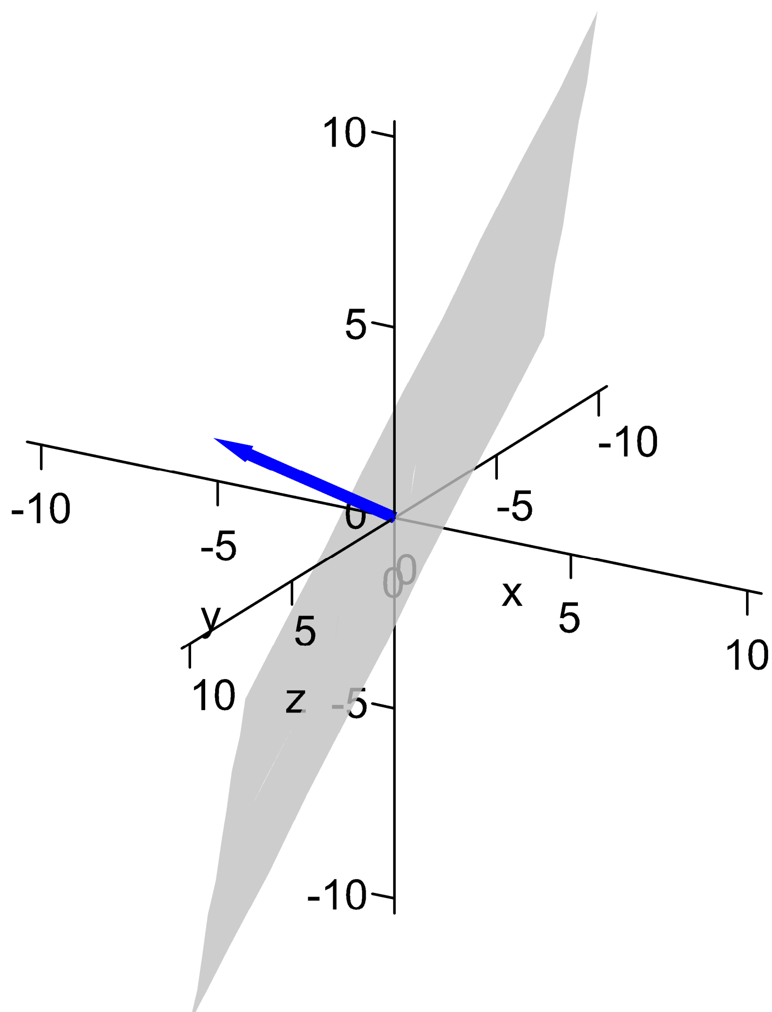
```
> restart;
> interface(warnlevel = 0) :                #      Maple 12
> with(plots) :
  with(LinearAlgebra) :
```

Vector Operations in 3D space

A Plane Through the Origin

```
> f := 2·x − 4·y + 2·z = 0 : # equation of a plane through the origin in 3D space
  p := implicitplot3d(f, x = -10 ..10, y = -10 ..10, z = -10 ..10, axes = normal,
                      style = patchnogrid, color = grey, transparency = 0.2) :
  n := arrow(⟨2, -4, 2⟩, color = blue) : # normal to the plane
  display([n, p], orientation = [30, 69], scaling = constrained, tickmarks = [4, 4, 4]);
```

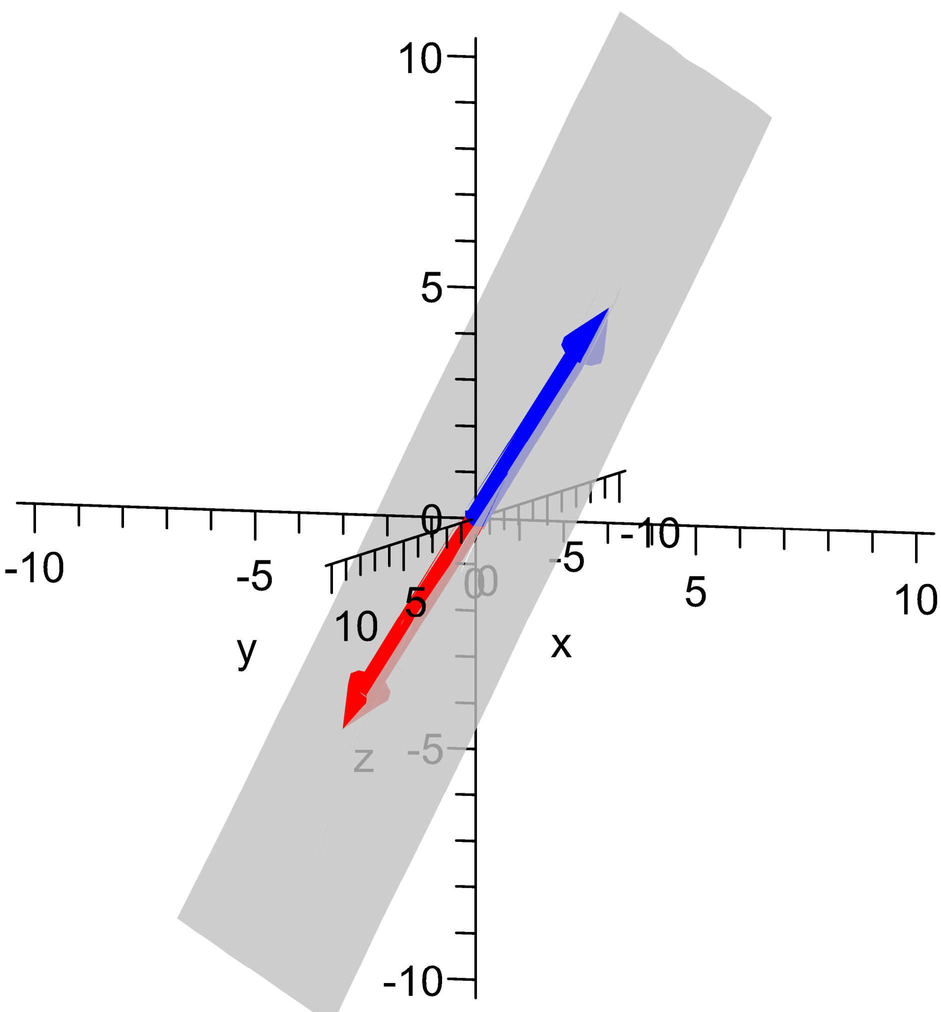


Scalar Multiplication

> $B := \langle 3, 4, 5 \rangle$; # vector B
> $C := -1 \cdot B$; # vector C
> $b := arrow(B, color = blue, width = 0.5)$:
> $c := arrow(C, color = red, width = 0.5)$:
> $display([p, b, c], orientation = [18, 84])$;

$$B := \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

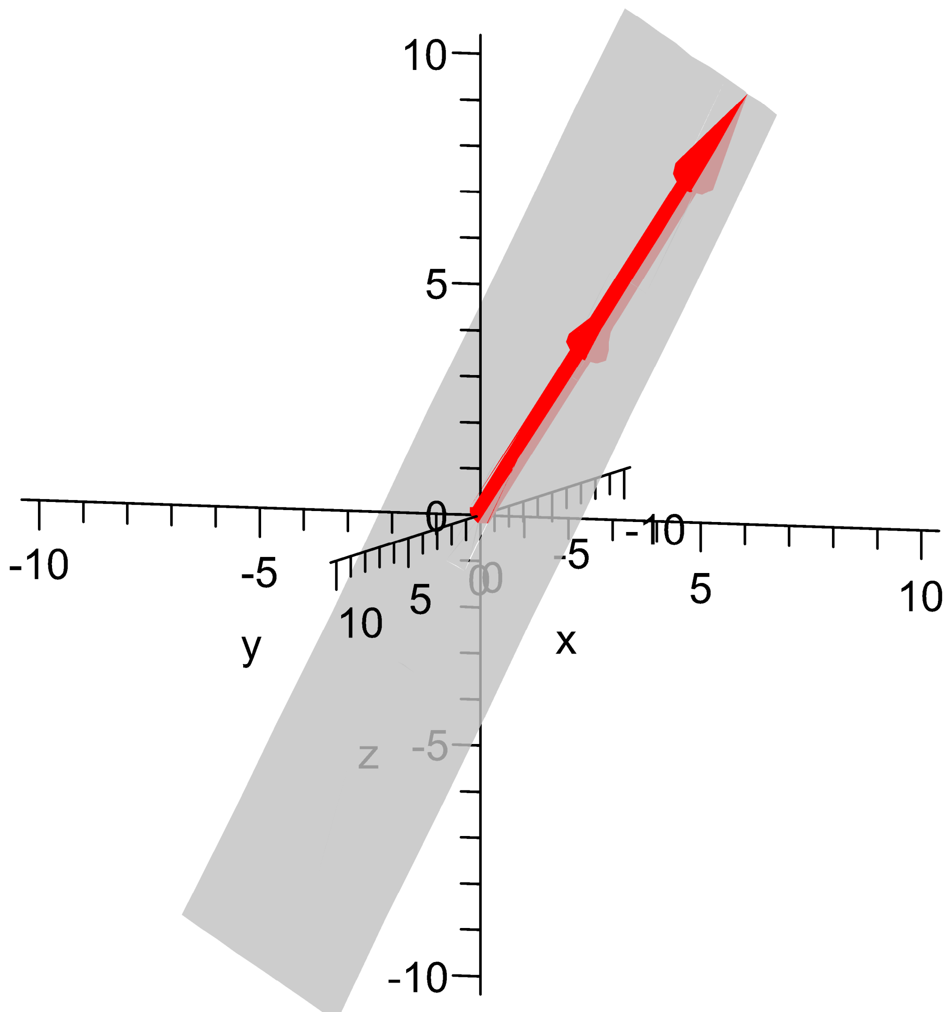
$$C := \begin{bmatrix} -3 \\ -4 \\ -5 \end{bmatrix}$$



> $B := \langle 3, 4, 5 \rangle;$
$C := 2 \cdot B;$
$b := arrow(B, color = red, width = 0.5) :$
$c := arrow(C, color = red, width = 0.5) :$
$display([p, c, b], orientation = [18, 84]);$

$$B := \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

$$C := \begin{bmatrix} 6 \\ 8 \\ 10 \end{bmatrix}$$



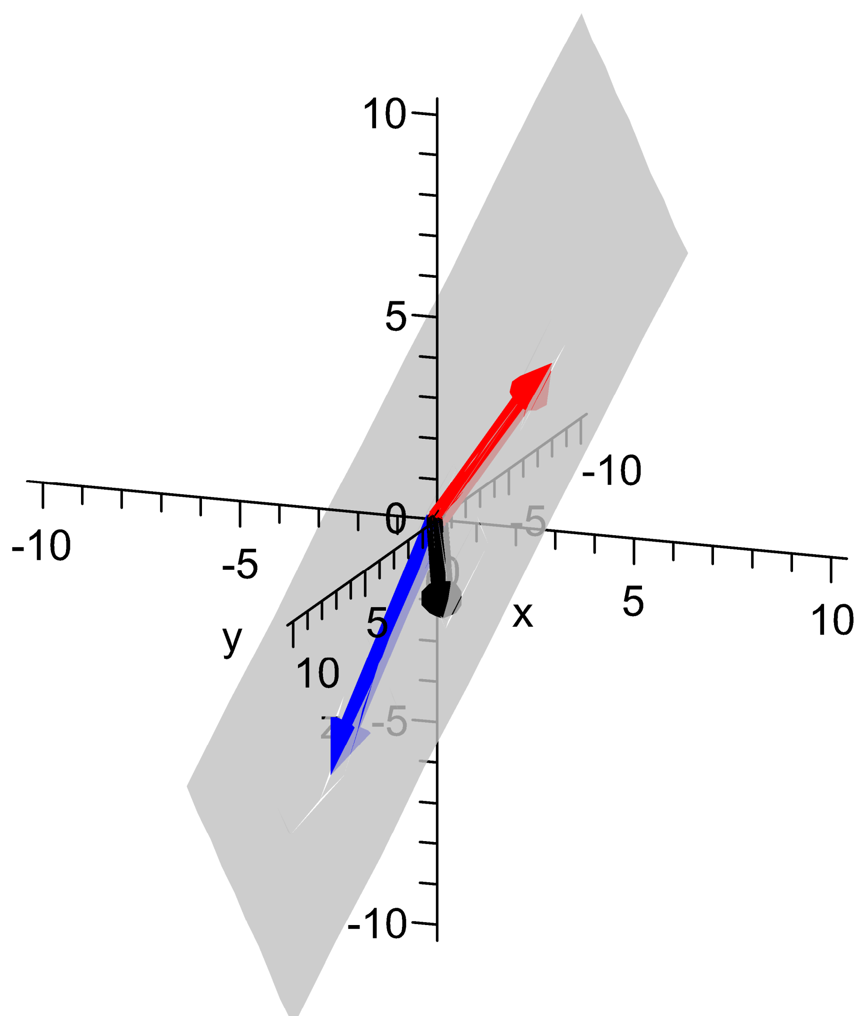
Vector Addition

> $A := \langle 2,-2,-6\rangle;\ B := \langle 3,4,5\rangle;\ C := A+B;$
> $a := arrow(A, color = blue, width = 0.5) : b := arrow(B, color = red, width = 0.5) :$
> $c := arrow(C, color = black, width = 0.5) :$
> $display([p, a, b, c], orientation = [20, 75]);$

$$A := \begin{bmatrix} 2 \\ -2 \\ -6 \end{bmatrix}$$

$$B := \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

$$C := \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$



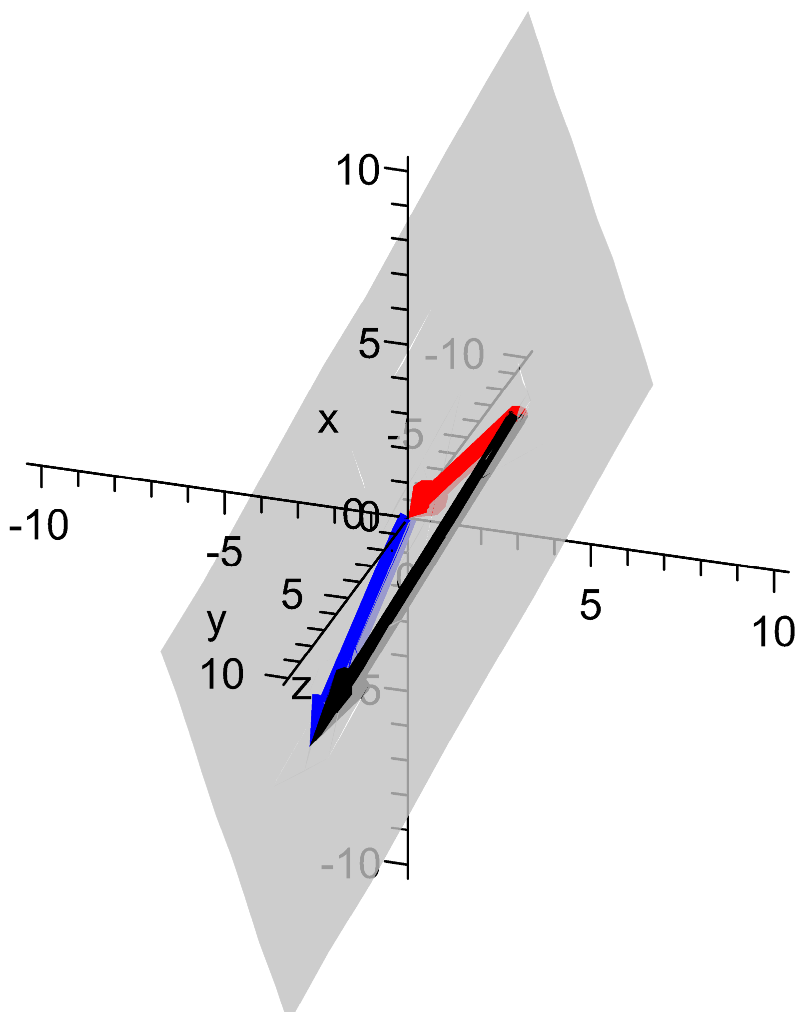
Vector Subtraction

> $A := \langle 2,-2,-6\rangle$; $B := \langle 3,4,5\rangle$; $C := A - B$; $a := arrow(A, color = blue, width = 0.5)$:
> $b := arrow(\langle 3, 4, 5\rangle, \langle -3, -4, -5\rangle, color = red, width = 0.5)$:
> $c := arrow(\langle 3, 4, 5\rangle, \langle -1, -6, -11\rangle, color = black, width = 0.5)$:
> $display([p, a, b, c], orientation = [18, 64])$;

$$A := \begin{bmatrix} 2 \\ -2 \\ -6 \end{bmatrix}$$

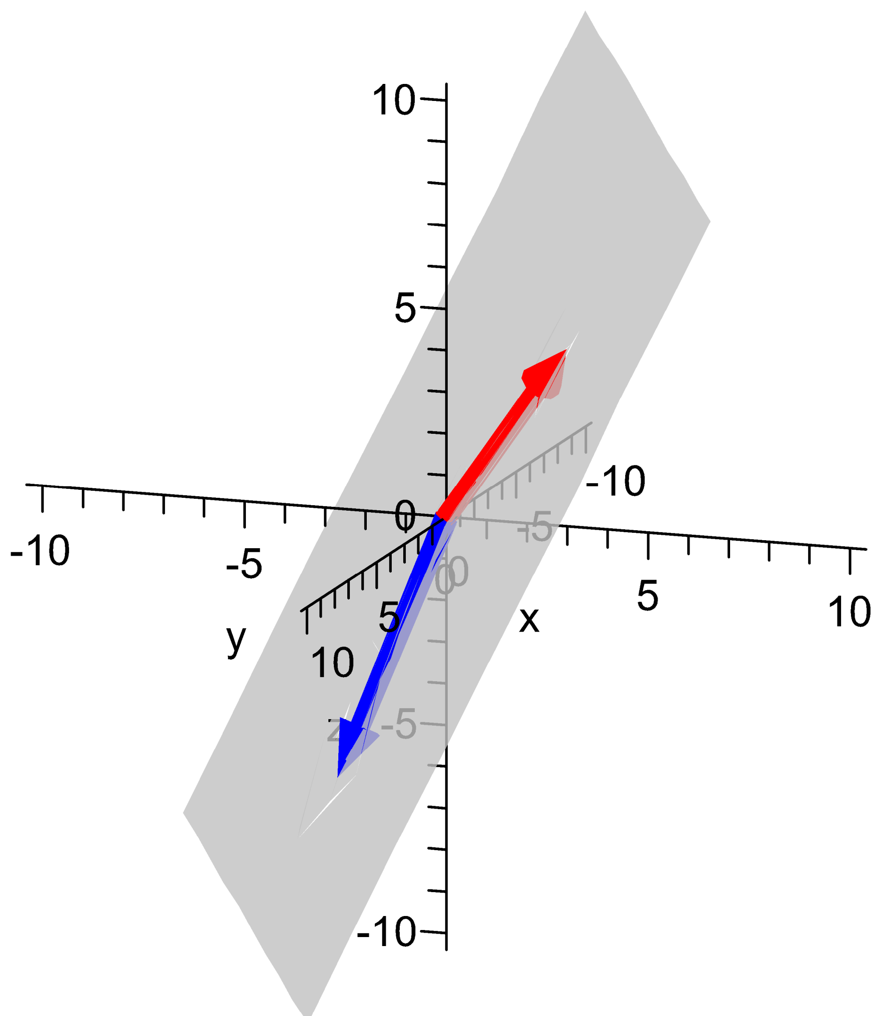
$$B := \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

$$C := \begin{bmatrix} -1 \\ -6 \\ -11 \end{bmatrix}$$



Vector Inner/Dot Product

$> A := \langle 2, -2, -6\rangle : B := \langle 3, 4, 5\rangle :$
$a := arrow(A, color = blue, width = 0.5) : b := arrow(B, color = red, width = 0.5) :$
$display([p, a, b], orientation = [19, 77]);$
$A \cdot B := DotProduct(A, B);$ # *Dot Product is a scalar quantity*
$\|A\| := VectorNorm(A, 2); \|B\| := VectorNorm(B, 2);$ # *magnitude of vectors A & B*
$\theta := evalf\left(\left(\frac{180}{\pi} \cdot \cos^{-1}\left(\frac{A \cdot B}{\|A\| \cdot \|B\|}\right)\right)\right);$ # *angle* θ *in degrees between vectors A and B*



$$A \cdot B := -32$$
$$\|A\| := 2\sqrt{11}$$
$$\|B\| := 5\sqrt{2}$$
$$\theta := 133.0191131 \quad \textbf{(5.1)}$$

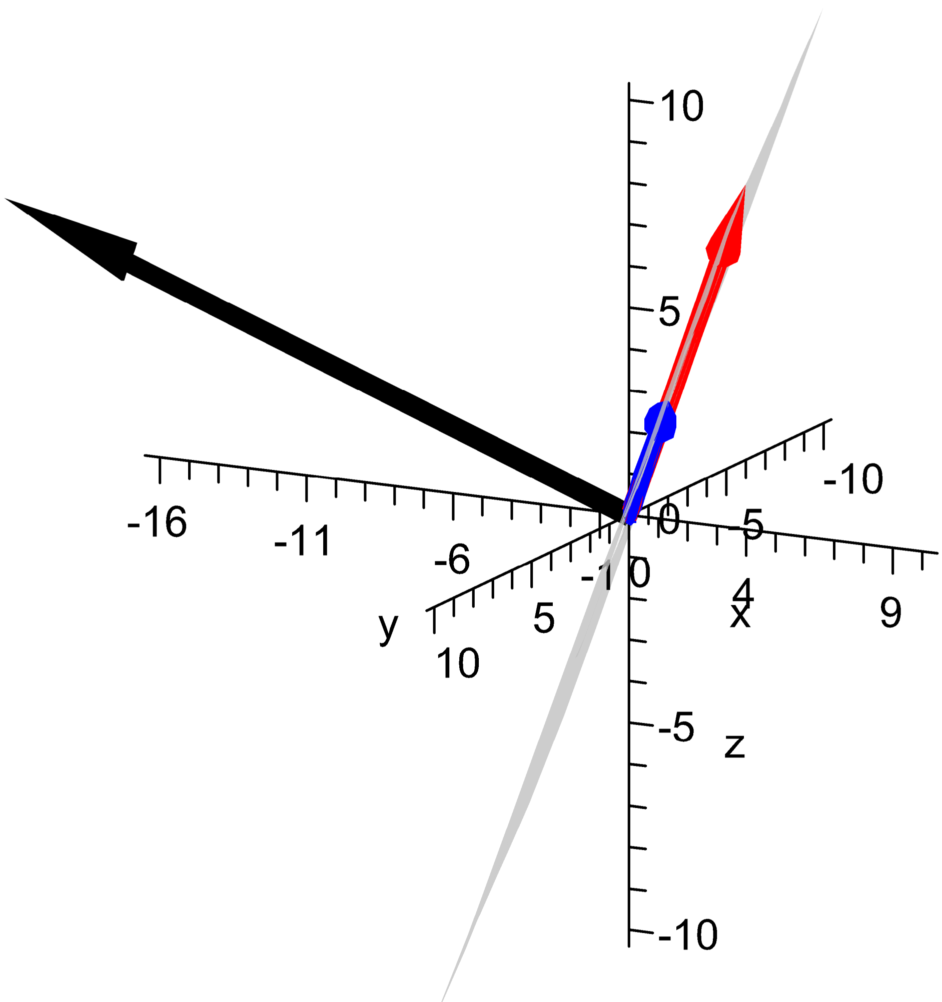
Vector Cross Product

> $A := \langle 4, 4, 4\rangle$; $B := \langle 6, 8, 10\rangle$; $C := CrossProduct(A, B)$;
$a := arrow(A, color = blue, width = 0.5)$: $b := arrow(B, color = red, width = 0.5)$:
$c := arrow(C, color = black, width = 0.5)$:
$display([p, a, b, c], orientation = [27, 76])$;

$$A := \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}$$

$$B := \begin{bmatrix} 6 \\ 8 \\ 10 \end{bmatrix}$$

$$C := \begin{bmatrix} 8 \\ -16 \\ 8 \end{bmatrix}$$



Equation of a Plane

Equation of a plane: ax + by + cz = d

A plane containing a point with a position vector R, which one can write as R-R_0, has a vector N normal or perpendicular to the plane. Mathematically,

$$(\mathbf{R} - \mathbf{R_0}) \bullet \mathbf{N} = 0$$

Let R = xi + yj +zk, R_0 = x_0i + y_0j + z_0k and N = ai + bj +ck.

$$(\mathbf{R} - \mathbf{R_0}) \bullet \mathbf{N} = a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$

$$(\mathbf{R} - \mathbf{R_0}) \bullet \mathbf{N} = ax + by + cz - (ax_0 + by_0 + cz_0) = 0$$

$$\mathbf{R} \bullet \mathbf{N} = ax + by + cz = 0, \quad \therefore\ d = 0$$

We can determine N by taking the cross products of two vectors in the plane. The plane containing the point and its position vector is described by equation ax + by + cz = d. Since $x_0 = y_0 = z_0 = 0$, the equation reduces to ax + by + cz = 0 describing a plane passing through the origin. When d ≠ 0 then d/|N| represents the distance of the plane to the origin. We can determine the x, y, and z intercepts:

$$x = \frac{d}{a}, \quad y = \frac{d}{b}, \quad z = \frac{d}{c}$$

The distance from the origin to the plane is given by:

$$D_0 = \frac{d}{|N|}$$

Three non-collinear points determine a plane. The procedure plane(p1,p2,p3) draws the position vectors R1, R2, R3, finds two vectors in the plane, and calculates the normal to the plane by taking the cross product of the two vectors in the plane.

```
> plane := proc(p1, p2, p3)
     local R1, R2, R3, A, B, N, f, d, no;
     R1 := ⟨p1⟩ : R2 := ⟨p2⟩ : R3 := ⟨p3⟩ :
     A := R3 − R1 ;                    # a vector from  P1 to P3
     B := R2 − R1;                     # a vector from P1 to P2
     print('A'= A); print('B'= B);
     N := CrossProduct(A, B); no := VectorNorm(N, 2);
     if no ≠ 0 then
        print(` A x B ` = N);      #` `vector ⊥ plane
        N := simplify(N/no) ;   #` `unit vector ⊥ plane
        d := N[1]·R1[1] + N[2]·R1[2] + N[3]·R1[3]; # distance to the origin
        f := N[1]·x + N[2]·y + N[3]·z = d :        # equation of the plane
        print(Equation of the plane is, factor(f) );
        print( The distance `from` the origin is, d);
        if N[1] ≠ 0 then print( The x intercept is, d/N[1] ) end if;   # the intercepts
        if N[2] ≠ 0 then print( The y intercept is, d/N[2] ) end if;
        if N[3] ≠ 0 then print( The z intercept is, d/N[3] ); end if;
     else print(We have `3` collinear points - this is a line); f := 0 end if ;
```

```
p := implicitplot3d(f, x =-10 ..10, y =-10 ..10, z =-10 ..10, axes = normal,
        style = patchnogrid, color = grey, transparency = 0.2 ) :
r1 := arrow(R1, color = green, width = 0.25 ) :      #position vectors
r2 := arrow(R2, color = magenta, width = 0.25 ) :
r3 := arrow(R3, color = yellow, width = 0.25 ) :
a := arrow(R1, A, color = black, width = 0.5 ) :      # vector A
b := arrow(R1, B, color = red, width = 0.5 ) :        # vector B
l := arrow(N, color = gray, length = d, shape = double_arrow , width = 1 ) :
n := arrow(R1, N, color = blue, width = 0.5, length = 5 ) : # normal to the plane
display( [r1, r2, r3, a, b, l, n, p ], orientation = [27, 85 ], tickmarks = [4, 4, 4 ]);
end:
> plane( [0, 0, 5 ], [ -5, 5, 5 ], [5, 5, 5 ]);
```

$$A = \begin{bmatrix} 5 \\ 5 \\ 0 \end{bmatrix}$$

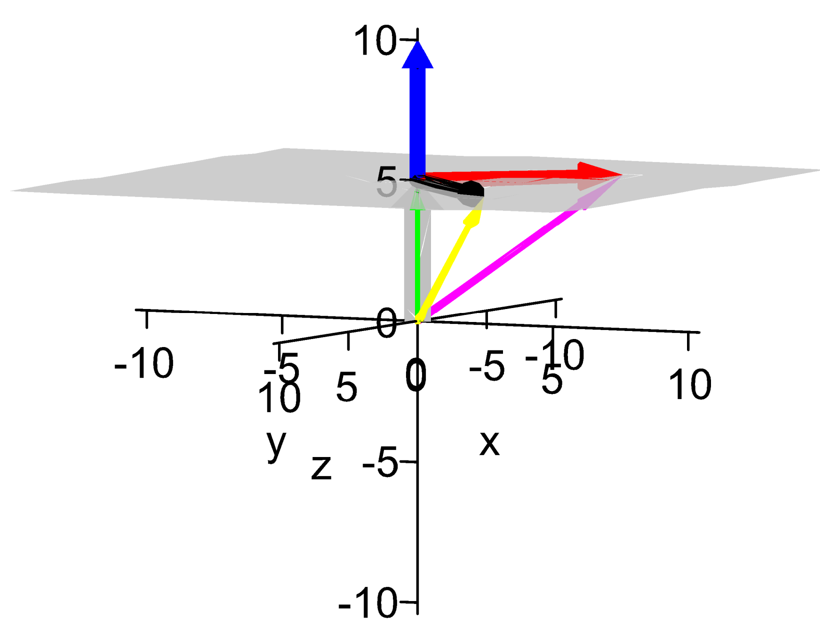
$$B = \begin{bmatrix} -5 \\ 5 \\ 0 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 0 \\ 0 \\ 50 \end{bmatrix}$$

Equation of the plane is, $z = 5$

The distance from the origin is, 5

The z intercept is, 5



> $plane([0, 0, 5], [-5, 5, 10], [5, 5, 0]);$

$$A = \begin{bmatrix} 5 \\ 5 \\ -5 \end{bmatrix}$$

$$B = \begin{bmatrix} -5 \\ 5 \\ 5 \end{bmatrix}$$

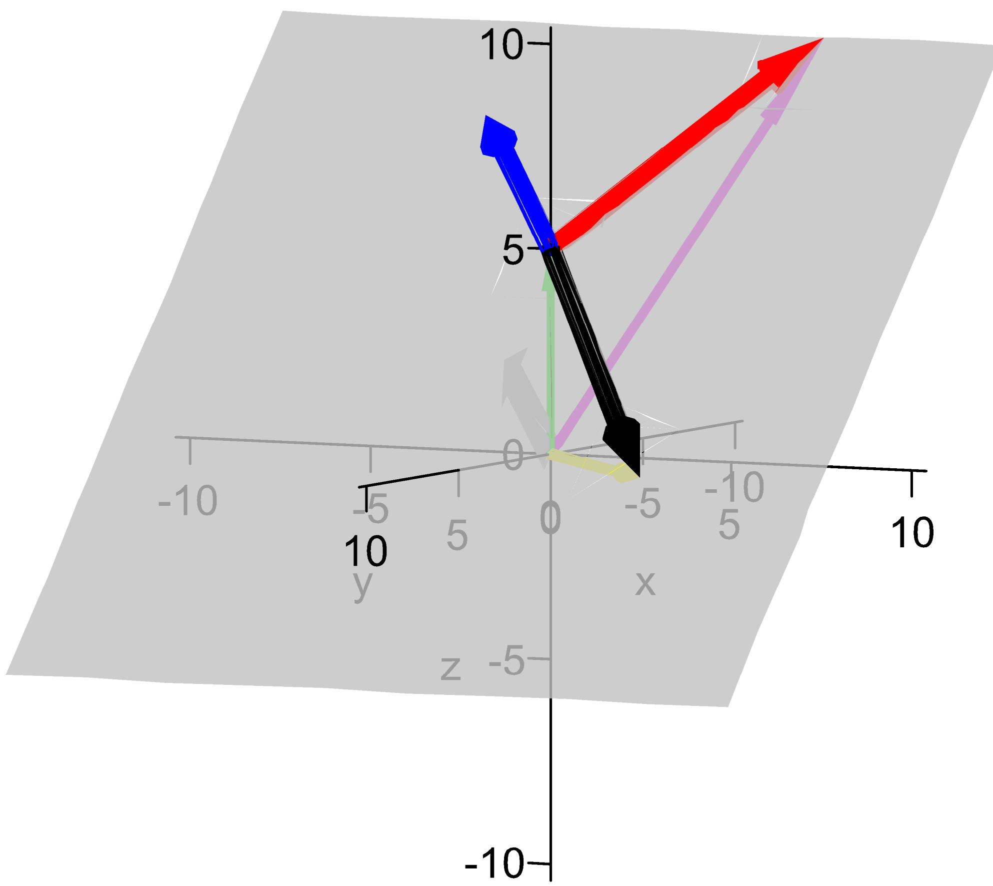
$$A \times B = \begin{bmatrix} 50 \\ 0 \\ 50 \end{bmatrix}$$

Equation of the plane is, $\frac{1}{2}\sqrt{2}\,(x+z) = \frac{5}{2}\sqrt{2}$

The distance from the origin is, $\frac{5}{2}\sqrt{2}$

The x intercept is, 5

The z intercept is, 5



> plane([0, 0, 0], [3, 4, 5], [2,-2,-6]) ;

$$A = \begin{bmatrix} 2 \\ -2 \\ -6 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 14 \\ -28 \\ 14 \end{bmatrix}$$

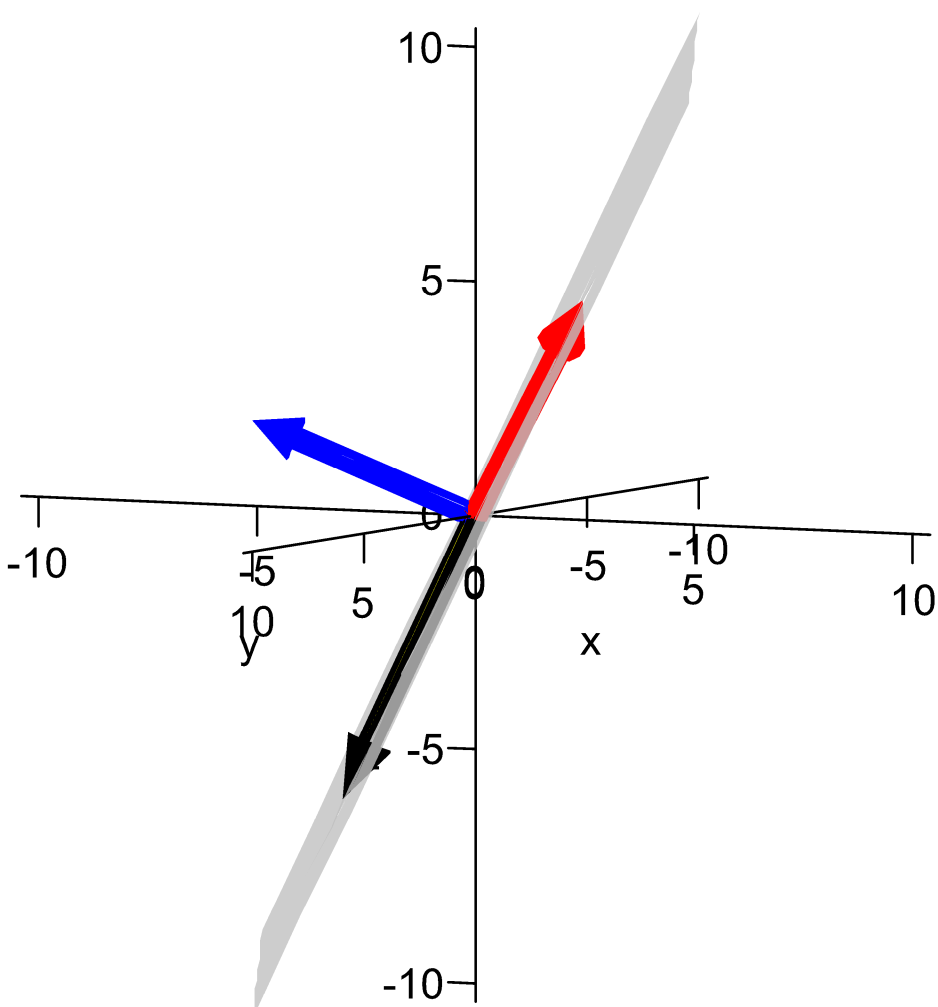
Equation of the plane is, $\frac{1}{6}\sqrt{6}\,(x - 2\,y + z) = 0$

The distance from the origin is, 0

The x intercept is, 0

The y intercept is, 0

The z intercept is, 0



> $plane([0, 0, 0], [-4, 0, 5], [4, 0, -5]);$

$$A = \begin{bmatrix} 4 \\ 0 \\ -5 \end{bmatrix}$$

$$B = \begin{bmatrix} -4 \\ 0 \\ 5 \end{bmatrix}$$

We have 3 collinear points — this is a line