```
> restart;
> interface(warnlevel = 0) :            # Maple 12
> with(LinearAlgebra) :
> with(plots) :
```

planes

```
> fxy := z = 0 :    # equation of an  x−y  plane in 3D space
  fzy := x = 0 :   # equation of an  z−y  plane in 3 D space
  fzx := y = 0 :   # equation of an  z−x  plane in 3 D space
  f2 := x + y + z = 0 :    # equation of a  plane whose normal vector  is (x + y + z)
  f3 := -x − y + 2 z = 0 :  #` `a plane` `containing vector (x + y + z)

> pxy := implicitplot3d(fxy, x =-2 ..2, y =-2 ..2, z =-2 ..2, axes = normal,
                         style = patchnogrid,  color = cyan, transparency = 0.6) :
  pzy := implicitplot3d(fzy, x =-2 ..2, y =-2 ..2, z =-2 ..2, axes = normal,
                         style = patchnogrid,  color = cyan, transparency = 0.6) :
  pzx := implicitplot3d(fzx, x =-2 ..2, y =-2 ..2, z =-2 ..2, axes = normal,
                         style = patchnogrid,  color = cyan, transparency = 0.6) :
  p2 := implicitplot3d(f2,  x =-2 ..2, y =-2 ..2, z =-2 ..2, axes = normal,
                         style = patchnogrid,  color = green, transparency = 0.6) :
  p3 := implicitplot3d(f3, x =-2 ..2, y =-2 ..2, z =-2 ..2, axes = normal,
                         style = patchnogrid,  color = blue, transparency = 0.6) :
```

The operators/matrices

```
> Rxy := Matrix([[1, 0, 0], [0, 1, 0], [0, 0,-1]]) : # a reflection matrix  across the x-y plane
  Rzy := Matrix([[-1, 0, 0], [0, 1, 0], [0, 0, 1]]) :# a reflection matrix  across the z-y plane
  Rzx := Matrix([[1, 0, 0], [0,-1, 0], [0, 0, 1]]) : # a reflection matrix  across the z-x plane
  I3 := IdentityMatrix(3) :
```

$$M1 := 2\left(Multiply\left(\frac{\langle 1, 1, 1\rangle}{\sqrt{3}}, Transpose\left(\frac{\langle 1, 1, 1\rangle}{\sqrt{3}}\right)\right)\right) - I3; \quad \#\quad 2\left(AA^T\right) - \mathrm{I}$$

$$M2 := I3 - 2\left(Multiply\left(\frac{\langle 1, 1, 1\rangle}{\sqrt{3}}, Transpose\left(\frac{\langle 1, 1, 1\rangle}{\sqrt{3}}\right)\right)\right); \quad \#\quad \mathrm{I} - 2\left(AA^T\right)$$

$$M1 := \begin{bmatrix} -\frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \end{bmatrix} \qquad (1)$$

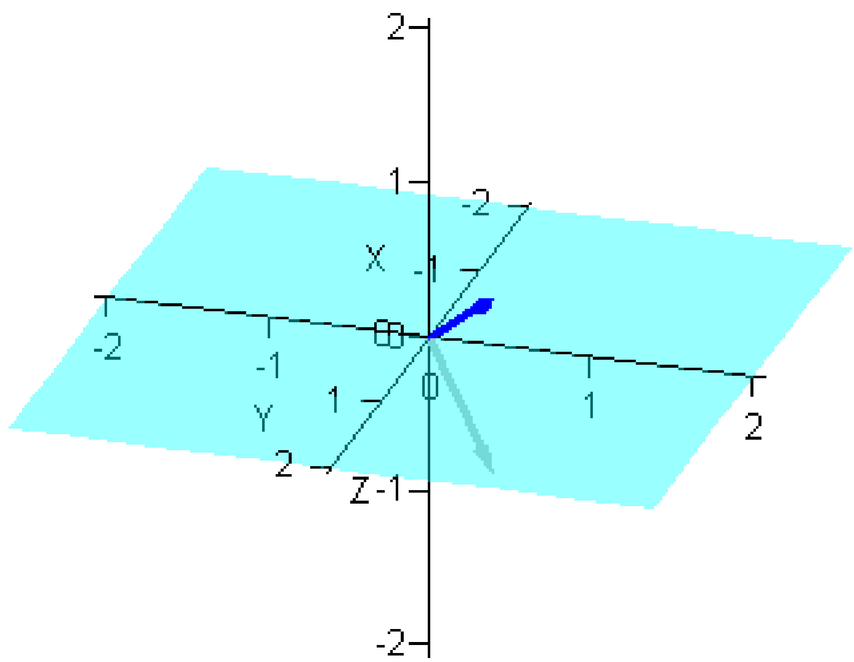
$$M2 := \begin{bmatrix} \frac{1}{3} & -\frac{2}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{bmatrix} \quad \textbf{(1)}$$

Reflection of Vector A = x + y + z across the x-y plane

> $A := \frac{1}{\sqrt{3}} \cdot \langle 1, 1, 1 \rangle$: # *vector A*
> A1 := Multiply(Rxy, A); # *vector A'*
> *a* := *arrow*(*A*, *color* = *blue*) : *a1* := *arrow*(A1, color = gray) :
> $\theta := evalf\left(\frac{180}{\pi} \cdot \cos^{-1}(DotProduct(A, A1))\right)$; # *angle between A and A1*
> *display*([a, a1, pxy], axes = normal, tickmarks = [4, 4, 4], scaling = constrained, labels = [X, Y, Z], *orientation* = [17, 66]) ;

$$A1 := \begin{bmatrix} \frac{1}{3}\sqrt{3} \\ \frac{1}{3}\sqrt{3} \\ -\frac{1}{3}\sqrt{3} \end{bmatrix}$$

$$\theta := 70.52877934$$



Reflection of Vector A = **x** + **y** + **z** **across the z-y plane**

$$> A := \frac{1}{\sqrt{3}} \cdot \langle 1, 1, 1 \rangle : \qquad \# \ vector\ A$$

A2 := Multiply(Rzy, A); # *vector A'*

a := *arrow*(*A*, *color* = *blue*) :

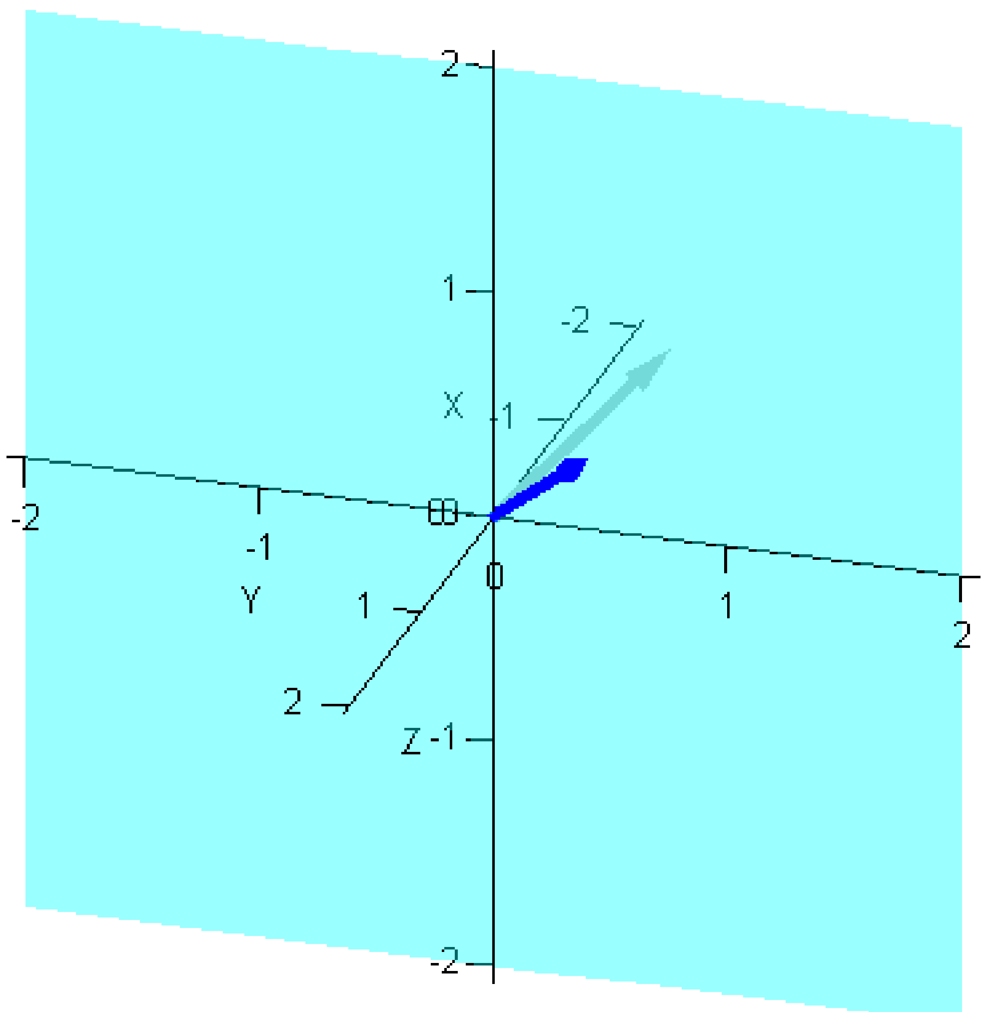
a2 := *arrow*(A2, color = gray) :

$$\theta := evalf\left(\frac{180}{\pi} \cdot \cos^{-1}(DotProduct(A, A2))\right); \ \# \ angle\ between\ A\ and\ A2$$

display([a, a2, pzy], axes = normal, tickmarks = [4, 4, 4], scaling = constrained, labels = [X, Y, Z], *orientation* = [17, 66]) ;

$$A2 := \begin{bmatrix} -\frac{1}{3}\sqrt{3} \\ \frac{1}{3}\sqrt{3} \\ \frac{1}{3}\sqrt{3} \end{bmatrix}$$

$$\theta := 70.52877934$$



Reflection of Vector A = **x** + **y** + **z** **across the z-x plane**

$$> \mathrm{A} := \frac{1}{\sqrt{3}} \cdot \langle 1, 1, 1 \rangle : \quad \# \ \textit{vector A}$$

$$\mathrm{A3} := \mathrm{Multiply}(\mathrm{Rzx}, \mathrm{A}); \ \# \ \textit{vector A'}$$

$$a := arrow(A, color = blue) :$$

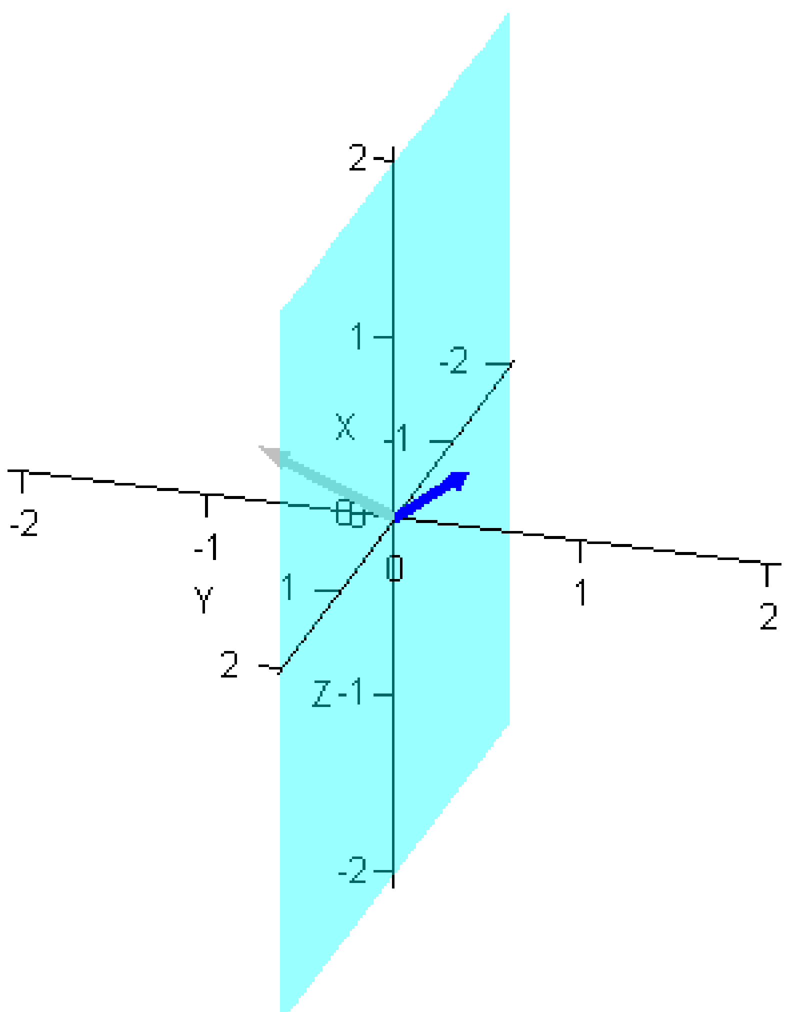
$$a3 := arrow(\mathrm{A3}, \mathrm{color} = \mathrm{gray}) :$$

$$\theta := evalf\left(\frac{180}{\pi} \cdot \cos^{-1}(DotProduct(A, A3)) \right); \ \# \ \textit{angle between A and A3}$$

$$display([\mathrm{a}, \mathrm{a3}, \mathrm{pzx}], \mathrm{axes} = \mathrm{normal}, \mathrm{tickmarks} = [4, 4, 4], \mathrm{scaling} = \mathrm{constrained}, \mathrm{labels} = [\mathrm{X}, \mathrm{Y}, \mathrm{Z}], orientation = [17, 66]);$$

$$A3 := \begin{bmatrix} \frac{1}{3}\sqrt{3} \\ -\frac{1}{3}\sqrt{3} \\ \frac{1}{3}\sqrt{3} \end{bmatrix}$$

$$\theta := 70.52877934$$

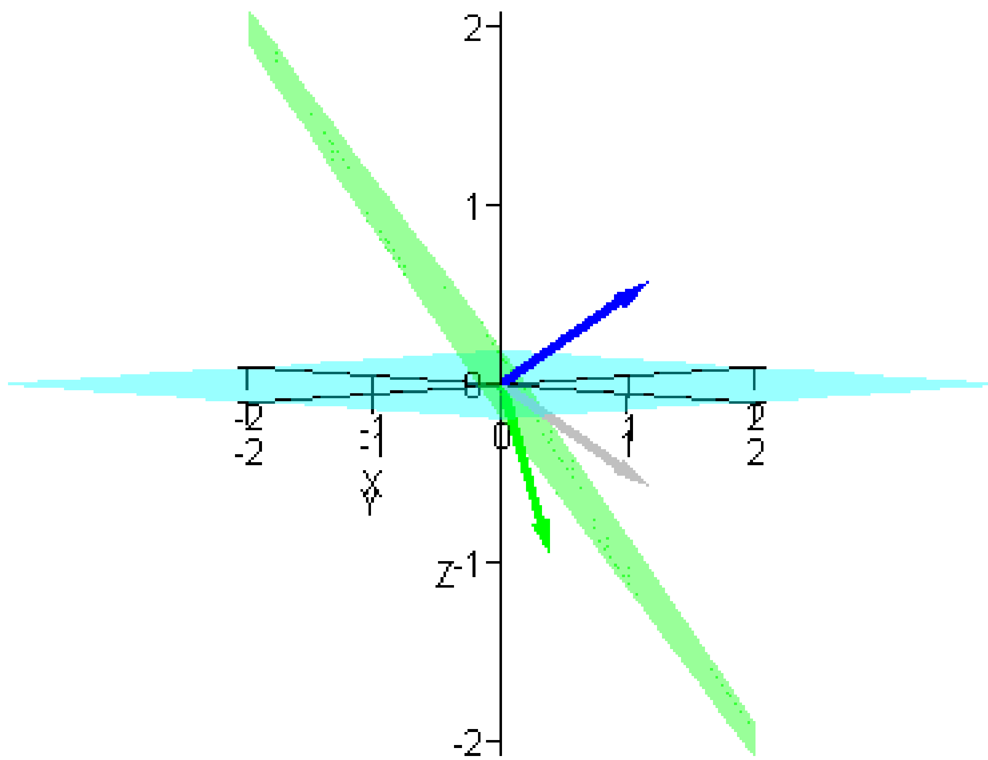


Reflection of Vector A' = x + y - z by $(I - 2(AA^T))$

> $A4 := factor(Multiply(M2, A1));$ # using $\mathrm{I} - 2(AA^T)$. *Reflection of A1 across plane f2*
> $a4 := arrow(A4, color = green):$
> $\theta := evalf\left(\frac{180}{\pi} \cdot \cos^{-1}(DotProduct(A, A4))\right);$ # *angle between A and A4*
> *display*([a, a1, a4, pxy, p2], axes = normal, tickmarks = [4, 4, 4], scaling = constrained, labels = [X, Y, Z], *orientation* = [-45, 86]) ;

$$A4 := \begin{bmatrix} \frac{1}{9}\sqrt{3} \\ \frac{1}{9}\sqrt{3} \\ -\frac{5}{9}\sqrt{3} \end{bmatrix}$$

$$\theta := 109.4712207$$

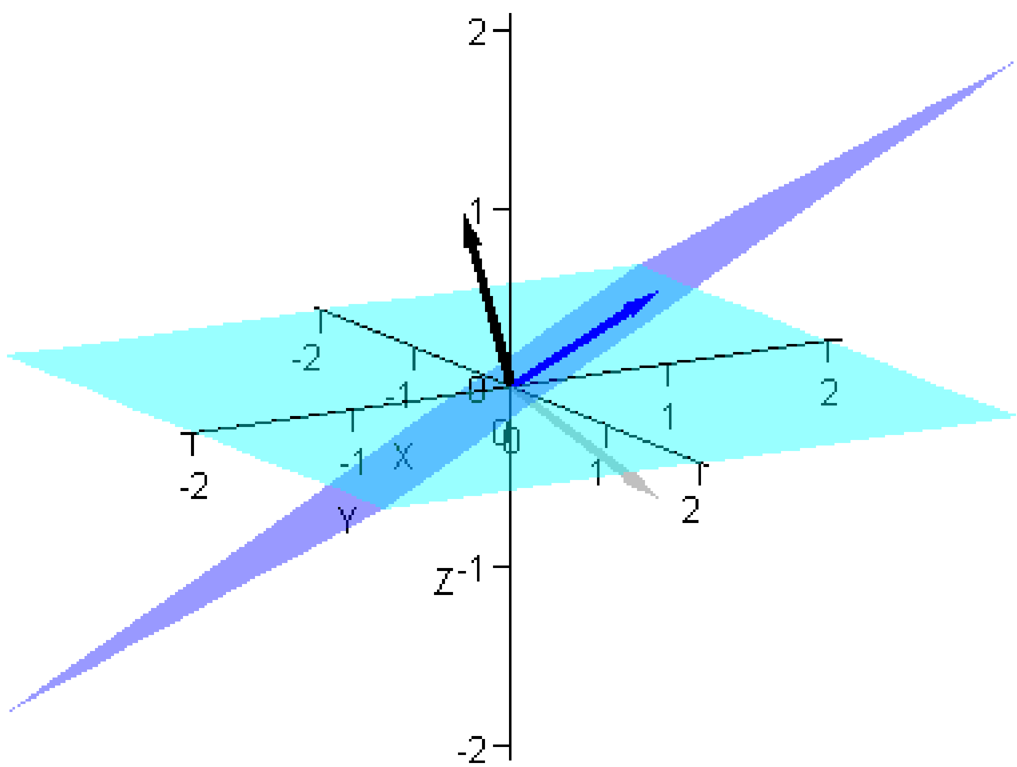


Reflection of Vector A' = x + y - z by $(2(AA^T) - I)$

> $A4 := Multiply(M1, A1)$; # *using* $2(AA^T) - \mathrm{I}$. *Reflection of A1 across vector A; plane f3*
$a4 := arrow(A4, color = black)$:
$\theta := evalf\left(\frac{180}{\pi} \cdot \cos^{-1}(DotProduct(A, A4))\right)$; # *angle between A and A4*
display([a, a1, a4, p3, pxy], axes = normal, tickmarks = [4, 4, 4], scaling = constrained, labels = [X, Y, Z], *orientation* = [-31, 76]) ;

$$A4 := \begin{bmatrix} -\frac{1}{9}\sqrt{3} \\ -\frac{1}{9}\sqrt{3} \\ \frac{5}{9}\sqrt{3} \end{bmatrix}$$

$$\theta := 70.52877934$$

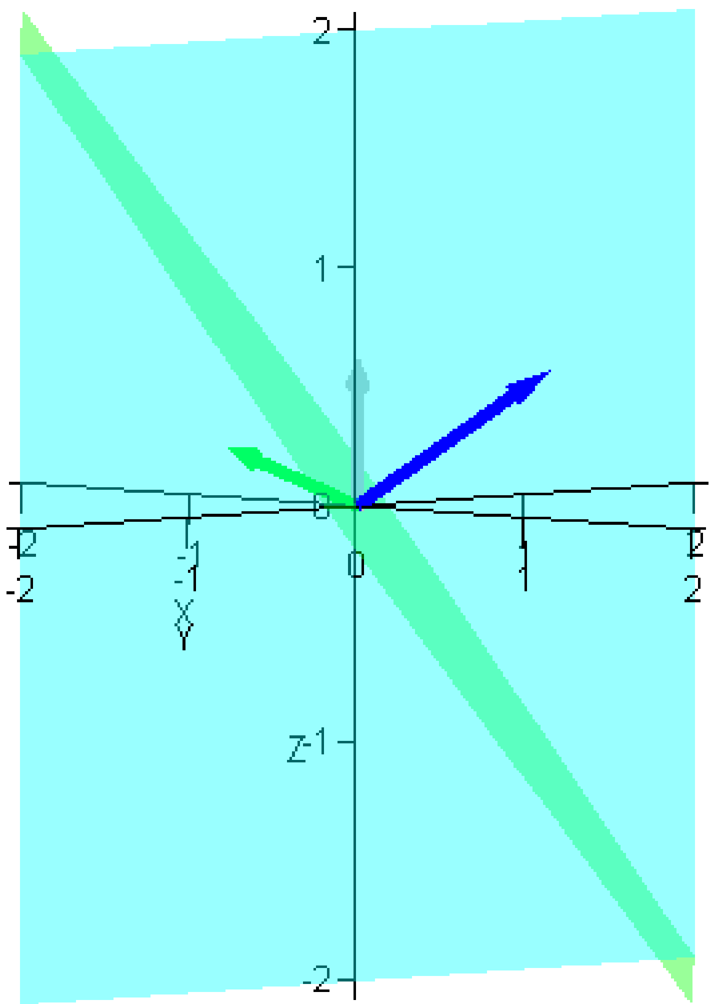


Reflection of Vector A' = -x + y + z by $\left(I - 2\left(AA^{T}\right)\right)$

> $A5 := factor(Multiply(M2, A2))$; # using $I - 2\left(AA^{T}\right)$. *Reflection of* A2 across *plane f2*
$a5 := arrow(A5, color = green)$:
$\theta := evalf\left(\frac{180}{\pi} \cdot \cos^{-1}(DotProduct(A, A5))\right)$; # *angle between A and A5*
display([a, a2, a5, p2, pzy], axes = normal, tickmarks = [4, 4, 4], scaling = constrained, labels = [X, Y, Z], *orientation* = [−45, 86]) ;

$$A5 := \begin{bmatrix} -\frac{5}{9}\sqrt{3} \\ \frac{1}{9}\sqrt{3} \\ \frac{1}{9}\sqrt{3} \end{bmatrix}$$

$$\theta := 109.4712207$$

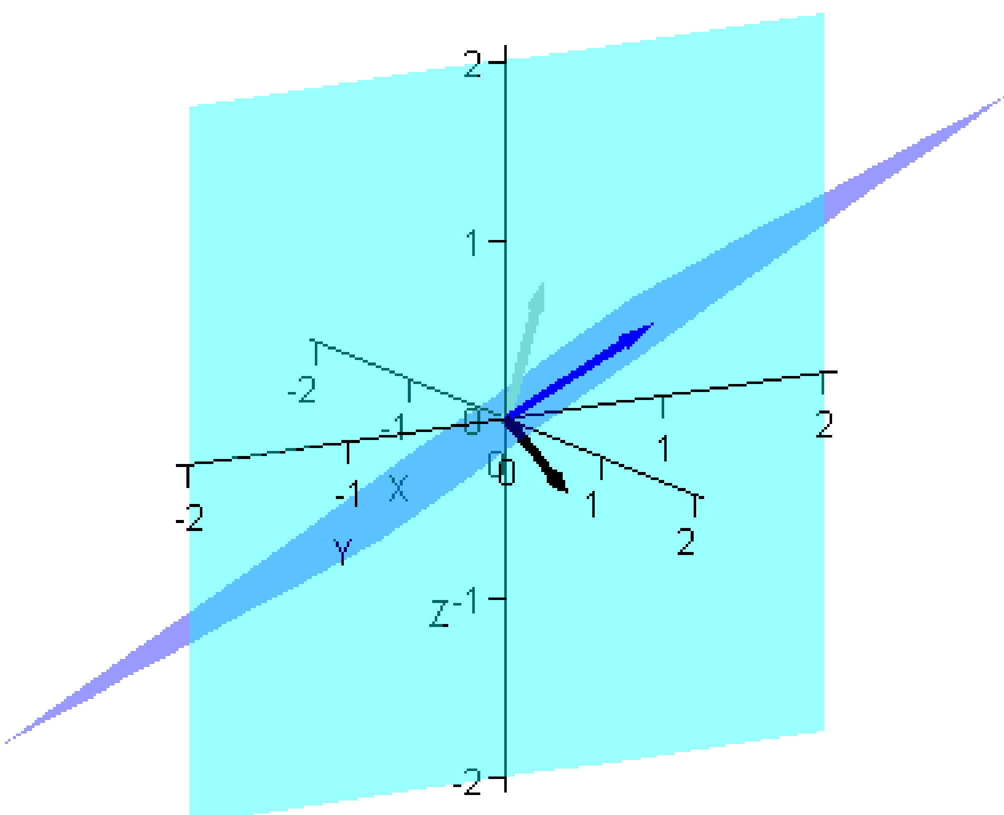


Reflection of Vector A' = -x + y + z by $(2(AA^T) - I)$

```
> A5 := Multiply(M1, A2);  # using 2(AA^T) - I. Reflection of A2 across vector A; plane f3
  a5 := arrow(A5, color = black) :
  θ := evalf(180/π · cos^(-1)(DotProduct(A, A5))); # angle between A and A5
  display([a, a2, a5, p3, pzy], axes = normal, tickmarks = [4, 4, 4], scaling = constrained, labels = [X, Y,
     Z],                         orientation = [-31, 76]) ;
```

$$A5 := \begin{bmatrix} \frac{5}{9}\sqrt{3} \\ -\frac{1}{9}\sqrt{3} \\ -\frac{1}{9}\sqrt{3} \end{bmatrix}$$

$$\theta := 70.52877934$$

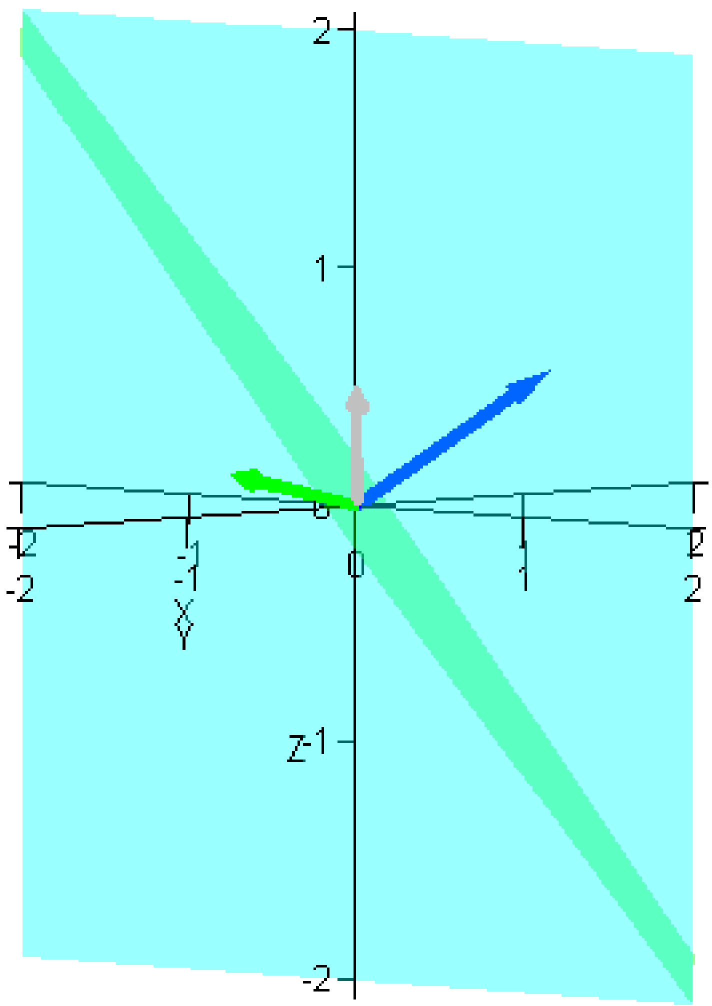


Reflection of Vector A' = x - y + z by $(I - 2(AA^T))$

> $A6 := factor(Multiply(M2, A3));$ # using $I - 2(AA^T)$. *Reflection of* A3 across *plane f2*
$a6 := arrow(A6, color = green):$
$\theta := evalf\left(\frac{180}{\pi} \cdot \cos^{-1}(DotProduct(A, A6))\right);$ # *angle between A and A6*
display([a, a3, a6, p2, pzx], axes = normal, tickmarks = [4, 4, 4], scaling = constrained, labels = [X, Y, Z], *orientation* = [−45, 86]);

$$A6 := \begin{bmatrix} \frac{1}{9}\sqrt{3} \\ -\frac{5}{9}\sqrt{3} \\ \frac{1}{9}\sqrt{3} \end{bmatrix}$$

$$\theta := 109.4712207$$



Reflection of Vector A' = x - y + z by $(2(AA^T) - I)$

> $A6 := Multiply(M1, A3);$ # *using* $2(AA^T) - I$. *Reflection of A3 across vector A; plane f3*
$a6 := arrow(A6, color = black):$
$\theta := evalf\left(\frac{180}{\pi} \cdot \cos^{-1}(DotProduct(A, A6))\right);$ # *angle between A and A6*
display([a, a3, a6, p3, pzx], axes = normal, tickmarks = [4, 4, 4], scaling = constrained, labels = [X, Y, Z], *orientation* = [-31, 76]) ;

$$A6 := \begin{bmatrix} -\frac{1}{9}\sqrt{3} \\ \frac{5}{9}\sqrt{3} \\ -\frac{1}{9}\sqrt{3} \end{bmatrix}$$

$$\theta := 70.52877934$$