrix} \cos(2\,\alpha) & \sin(2\,\alpha) \\ \sin(2\,\alpha) & -\cos(2\,\alpha) \end{bmatrix}$$

> $\mathrm{R} := \theta \rightarrow Matrix([[\cos(2 \cdot \theta), \sin(2 \cdot \theta)], [\sin(2 \cdot \theta), -\cos(2 \cdot \theta)]]):$

Vector M is the reflection of Vector V_0 about the x-axis

$$\mathbf{M} = \cos(\alpha)\mathbf{x} - \sin(\alpha)\mathbf{y}$$

```
> 'V[0]' = V[0]; M := simplify(Multiply(U(π), V[0])) : ' U(π) · V[0] ' = M;
```

$$V_0 = \begin{bmatrix} 0.9960908483 \\ 0.08833471511 \end{bmatrix}$$

$$U(\pi)\ V_0 = \begin{bmatrix} 0.9960908483 \\ -0.08833471511 \end{bmatrix} \tag{2}$$

Vector N is the reflection of Vector M about initial V_0 vector

$$\mathbf{M} = \cos(\alpha)\mathbf{x} - \sin(\alpha)\mathbf{y}$$

```
> N := simplify(Multiply(R(α), M)) : ' R(α) ·M' = N;
```

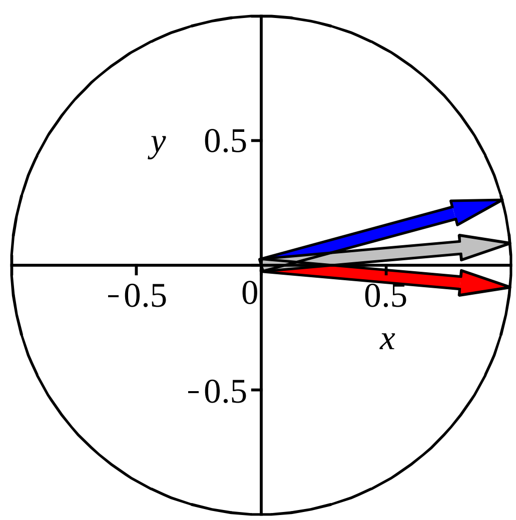
$$R(\alpha)\ M = \begin{bmatrix} 0.9650007735 \\ 0.2622470344 \end{bmatrix} \tag{3}$$

Display the initial vector V_0 (gray), the reflection of the initial vector V_0 (red) about the x-axis and the vector(blue) resulting from the reflection of the reflection of the initial vector V_0(red) about the initial vector V_0(gray) .

```
> k := 1; a1 := arrow(V[0], color = gray) : b1 := arrow(M, color = red) : c1 := arrow(N, color = blue) :
P := N[2]^2;   # square of the coefficient of the y-component
display([circle, a1, b1, c1], axes = normal, scaling = constrained, tickmarks = [4, 4]);
θ := evalf((180/π) · cos^(-1)(DotProduct(N, Vector([cos(α), sin(α)]))));
    # angle between N and V0
```

$$k := 1$$

$$P := 0.06877350705$$



$$\theta := 10.13562340 \tag{4}$$

Repeat the preceding reflection steps until the square of the coefficient of the y-component reaches a maximum. This is equivalent to a series of CCW rotations of the initial vector V_0.

```
> a1 := arrow(V_0, color = black, shape = harpoon) : # this is a line in the direction of  V_0
   do
     Pold := N[2]^2 :   # square of the coefficient of the y-component
     k := k + 1;
     M := simplify(Multiply(U(π), N)) :
     N := simplify(Multiply(R(α), M)) :
     Pnew := N[2]^2 :
     if  Pnew < Pold then
       printf("\n     The Maximum Value is %f, after %d iterations\n", Pold, k − 1);
       quit(0);
     end if;
     a2 := arrow(M,  color = red) :     # display the vectors
     a3 := arrow(N, color = blue) :
     display([circle, a1, a2, a3], axes = normal, scaling = constrained,  tickmarks = [4, 4]);
     θ := evalf((180/π)·cos^(−1)(DotProduct(N, Vector([cos(α), sin(α)]))));
     print( ); print( );
   end do;
```

$$Pold := 0.06877350705$$

$$k := 2$$

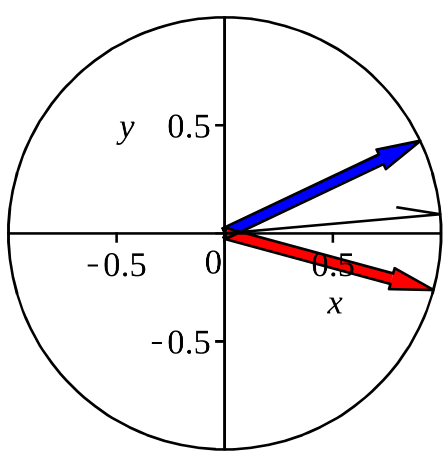
$$M := \begin{bmatrix} 0.9650007735 \\ -0.2622470344 \end{bmatrix}$$

$$N := \begin{bmatrix} 0.9037910101 \\ 0.4279740763 \end{bmatrix}$$

$$Pnew := 0.1831618100$$

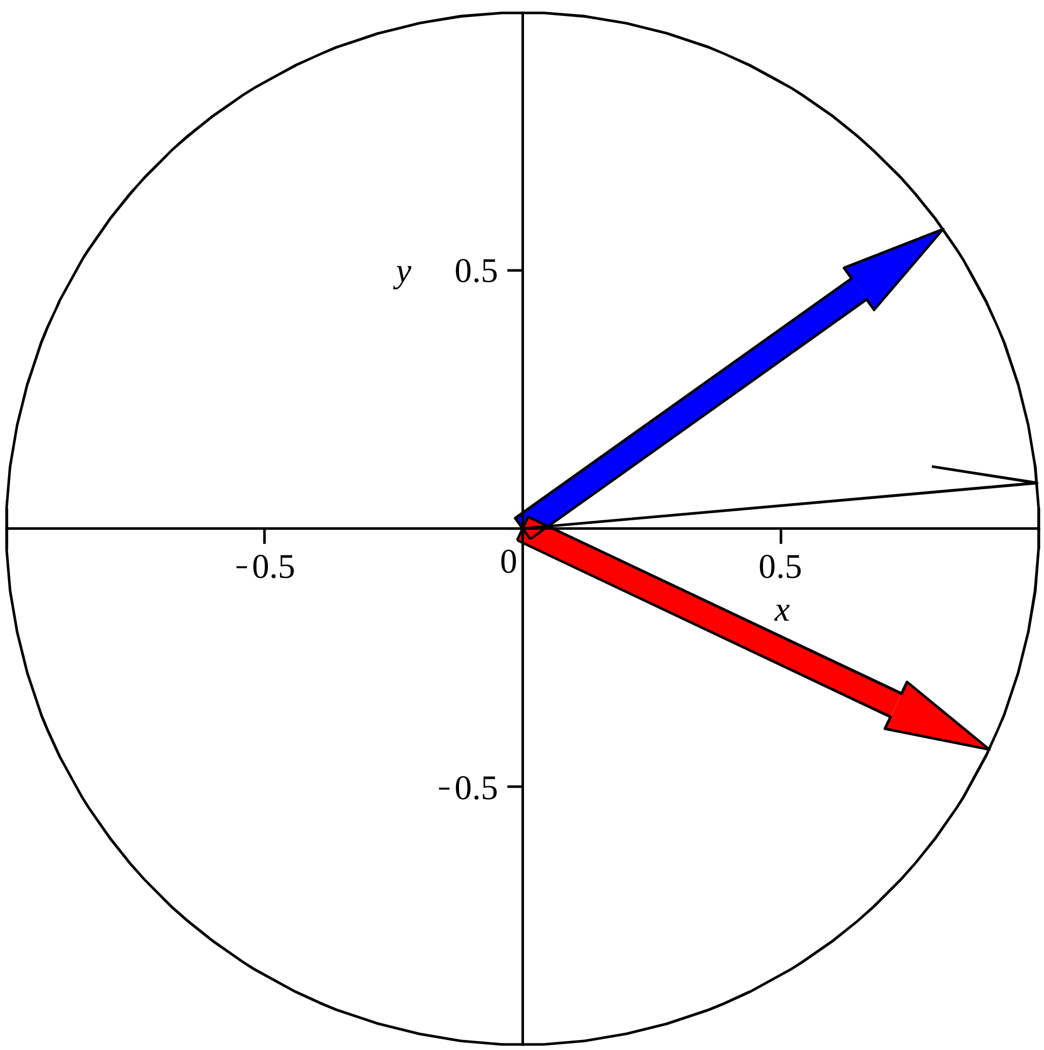
$$a2 := PLOT(\ldots)$$

$$a3 := PLOT(\ldots)$$



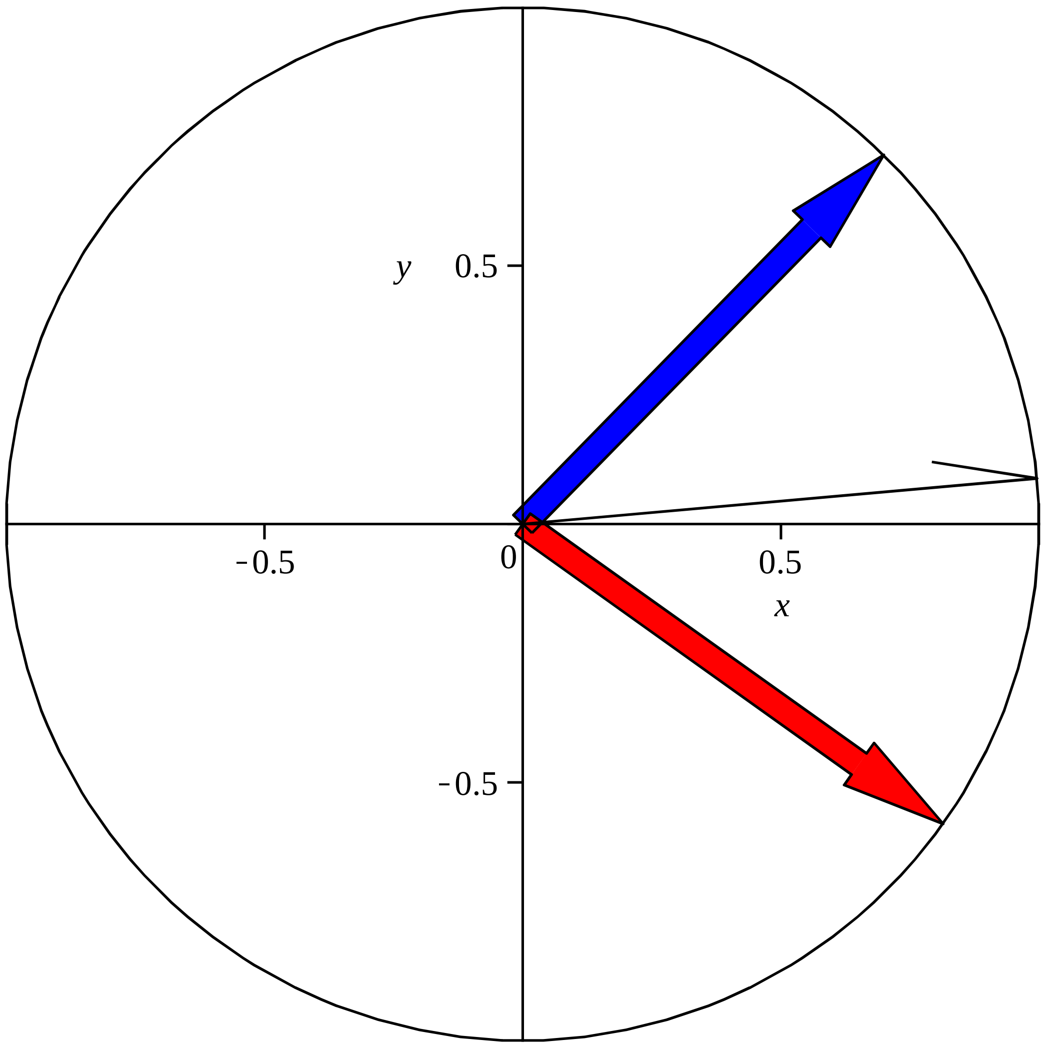
$$\theta := 20.27124680$$

$$Pold := 0.1831618100$$
$$k := 3$$
$$M := \begin{bmatrix} 0.9037910101 \\ -0.4279740763 \end{bmatrix}$$
$$N := \begin{bmatrix} 0.8143720425 \\ 0.5803431539 \end{bmatrix}$$
$$Pnew := 0.3367981763$$
$$a2 := PLOT(\ldots)$$
$$a3 := PLOT(\ldots)$$



$$\theta := 30.40687018$$

$$Pold := 0.3367981763$$
$$k := 4$$
$$M := \begin{bmatrix} 0.8143720425 \\ -0.5803431539 \end{bmatrix}$$
$$N := \begin{bmatrix} 0.6995348234 \\ 0.7145985101 \end{bmatrix}$$
$$Pnew := 0.5106510306$$
$$a2 := PLOT(\ldots)$$
$$a3 := PLOT(\ldots)$$



$$\theta := 40.54249357$$

$Pold := 0.5106510306$

$k := 5$

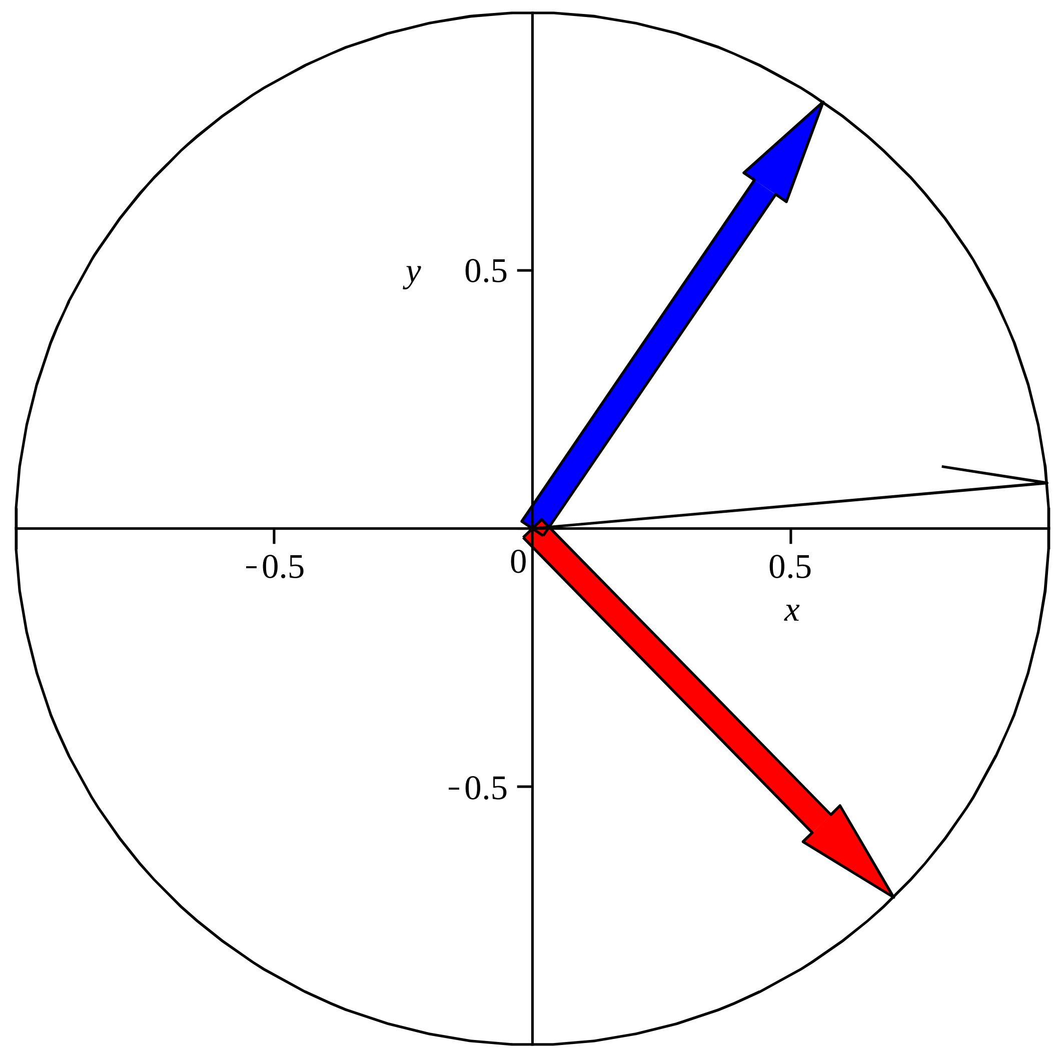
$$M := \begin{bmatrix} 0.6995348234 \\ -0.7145985101 \end{bmatrix}$$

$$N := \begin{bmatrix} 0.5628636622 \\ 0.8265497550 \end{bmatrix}$$

$Pnew := 0.6831844975$

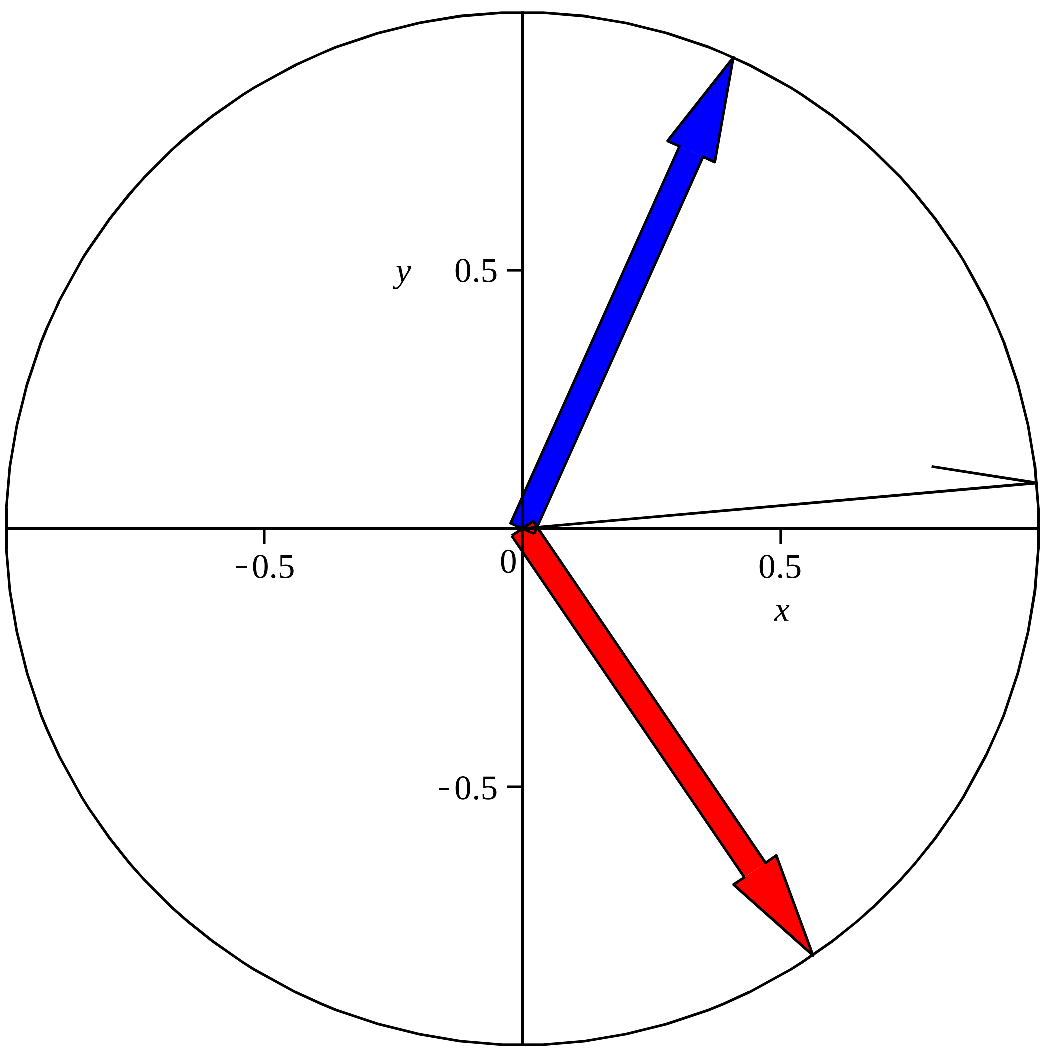
$a2 := PLOT(...)$

$a3 := PLOT(...)$



$\theta := 50.67811697$

$$Pold := 0.6831844975$$
$$k := 6$$
$$M := \begin{bmatrix} 0.5628636622 \\ -0.8265497550 \end{bmatrix}$$
$$N := \begin{bmatrix} 0.4086243511 \\ 0.9127026566 \end{bmatrix}$$
$$Pnew := 0.8330261394$$
$$a2 := PLOT(\ldots)$$
$$a3 := PLOT(\ldots)$$



$$\theta := 60.81374036$$

$Pold := 0.8330261394$

$k := 7$

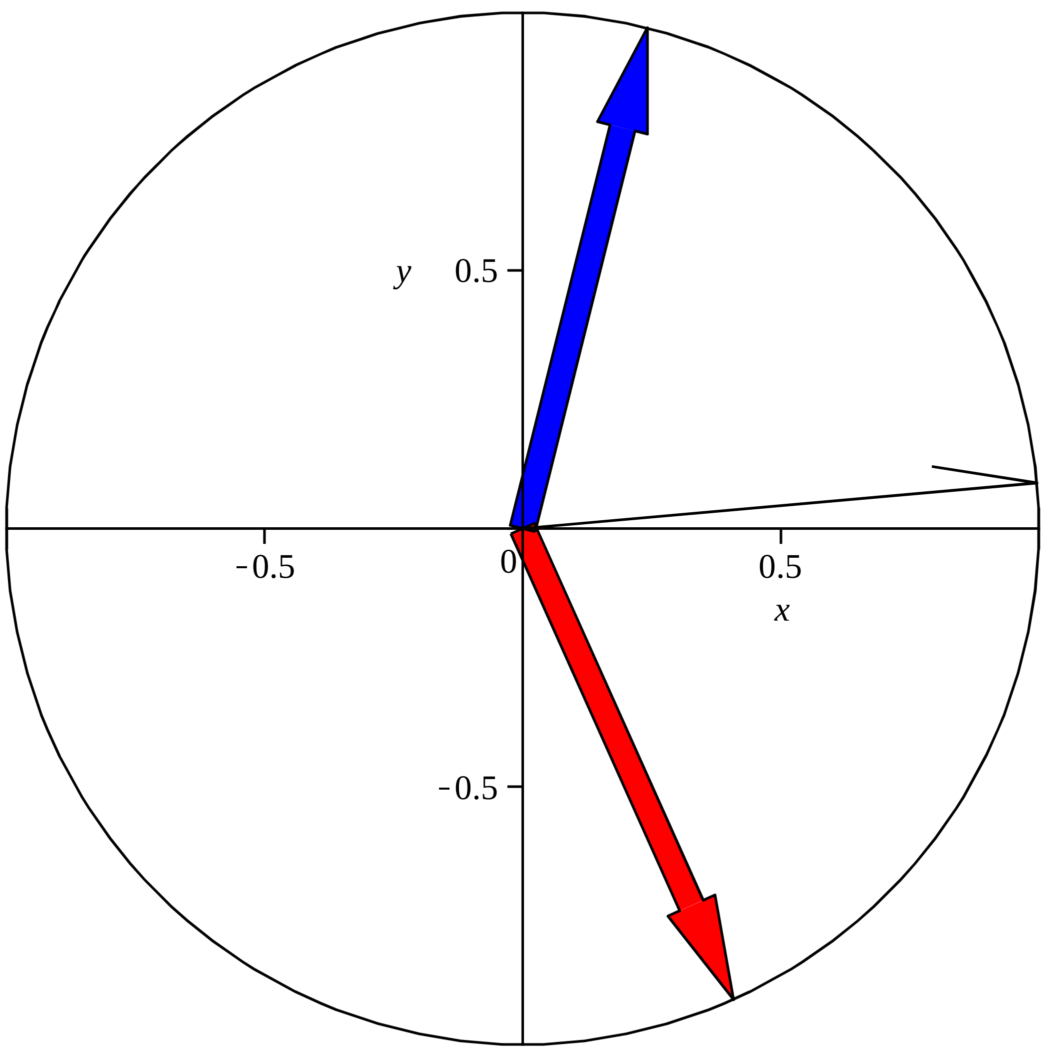
$$M := \begin{bmatrix} 0.4086243511 \\ -0.9127026566 \end{bmatrix}$$

$$N := \begin{bmatrix} 0.2416310209 \\ 0.9703682030 \end{bmatrix}$$

$Pnew := 0.9416144494$

$a2 := PLOT(\ldots)$

$a3 := PLOT(\ldots)$



$\theta := 70.94936375$

$$Pold := 0.9416144494$$

$$k := 8$$

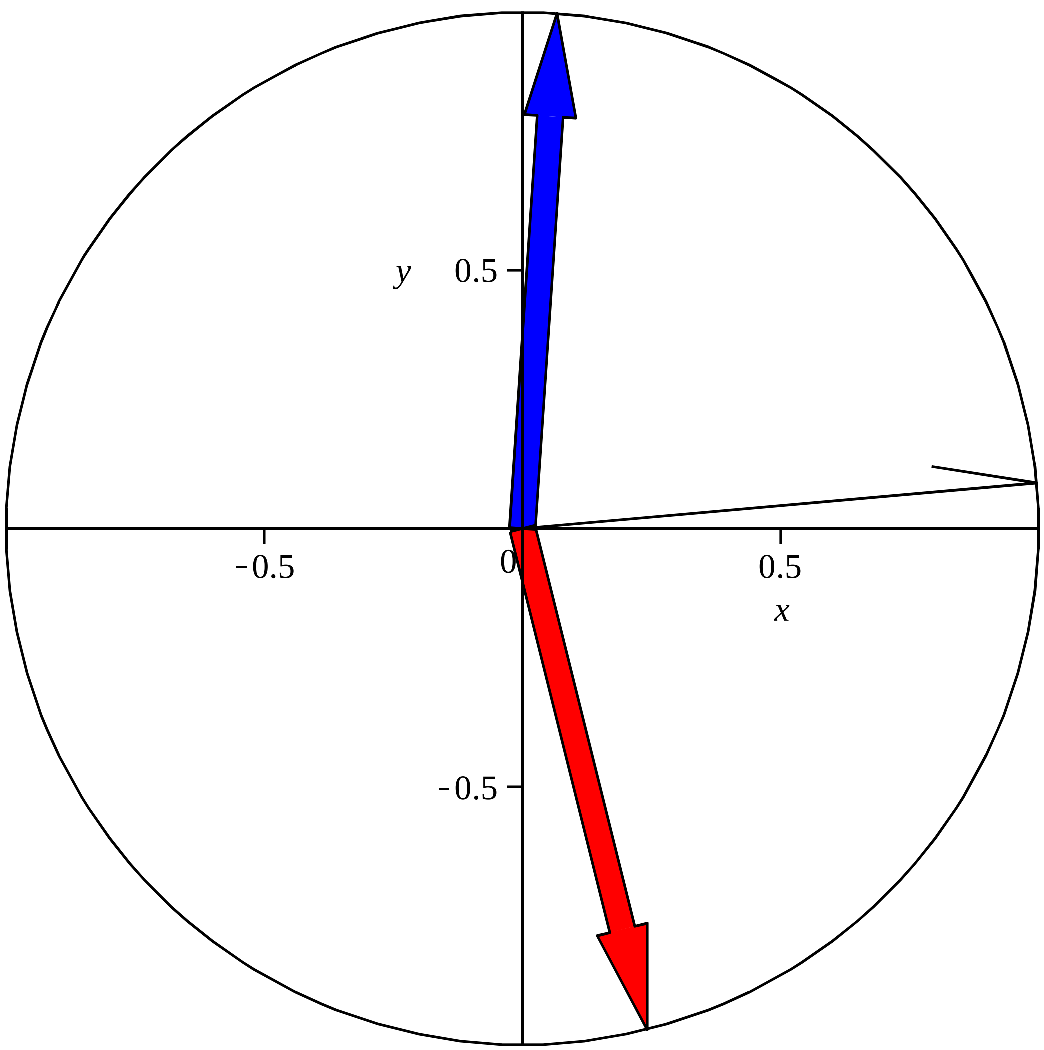
$$M := \begin{bmatrix} 0.2416310209 \\ -0.9703682030 \end{bmatrix}$$

$$N := \begin{bmatrix} 0.06709588216 \\ 0.9977465321 \end{bmatrix}$$

$$Pnew := 0.9954981423$$

$$a2 := PLOT(...)$$

$$a3 := PLOT(...)$$



$$\theta := 81.08498715$$

$$Pold := 0.9954981423$$

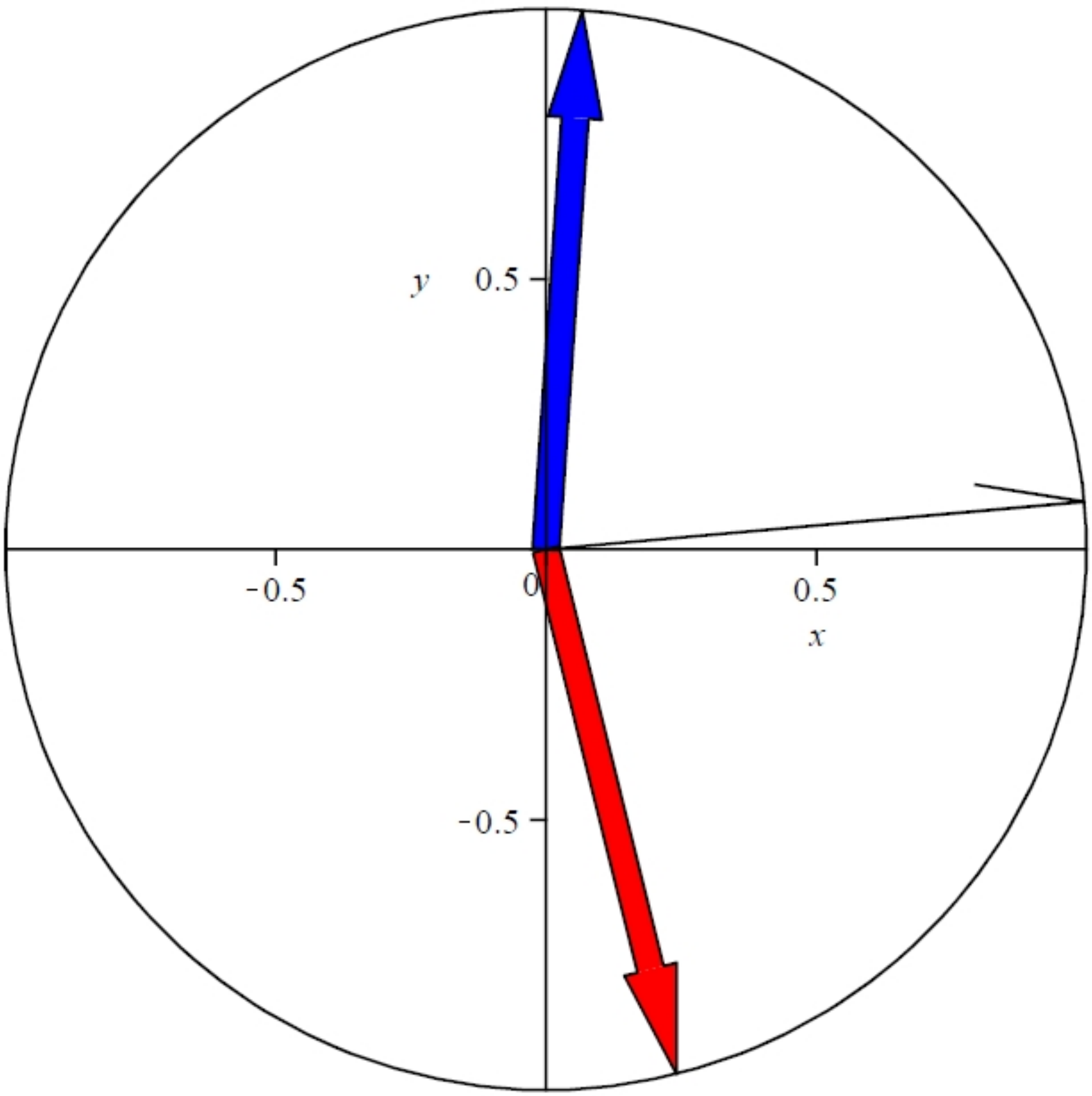
$$k := 9$$

$$M := \begin{bmatrix} 0.06709588216 \\ -0.9977465321 \end{bmatrix}$$

$$N := \begin{bmatrix} -0.1095334591 \\ 0.9939831090 \end{bmatrix}$$

$$Pnew := 0.9880024210$$

```
The Maximum Value is 0.995498, after 8 iterations
```



$$\theta := 81.08498715$$