> *restart*;
> *with*(*inttrans*) :
> *_EnvUseHeavisideAsUnitStep* := *true* :

Fourier Transforms

$$g(\nu) = \mathcal{F}\{f(t)\} \equiv \int_{-\infty}^{\infty} f(t) \cdot e^{-i\,2\,\pi\,\nu\,t}\,dt$$

$$f(t) = \mathcal{F}^{-1}\{g(\nu)\} \equiv \int_{-\infty}^{\infty} g(\nu) \cdot e^{i\,2\,\pi\,\nu\,t}\,d\nu$$

Using Maple's Fourier Transforms

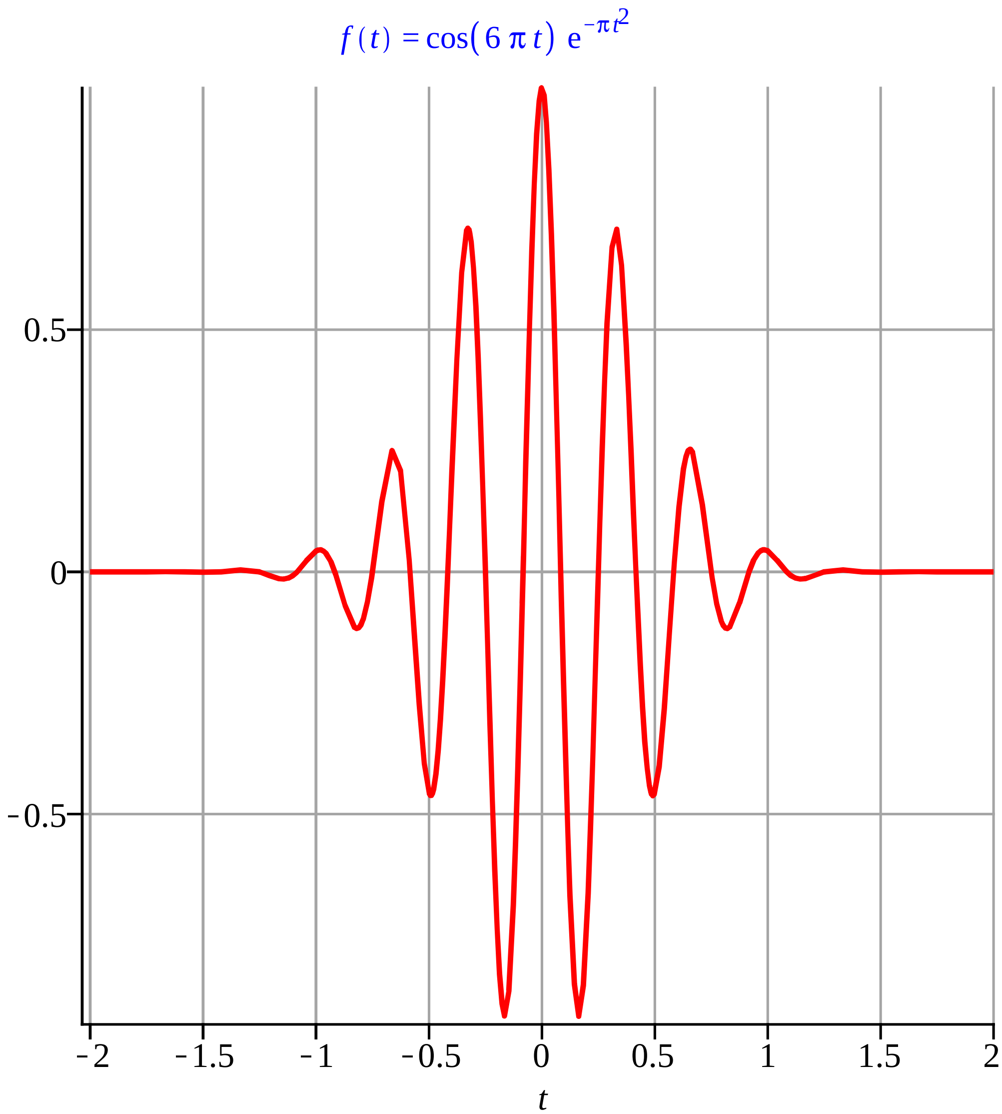
$$g(w) = \mathcal{F}\{f(t)\} = \text{fourier}(f(t), t, \omega) \equiv \int_{-\infty}^{\infty} f(t) \cdot e^{-i\,\omega\,t}\,dt$$

$$f(t) = \mathcal{F}^{-1}\{g(\omega)\} = \text{invfourier}(g(\omega), \omega, t) \equiv \frac{1}{2\,\pi} \int_{-\infty}^{\infty} g(\omega) \cdot e^{i\,t\,\omega}\,d\omega$$

since $\omega = 2\pi\nu$

Symmetrical function f(t) = $\cos(6\,\pi \cdot t) \cdot e^{-\pi \cdot t^2}$

> $f := t \rightarrow \cos(\,6\,\pi \cdot t) \cdot e^{-\pi \cdot t^2}$: $'f(t)' = f(t)$;
plot(f(t), t =− 2 ..2, thickness = 2, axes = frame, gridlines = true, tickmarks = [10, 5]);

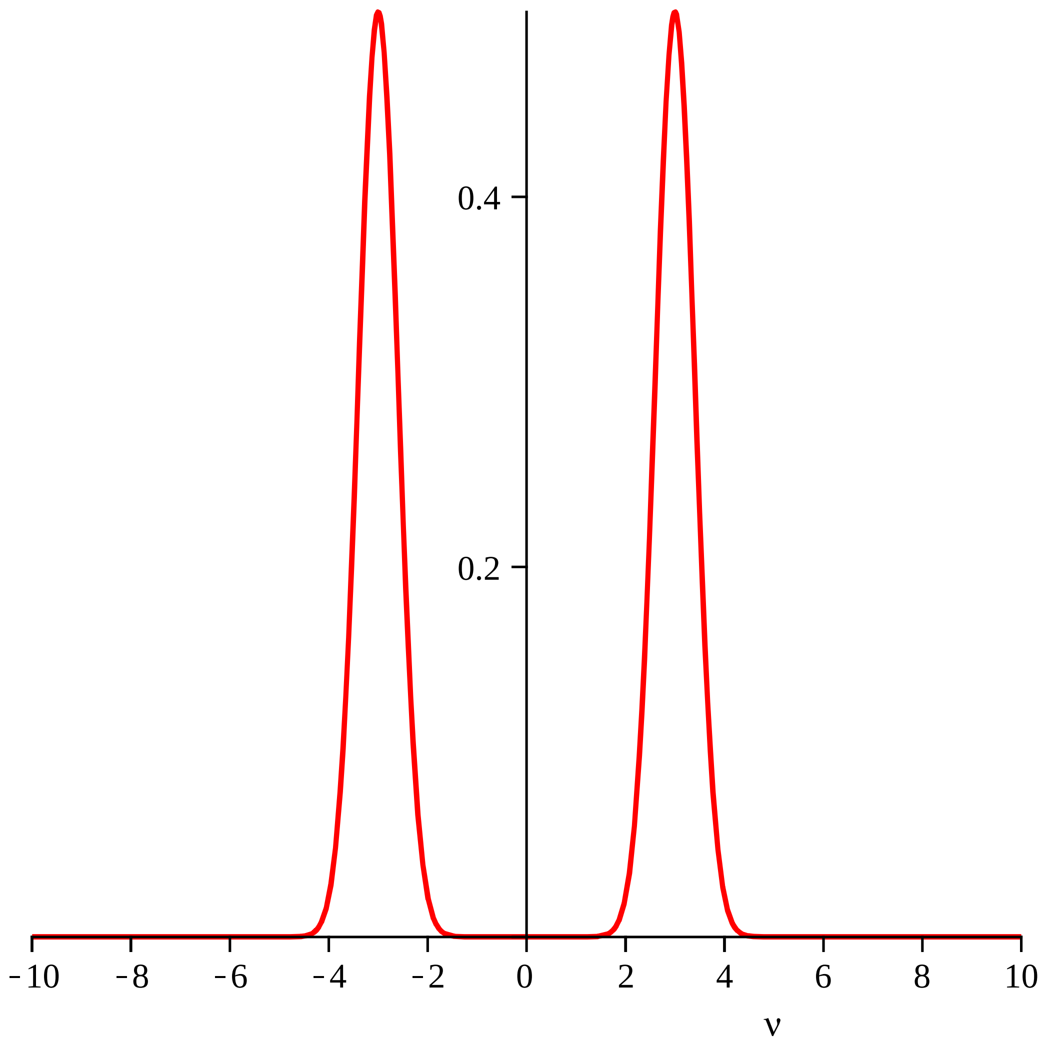


Fourier transform $\mathcal{F}\{f(t)\}$

> $g := \nu \rightarrow fourier(f(t), t, 2\pi\cdot\nu) : '\mathcal{F}\{f(t)\}' = g(\nu); 'g(\nu)' = g(\nu);$
> $plot(g(\nu), \text{thickness} = 2, \ \text{tickmarks} = [10, 3]);$

$$\mathcal{F}\{f(t)\} = \cosh(6\pi\nu)\, e^{-9\pi - \pi\nu^2}$$

$$g(\nu) = \cosh(6\pi\nu)\, e^{-9\pi - \pi\nu^2}$$

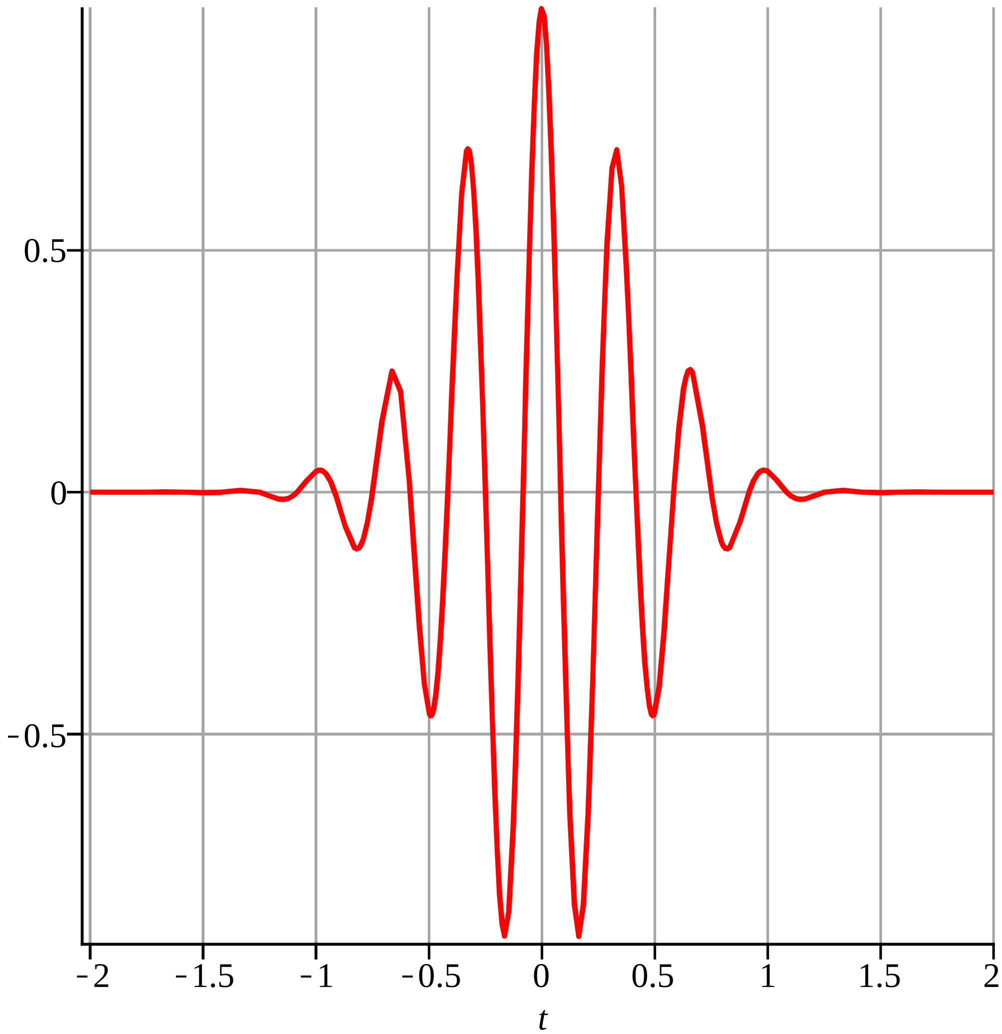


Inverse Fourier Transform $\mathcal{F}^{-1}\{g(\nu)\}$

```
> z := t→2 π·invfourier(g(ν), ν, 2 π·t) :
  'InvF{g(ν)}' = z(t); 'f(t)' = z(t);
  plot(z(t), t = -2 ..2, thickness = 2, axes = frame, gridlines = true, tickmarks = [10, 3]);
```

$$InvF\{g(\nu)\} = \cos(6\pi t)\, e^{-\pi t^2}$$

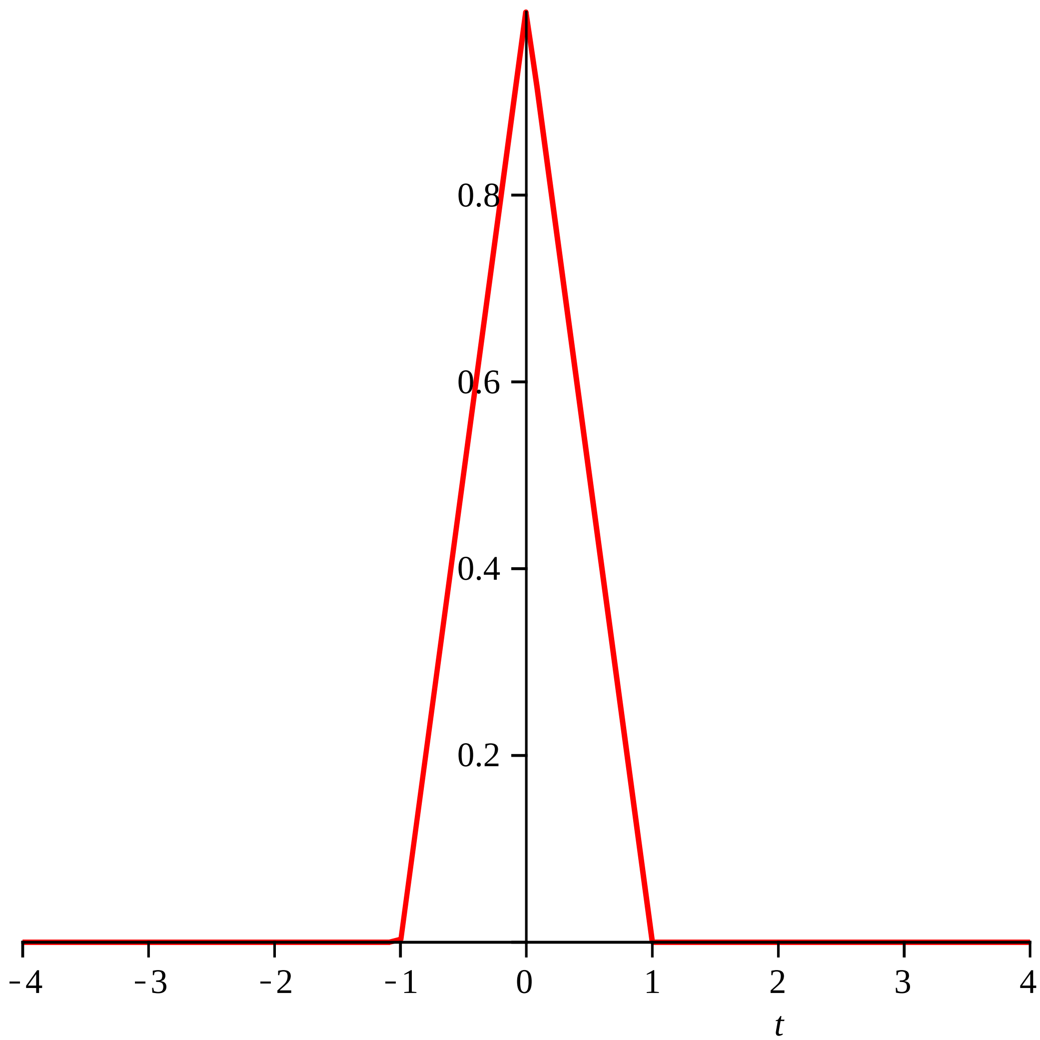
$$f(t) = \cos(6\pi t)\, e^{-\pi t^2}$$



Triangle function Λ(t)

> $f := t \to piecewise(|t| \le 1, 1 - |t|) : 'f(t)' = f(t);$
> $plot(f(t), t = -4..4, thickness = 2, tickmarks = [10, 5]);$

$$f(t) = \begin{cases} 1 - |t| & |t| \le 1 \\ 0 & otherwise \end{cases}$$

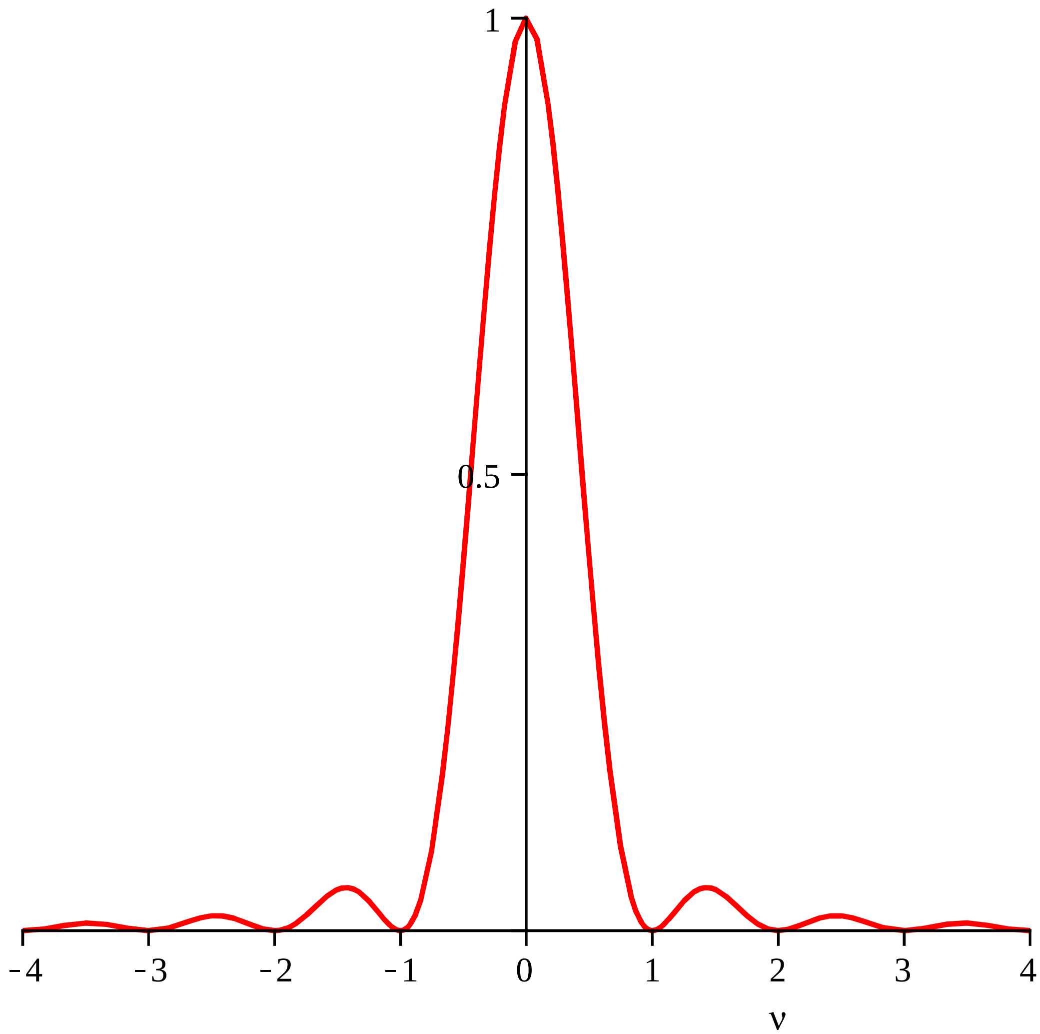


Fourier Transform of the Triangle function Λ(t)

> $g := \nu \rightarrow fourier(f(t), t, 2\pi\cdot\nu)$: $'\mathcal{F}\{f(t)\}' = g(\nu)$; $'g(\nu)' = g(\nu)$;
$plot(g(\nu), \nu = -4..4, \text{thickness} = 2, \text{tickmarks} = [10, 3])$;

$$\mathcal{F}\{f(t)\} = \frac{\sin(\pi\nu)^2}{\pi^2\nu^2}$$

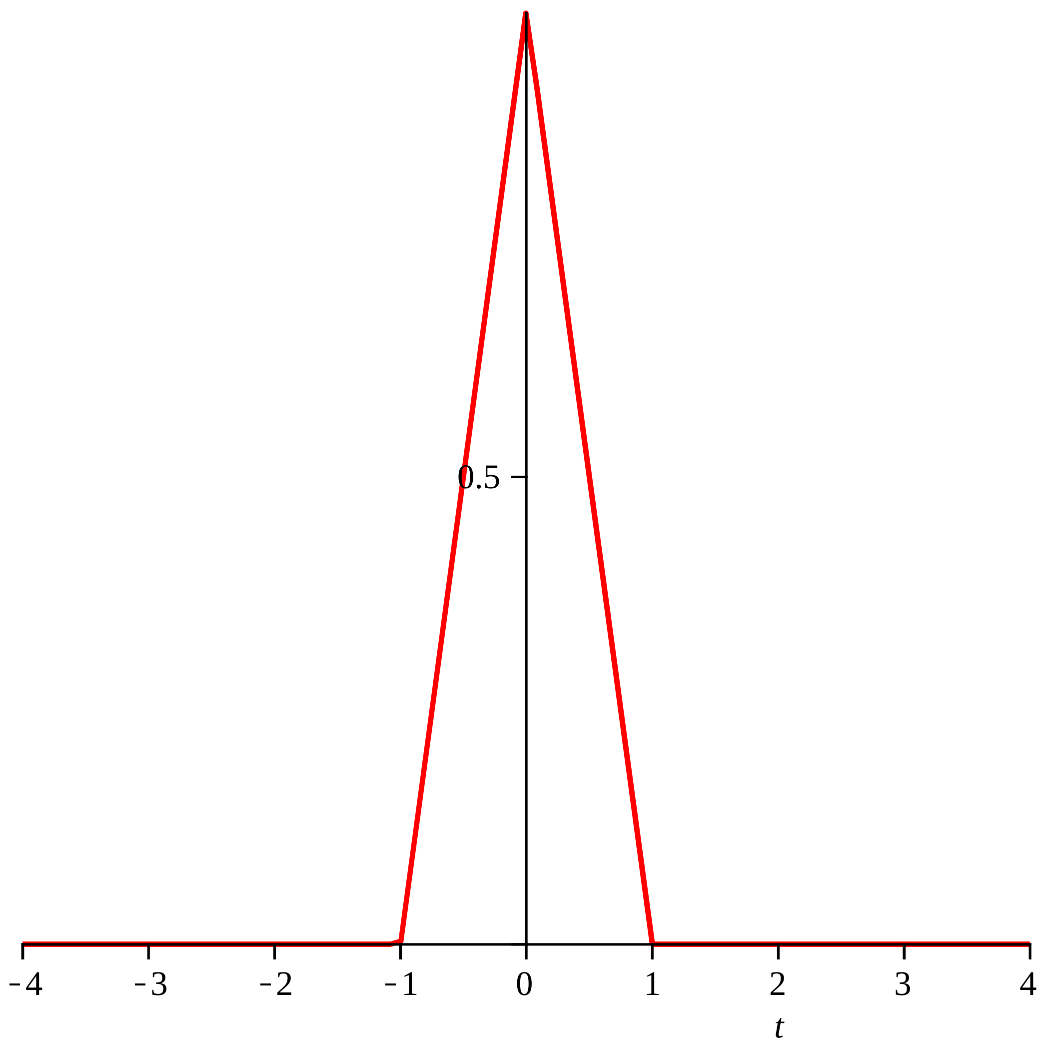
$$g(\nu) = \frac{\sin(\pi\nu)^2}{\pi^2\nu^2}$$



Inverse Fourier Transform $\mathcal{F}^{-1}\{g(\nu)\}$

> $z := t \to 2\,\pi \cdot invfourier(g(\nu), \nu, 2\,\pi \cdot t)$:
> $'Inv\mathcal{F}\{g(\nu)\}' = simplify(convert(z(t), piecewise))$;
> $plot(z(t), t = -4..4, thickness = 2, tickmarks = [10, 3])$;

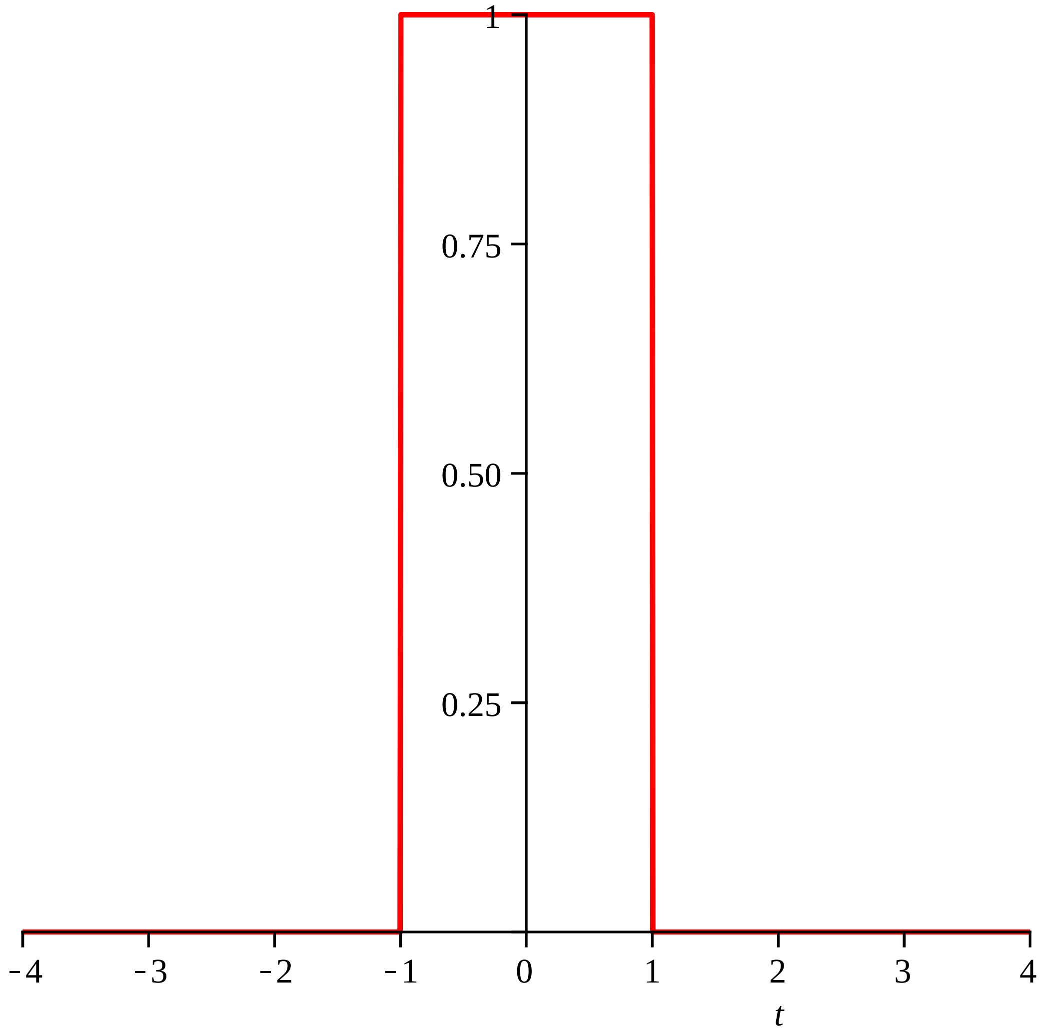
$$Inv\mathcal{F}\{g(\nu)\} = \begin{cases} 0 & t \le -1 \\ 1+t & t \le 0 \\ 1-t & t \le 1 \\ 0 & 1 < t \end{cases}$$



#1 The Box function Π(t)

> $f := t \rightarrow piecewise(|t| \leq 1, 1) : 'f(t)' = f(t);$
> $plot(f(t), t = -4..4, thickness = 2, tickmarks = [10, 5]);$

$$f(t) = \begin{cases} 1 & |t| \leq 1 \\ 0 & otherwise \end{cases}$$

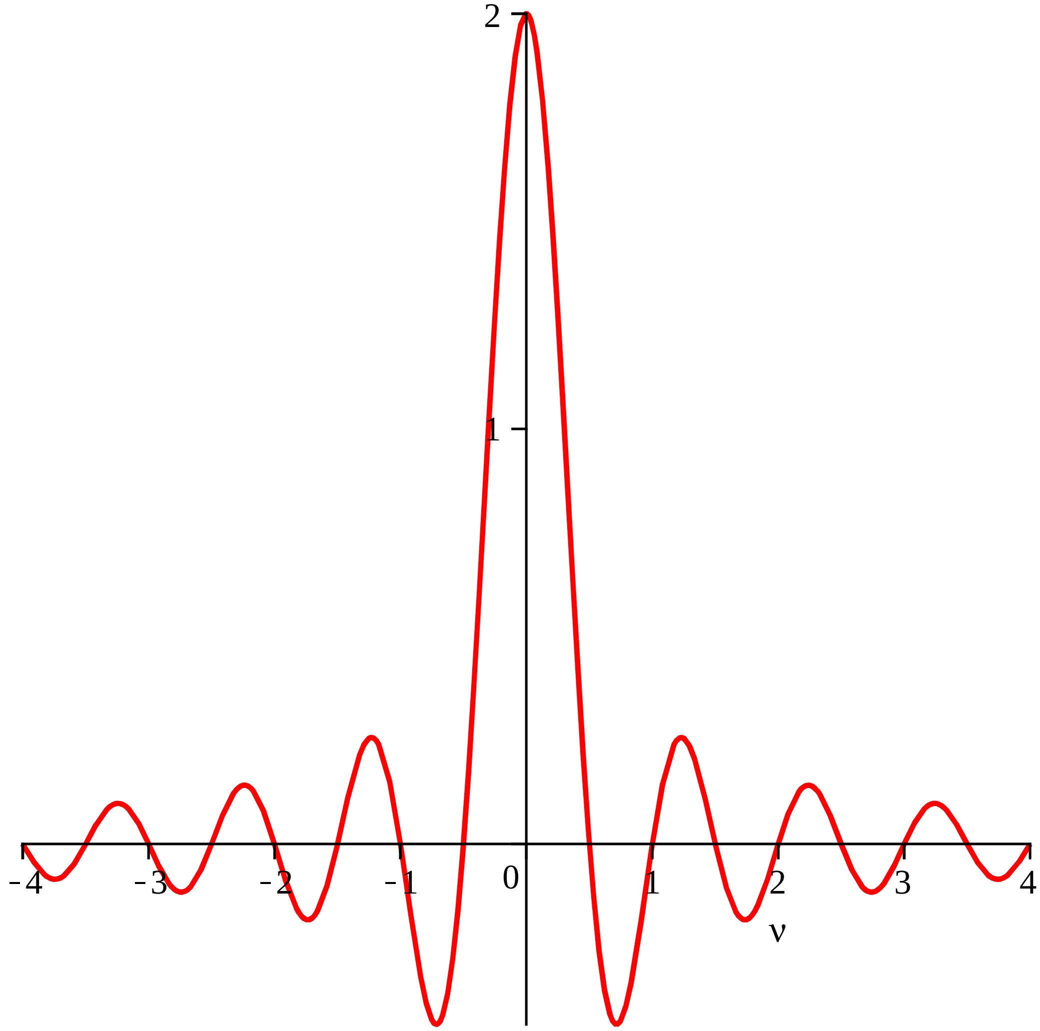


Fourier Transform of the Box function Π(t)

> $g := \nu \rightarrow fourier(f(t), t, 2\,\pi \cdot \nu) : '\mathcal{F}\{f(t)\}' = g(\nu); 'g(\nu)' = g(\nu);$
> $plot(\mathrm{g}(\nu), \nu = -4..4, \text{thickness} = 2, \text{ tickmarks} = [10, 3]);$

$$\mathcal{F}\{f(t)\} = \frac{\sin(2\,\pi\,\nu)}{\pi\,\nu}$$

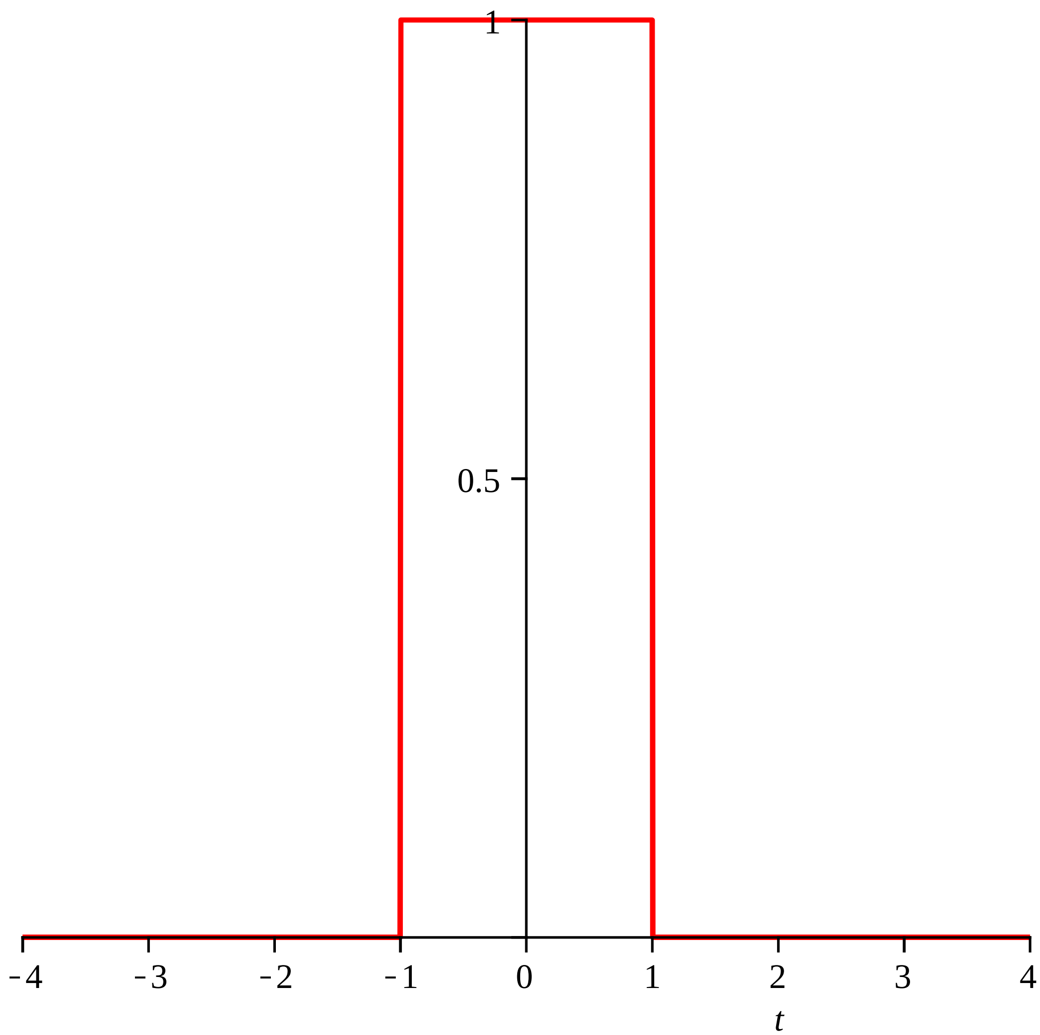
$$g(\nu) = \frac{\sin(2\,\pi\,\nu)}{\pi\,\nu}$$



Inverse Fourier Transform $\mathcal{F}^{-1}\{g(\nu)\}$

```
> z := t→2 π·invfourier(g(ν), ν, 2 π·t) :
  'Inv.ℱ{g(ν)}' = simplify(convert(z(t), piecewise));
  plot(z(t), t =−4 ..4, thickness = 2, tickmarks = [10, 3]);
```

$$Inv.\mathcal{F}\{g(\nu)\} = \begin{cases} 0 & t \le -1 \\ 1 & t \le 1 \\ 0 & 1 < t \end{cases}$$

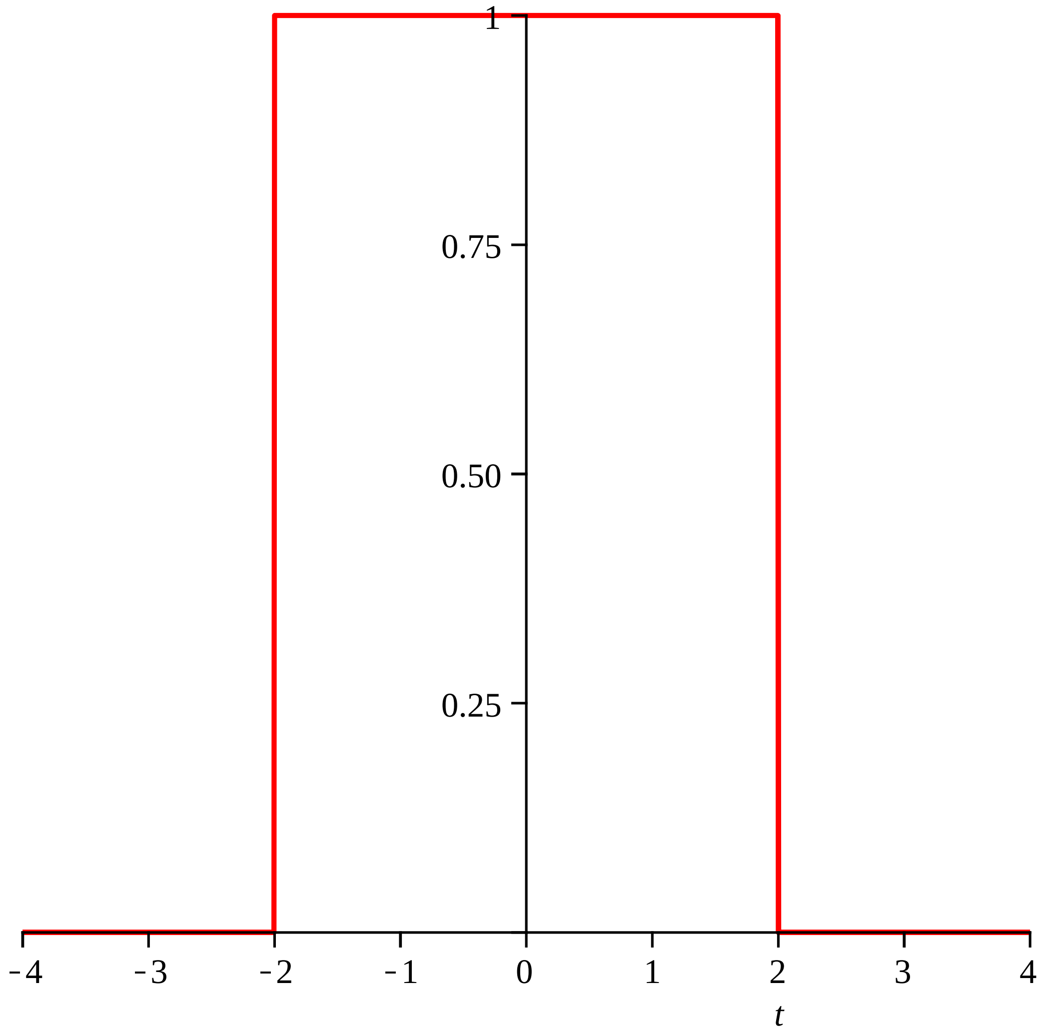


#2 The Box function Π(t)

compare the widths of f(t) and $\mathcal{F}$\{f(t)\}

> $f := t \rightarrow piecewise(|t| \leq 2, 1) : 'f(t)' = f(t);$
> $plot(f(t), t = -4..4, \text{thickness} = 2, \text{tickmarks} = [10, 5]);$

$$f(t) = \begin{cases} 1 & |t| \leq 2 \\ 0 & otherwise \end{cases}$$

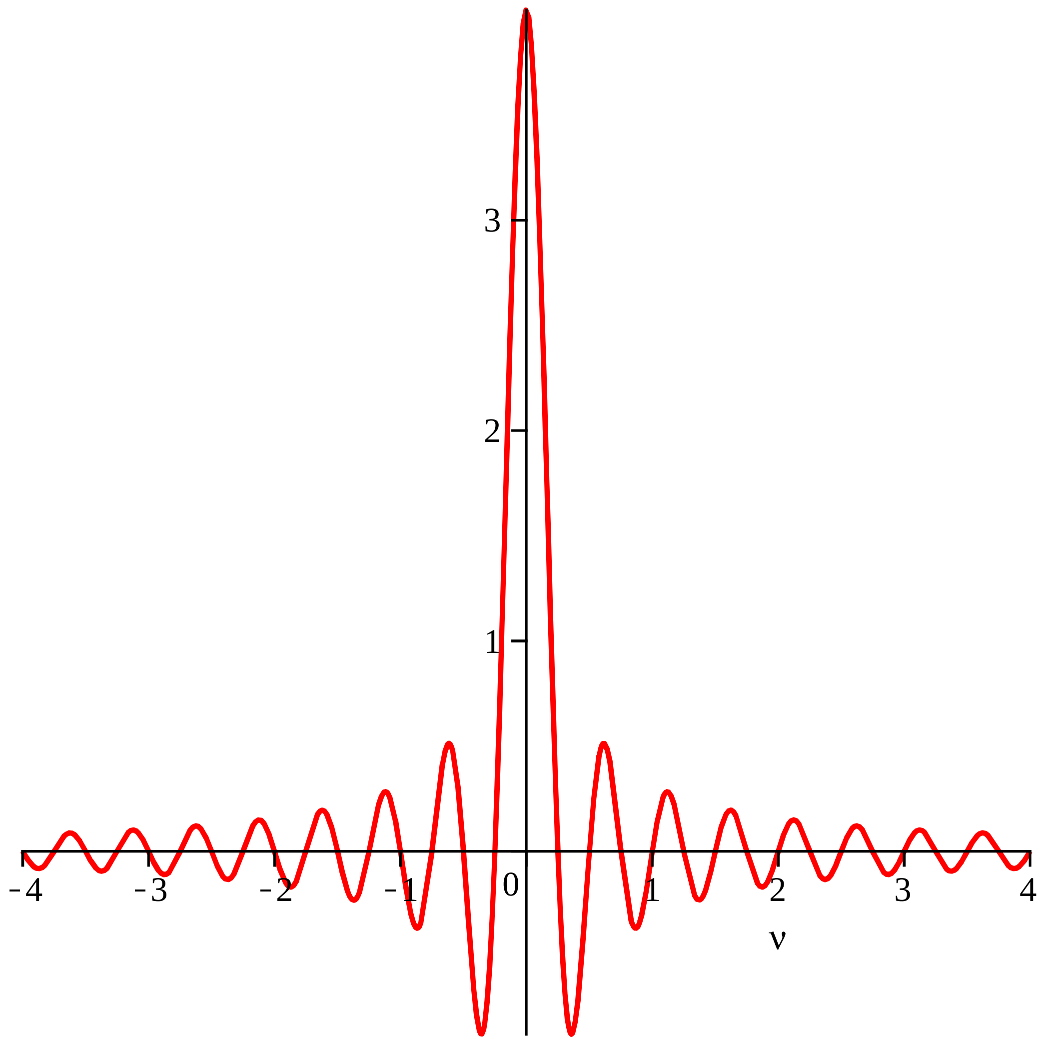


Fourier Transform of the Box function Π(t)

> $g := \nu \rightarrow fourier(f(t), t, 2\pi\cdot\nu) : '\mathcal{F}\{f(t)\}' = g(\nu); 'g(\nu)' = g(\nu);$
> $plot(g(\nu), \nu = -4..4, \text{thickness} = 2, \text{tickmarks} = [10, 3]);$

$$\mathcal{F}\{f(t)\} = \frac{\sin(4\pi\nu)}{\pi\nu}$$

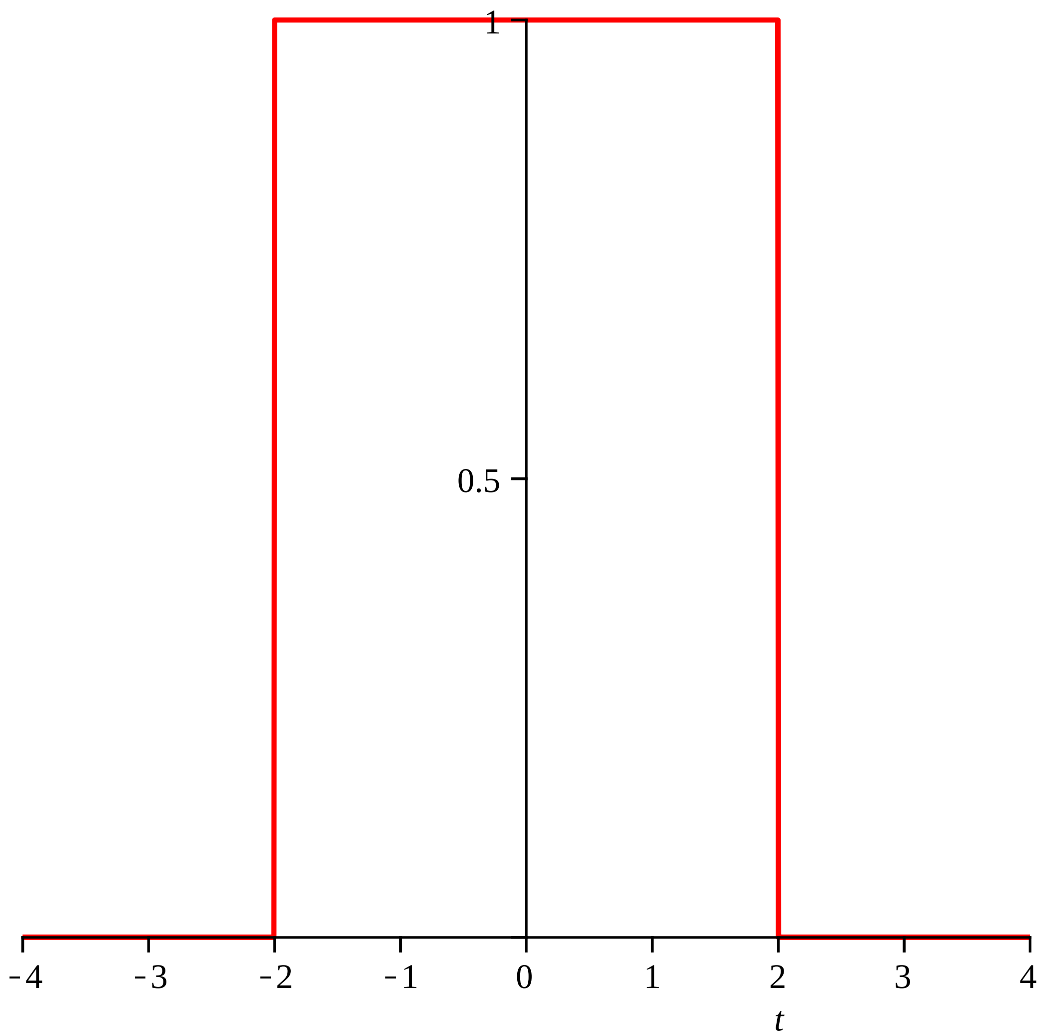
$$g(\nu) = \frac{\sin(4\pi\nu)}{\pi\nu}$$



Inverse Fourier Transform $\mathcal{F}^{-1}\{g(\nu)\}$

```
> z := t→2 π·invfourier(g(ν), ν, 2 π·t) :
  'Inv𝓕{g(ν)}' = simplify(convert(z(t), piecewise));
  plot(z(t), t = −4 ..4, thickness = 2, tickmarks = [10, 3]);
```

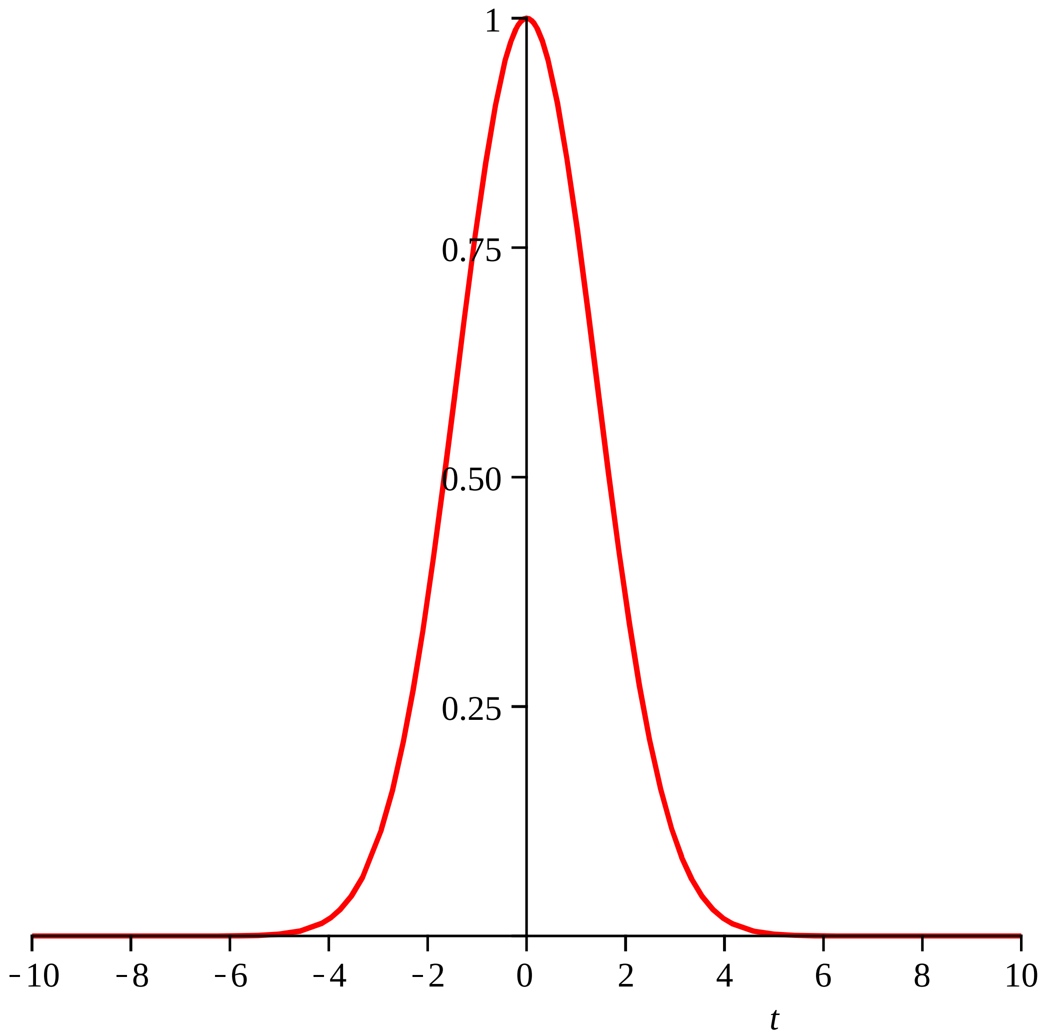
$$Inv\mathcal{F}\{g(\nu)\} = \begin{cases} 0 & t \le -2 \\ 1 & t \le 2 \\ 0 & 2 < t \end{cases}$$



Two Gaussian functions $e^{-\frac{x^2}{a^2}}$

> $f1 := t \to e^{-\frac{t^2}{4}}$: 'f1(t)'= f1(t); #` `**a= 2**
> plot(f1(t), thickness = 2, tickmarks = [10, 5]);

$$f1(t) = e^{-\frac{1}{4}t^2}$$

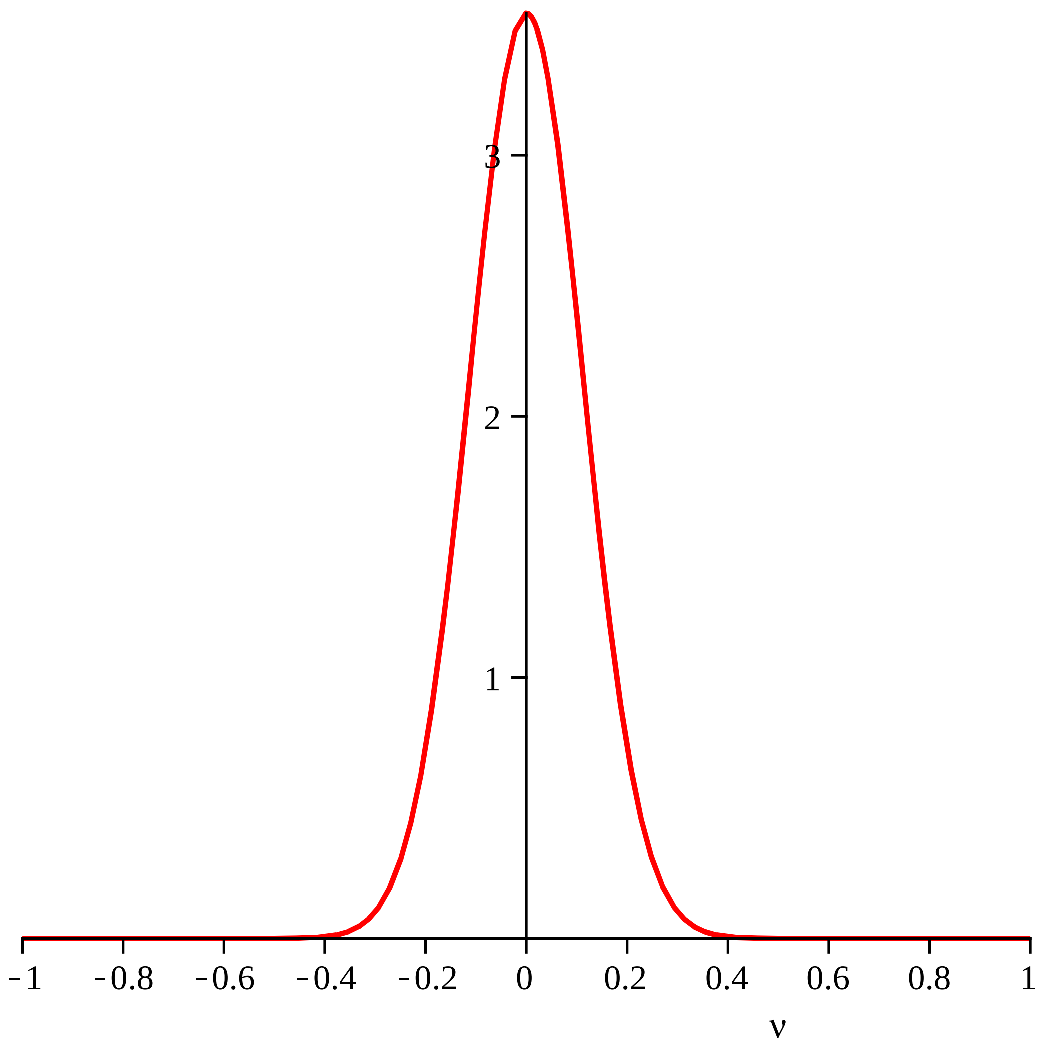


Fourier Transform of Gaussian functions $e^{-\frac{x^2}{4}}$

> $g1 := \nu \rightarrow fourier(f1(t), t, 2\pi \cdot \nu) : '\mathcal{F}\{f1(t)\}' = g1(\nu); 'g1(\nu)' = g1(\nu);$
> $plot(g1(\nu), \nu = -1..1, \text{thickness} = 2, \text{tickmarks} = [10, 3]);$

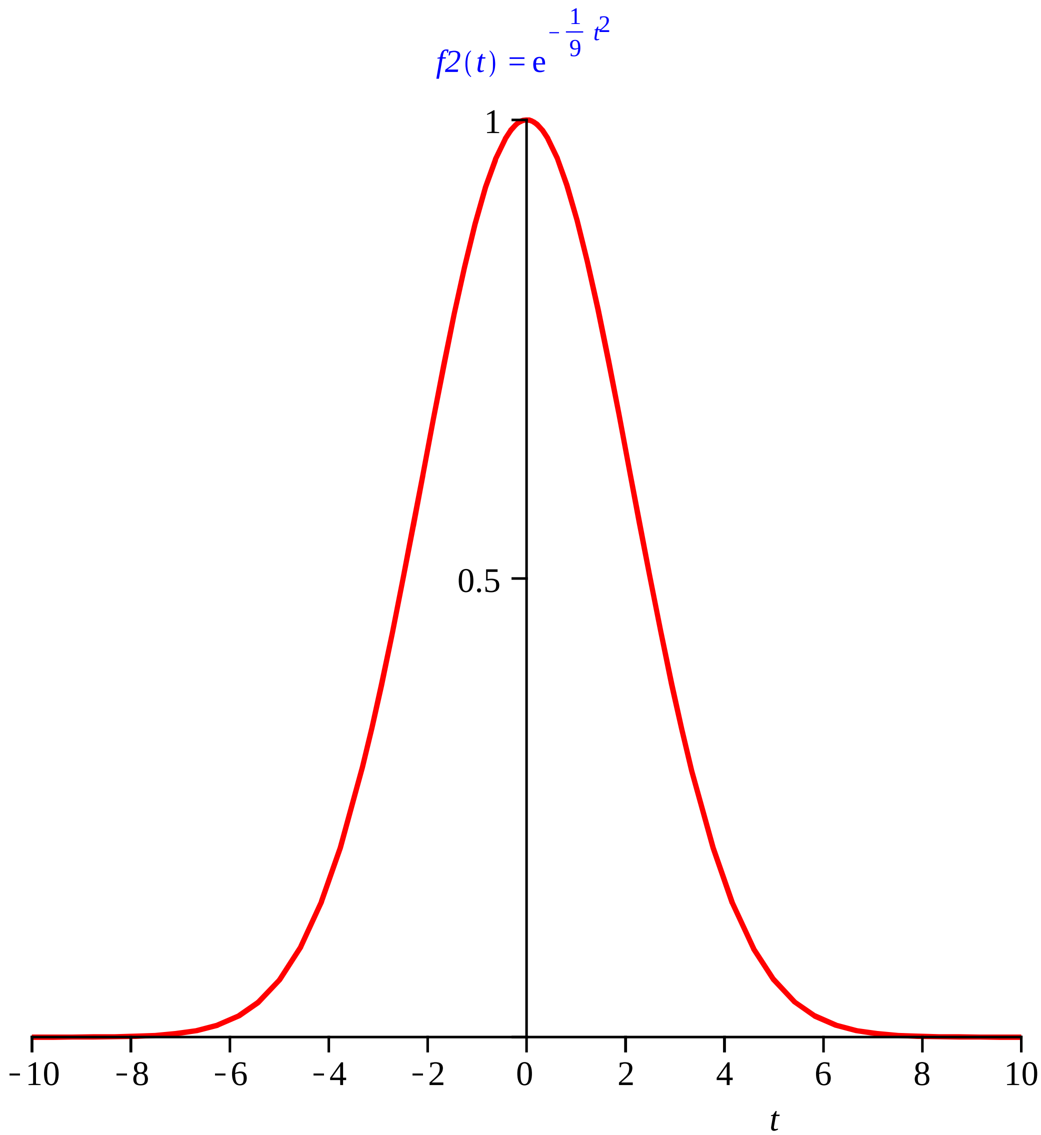
$$\mathcal{F}\{f1(t)\} = 2\,e^{-4\pi^2\nu^2}\sqrt{\pi}$$

$$g1(\nu) = 2\,e^{-4\pi^2\nu^2}\sqrt{\pi}$$



Gaussian functions $\mathbf{e}^{-\frac{x^2}{9}}$

> $f2 := t \rightarrow e^{-\frac{t^2}{9}}$: $'f2(t)' = f2(t)$; #` `**a = 3**
$plot(\, f2(\mathrm{t}), \mathrm{thickness} = 2, \mathrm{tickmarks} = [10, 3])$;

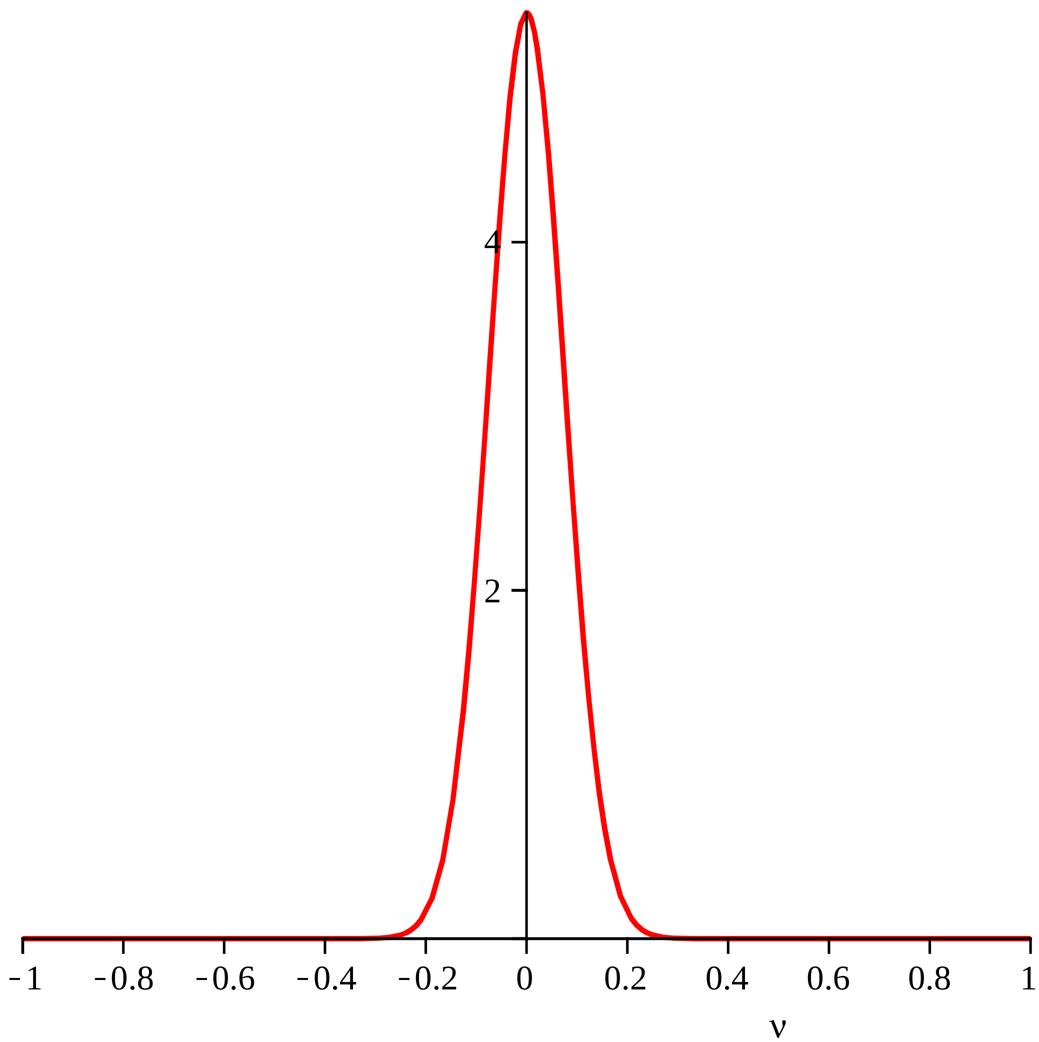


Fourier Transform of Gaussian functions $e^{-\frac{x^2}{9}}$

> $g2 := \nu \to fourier(f2(t), t, 2\pi\cdot\nu) : '\mathcal{F}\{f2(t)\}' = g2(\nu); 'g2(\nu)' = g2(\nu);$
$plot(g2(\nu), \nu = -1..1, \text{thickness} = 2, \text{tickmarks} = [10, 3]);$

$$\mathcal{F}\{f2(t)\} = 3\,e^{-9\pi^2\nu^2}\sqrt{\pi}$$

$$g2(\nu) = 3\,e^{-9\pi^2\nu^2}\sqrt{\pi}$$



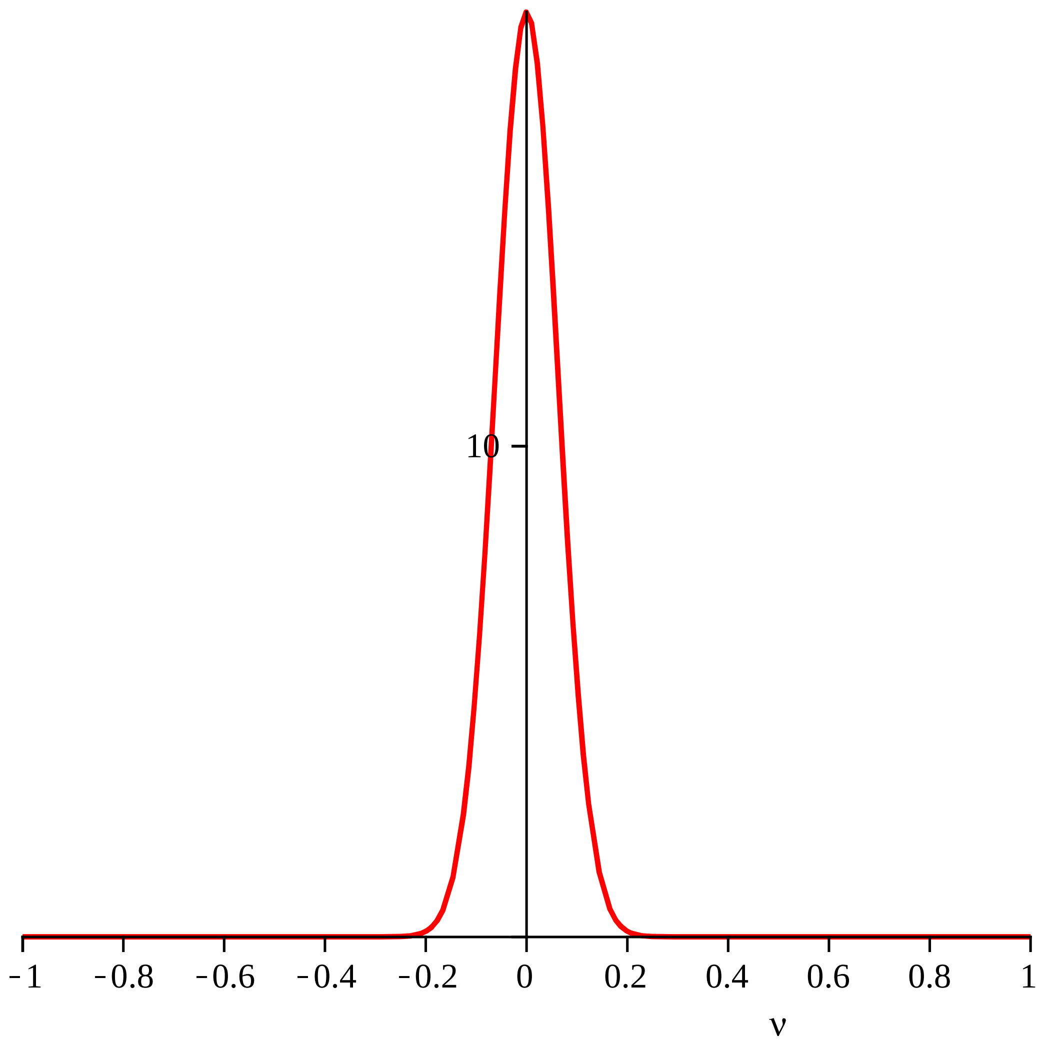
Convolution of two Gaussian

$f1(x) * f2(x) = \mathcal{F}\{f1(x)\}\times\mathcal{F}\{f2(x)\} = g1(\nu)\times g2(\nu)$

$h(\nu) = g1(\nu)\times g2(\nu)$

```
> h := ν→g1(ν)·g2(ν) : 'h(ν)'=h(ν);
  plot(h(ν), ν =-1..1, thickness = 2,  tickmarks = [10, 3]);
```

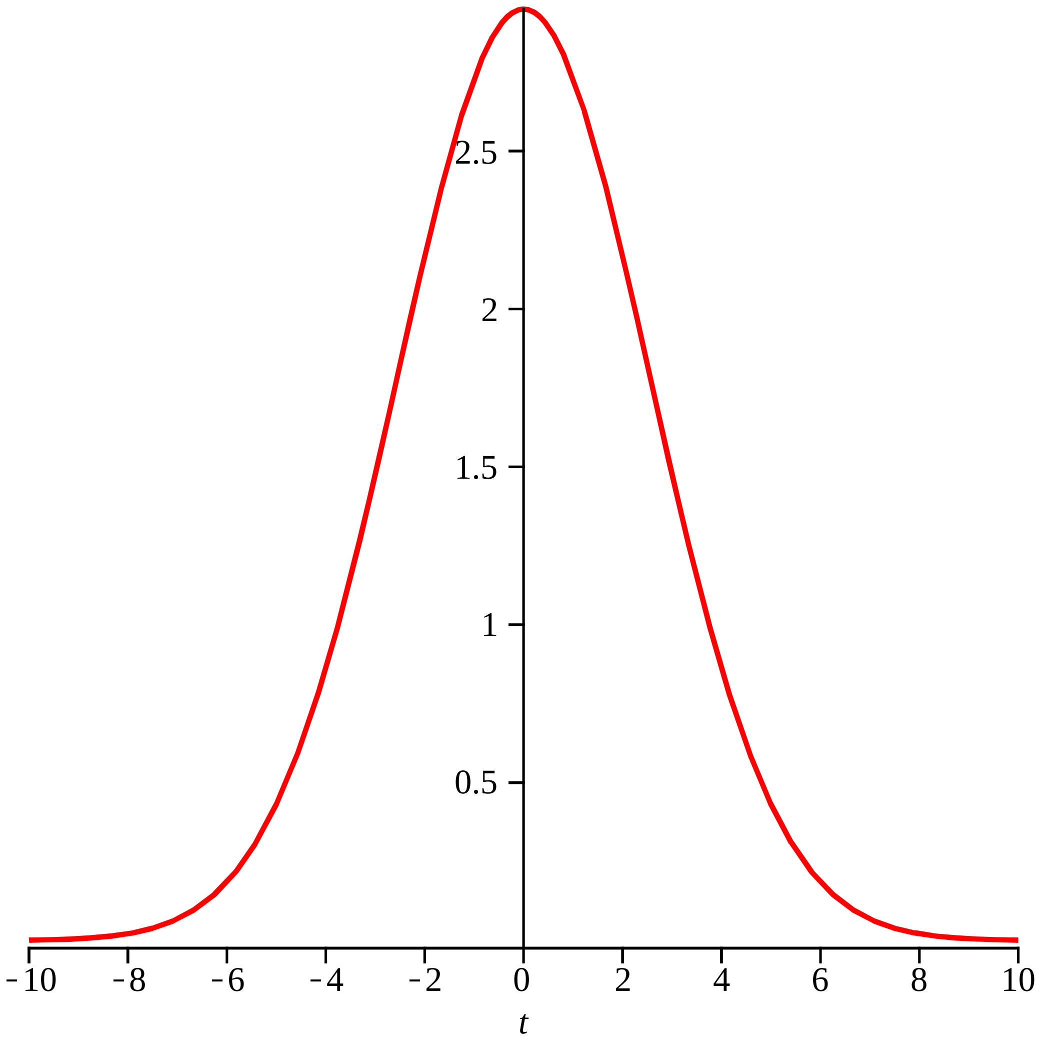
$$h(\nu) = 6\,e^{-4\pi^2\nu^2}\,\pi\,e^{-9\pi^2\nu^2}$$



f3(x) = $\mathcal{F}^{-1}$\{h(ν)\}

> $f3 := t \rightarrow simplify(2\,\pi \cdot invfourier(h(\nu), \nu, 2\,\pi \cdot t))$:'$f3(t)$'$= f3(t)$;
> $plot(\mathrm{f3}(\mathrm{t}), \mathrm{thickness} = 2, \ \mathrm{tickmarks} = [10, 5])$;

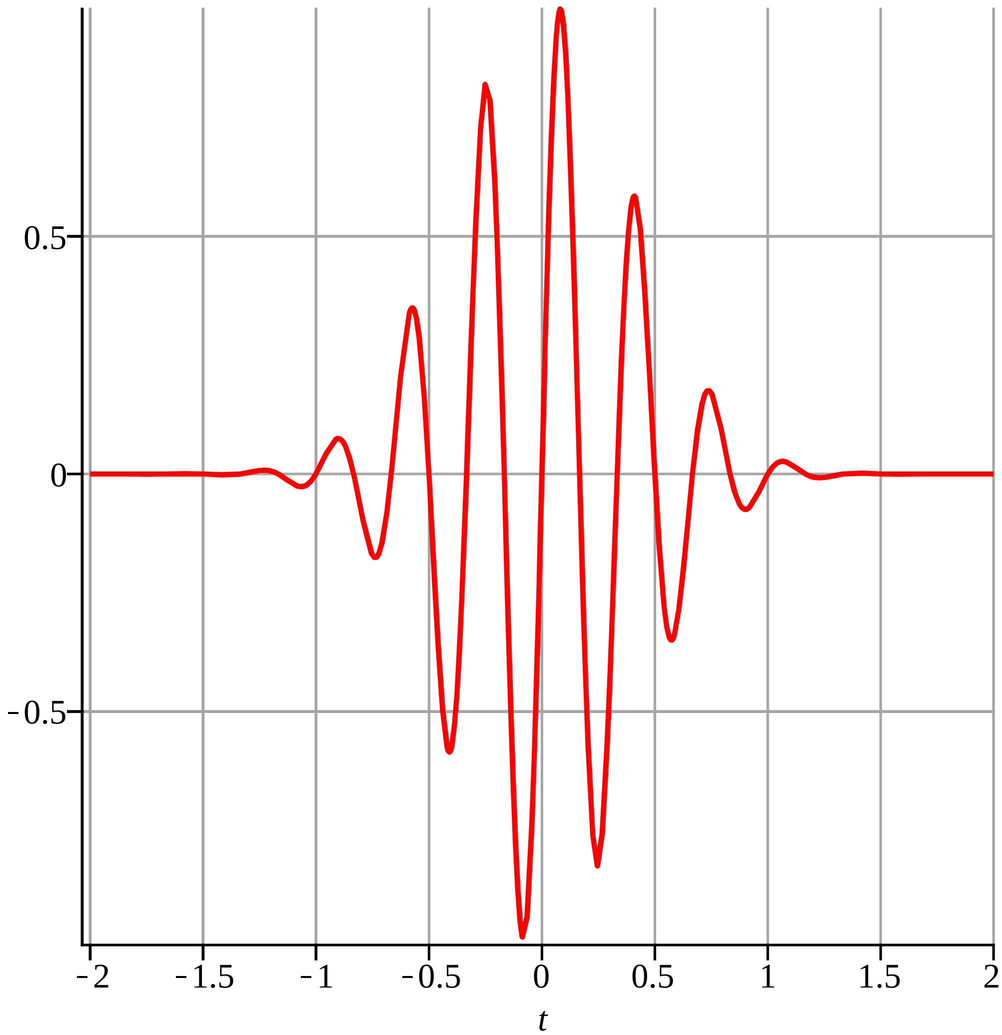
$$f3(t) = \frac{6}{13}\sqrt{\pi}\,\mathrm{e}^{-\frac{1}{13}t^2}\sqrt{13}$$



Unsymmetrical function f(t) = $\sin(6\pi t)\, e^{-\pi t^2}$

> $f := t \to \sin(6\pi \cdot t) \cdot e^{-\pi \cdot t^2}$: $'f(t)' = f(t)$;
plot(f(t), t = −2 ..2, thickness = 2, axes = frame, gridlines = true, tickmarks = [10, 5]);

$$f(t) = \sin(6\pi t)\, e^{-\pi t^2}$$

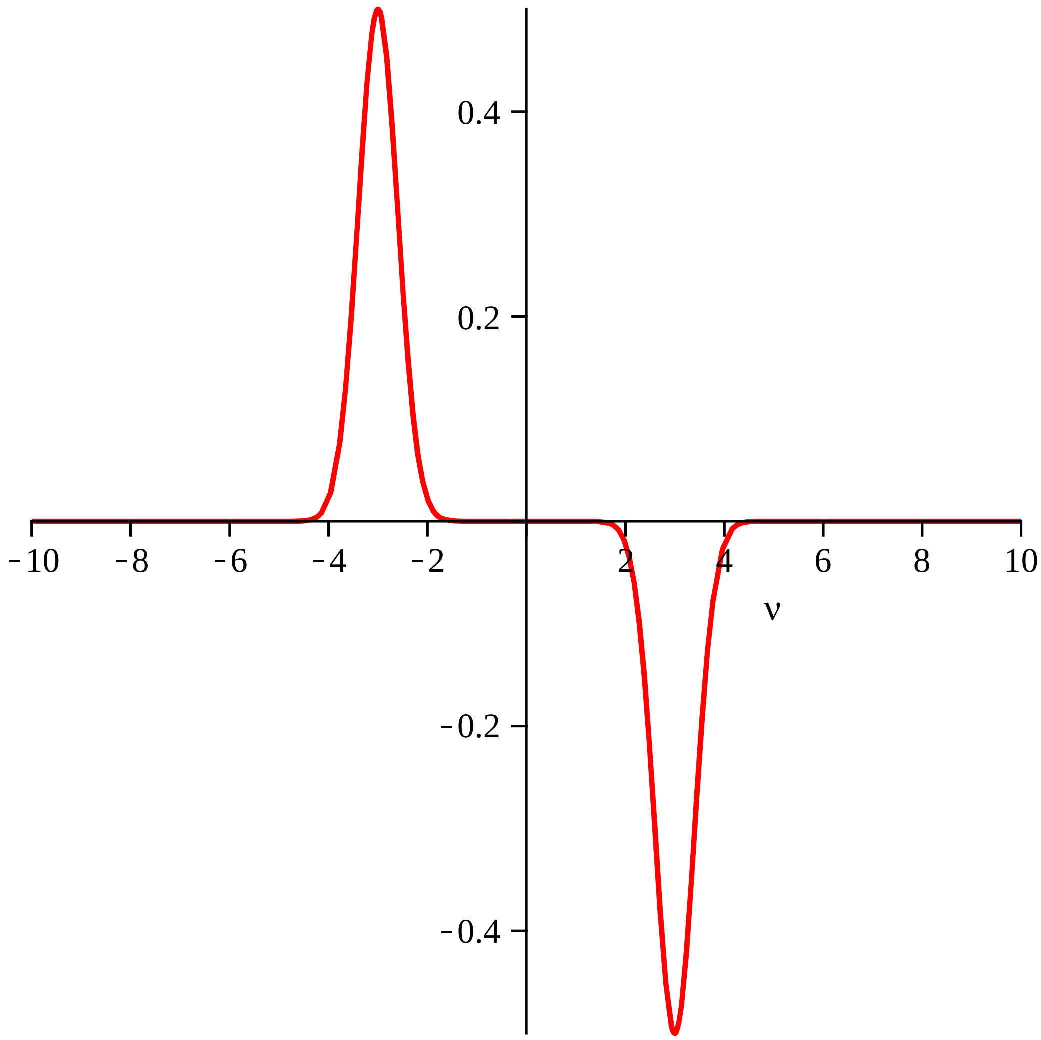


Fourier Transform of f(t) is complex

> $g := \nu \to fourier(f(t), t, 2\pi \cdot \nu)$:
> $'\mathcal{F}\{f(t)\}' = g(\nu)$; $'g(\nu)' = g(\nu)$;
> $plot(\Im(g(\nu)), \text{thickness} = 2, \text{tickmarks} = [10, 5])$; #\` **plotting the imaginary part**

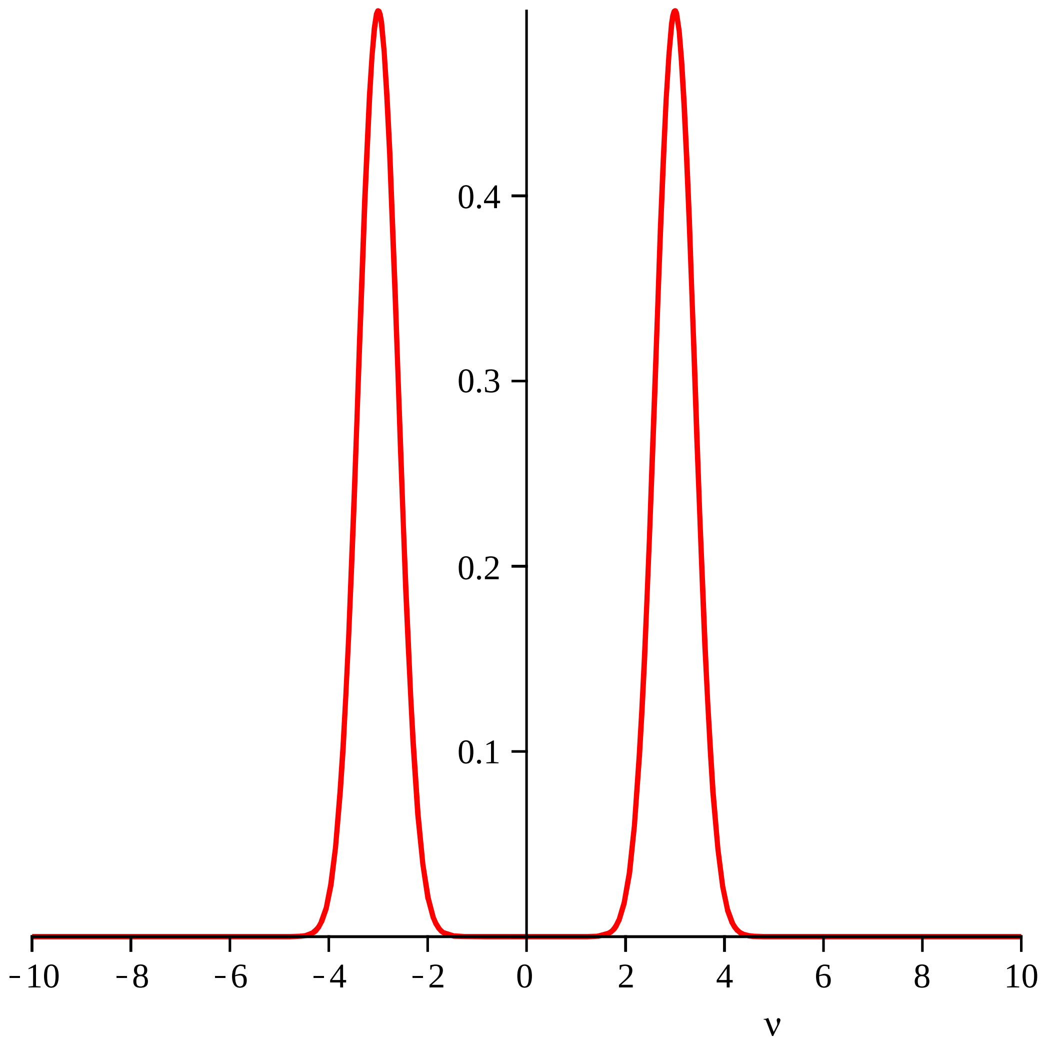
$$\mathcal{F}\{f(t)\} = -I \sinh(6\pi\nu)\, e^{-9\pi - \pi\nu^2}$$

$$g(\nu) = -I \sinh(6\pi\nu)\, e^{-9\pi - \pi\nu^2}$$



Plotting $\|\mathbf{g(\nu)}\| = \sqrt{g(\nu)\cdot g(\nu)^{*}}$

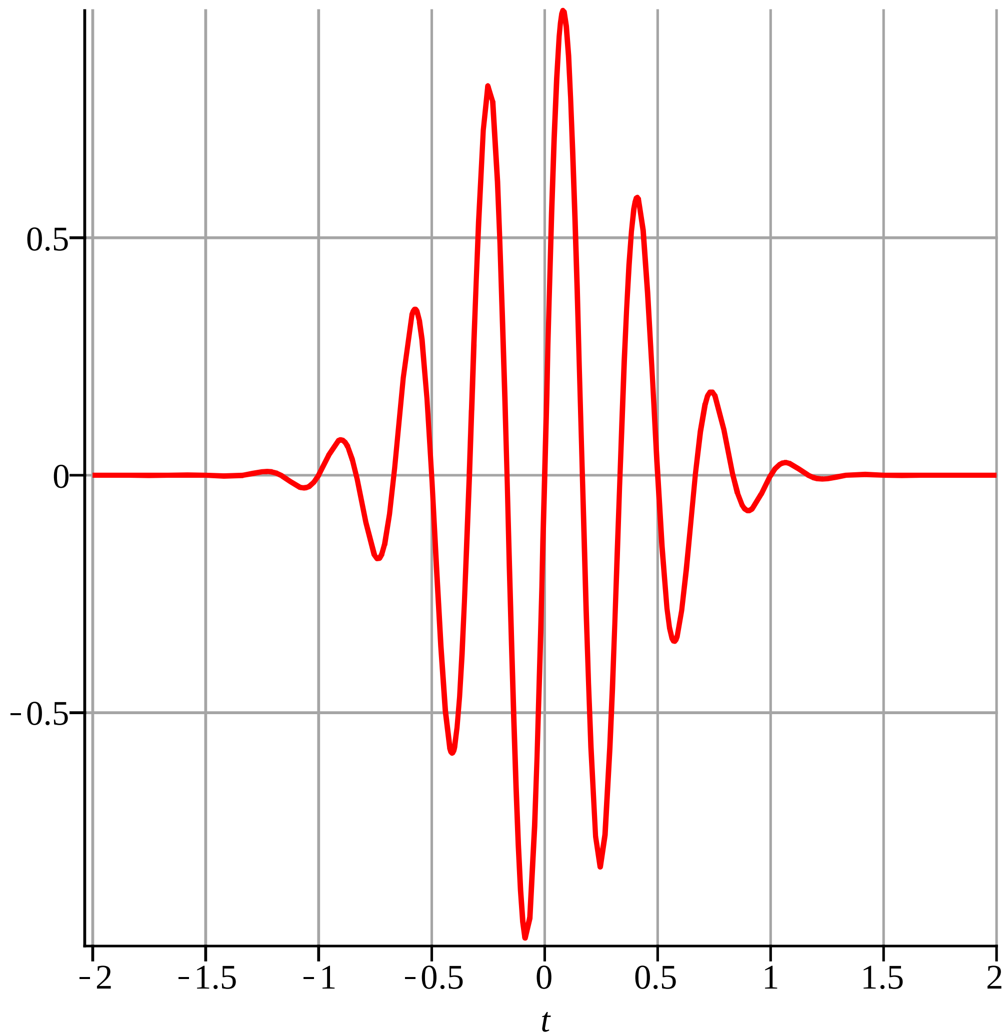
> $plot\left(\sqrt{g(\nu)\cdot conjugate(g(\nu))}\right.$, thickness = 2, tickmarks = [10, 5]$\left.\right)$;



> $z := t \to 2\,\pi \cdot invfourier(g(\nu), \nu, 2\,\pi \cdot t)$:
'$Inv\mathscr{F}\{g(\nu)\}$' = z(t); '$f(t)$' = z(t);
plot(z(t), t = -2 ..2, thickness = 2, axes = frame, gridlines = true, tickmarks = [10, 3]);

$$Inv\mathscr{F}\{g(\nu)\} = \sin(6\,\pi\,t)\; \mathrm{e}^{-\pi t^2}$$

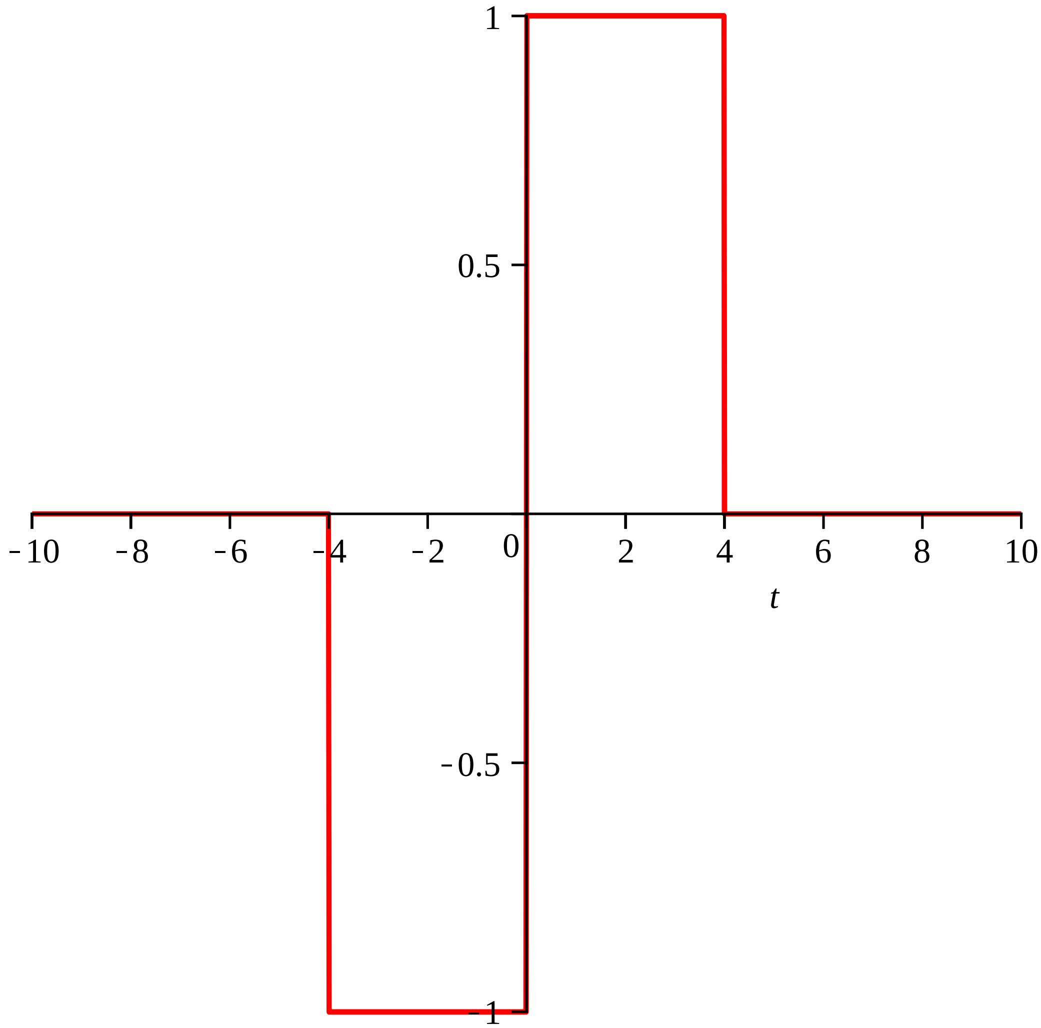
$$f(t) = \sin(6\,\pi\,t)\; \mathrm{e}^{-\pi t^2}$$



Unsymmetrical function f(t)

> $f := t \to piecewise(-4 \le t < 0, -1, 0 \le t < 4, 1, 0) : 'f(t)' = f(t);$
> $plot(f(t), \text{thickness} = 2, \text{tickmarks} = [10, 5]);$

$$f(t) = \begin{cases} -1 & -4 \le t \textbf{ and } t < 0 \\ 1 & 0 \le t \textbf{ and } t < 4 \\ 0 & \textit{otherwise} \end{cases}$$

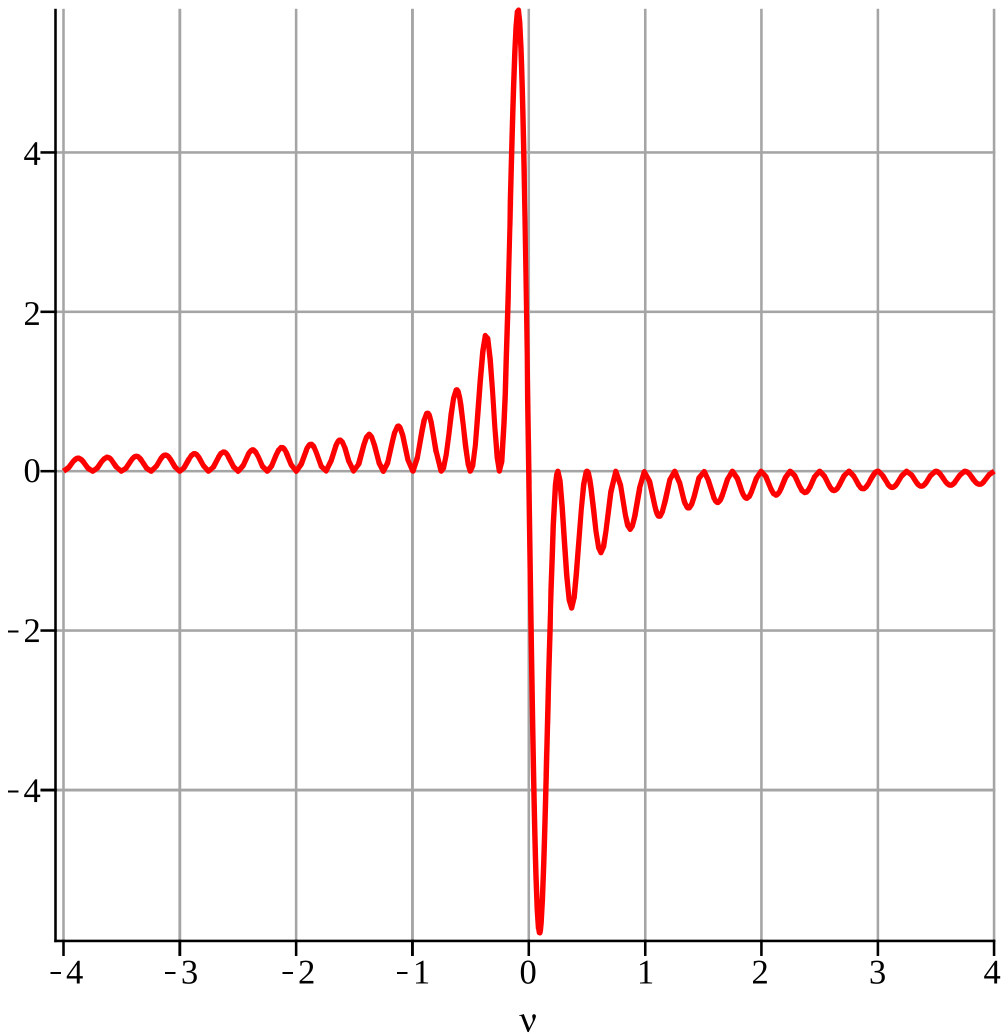


Fourier Transform of f(t) is complex

> $g := \nu \to fourier(f(t), t, 2\pi \cdot \nu)$:
$'\mathcal{F}\{f(t)\}' = g(\nu)$; $'g(\nu)' = g(\nu)$;
$plot(\Im(g(\nu)), \nu = -4..4$, thickness = 2, axes = frame, gridlines = true, tickmarks = [10, 5]);
#` **plottting the imaginary part**

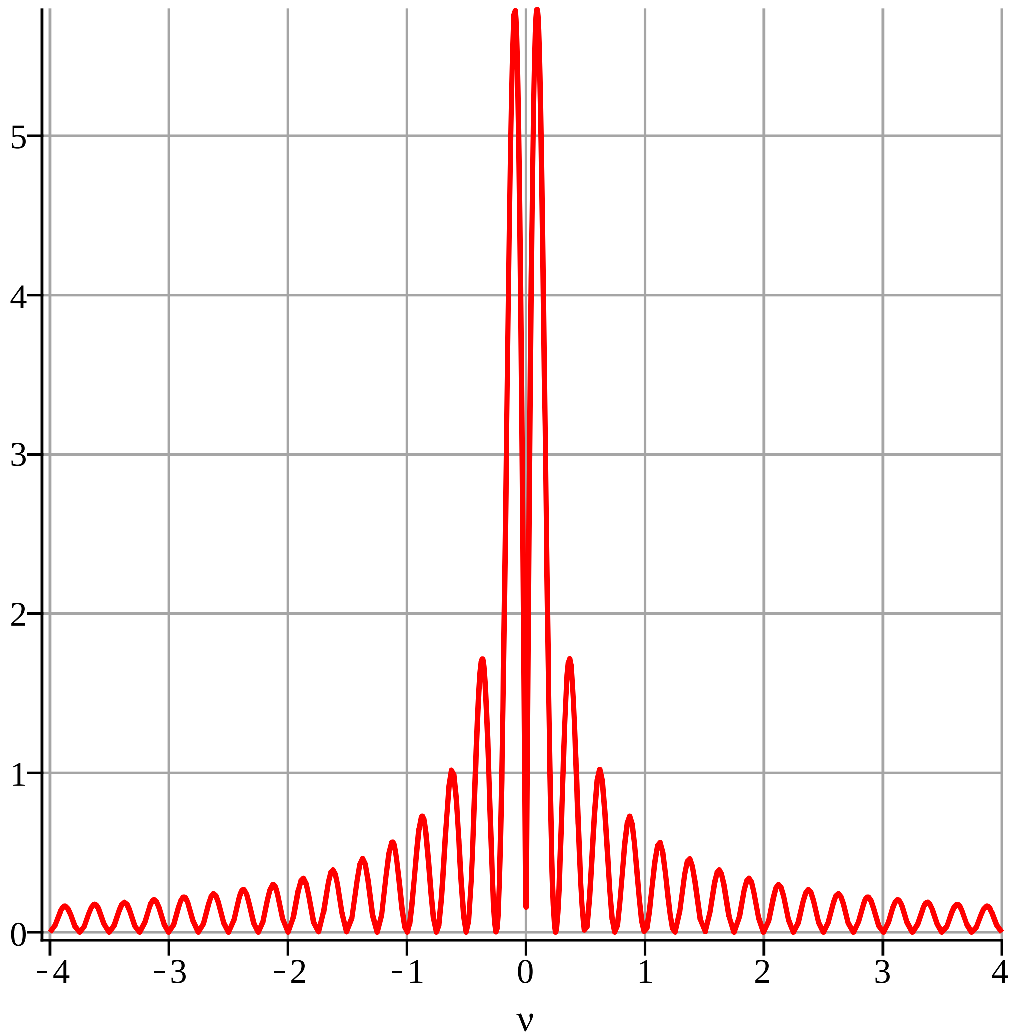
$$\mathcal{F}\{f(t)\} = -\frac{2\,I\sin(4\pi\nu)^2}{\pi\nu}$$

$$g(\nu) = -\frac{2\,I\sin(4\pi\nu)^2}{\pi\nu}$$



Plotting $\|\mathbf{g}(\nu)\| = \sqrt{g(\nu) \cdot g(\nu)^*}$

```
> plot(sqrt(g(ν)·conjugate(g(ν))), ν = -4..4, axes = frame, gridlines = true, thickness = 2, tickmarks = [10, 5]);
```



> $z := t \to 2\,\pi \cdot invfourier\big(g(\nu), \nu, 2\,\pi \cdot t\big)$:
$'Inv\mathscr{F}\{g(\nu)\}' = convert(z(t), piecewise)$;
$plot(\mathrm{z}(\mathrm{t}), \mathrm{thickness} = 2, \mathrm{tickmarks} = [10, 3])$;

$$Inv\mathscr{F}\{g(\nu)\} = \begin{cases} 0 & t \leq -4 \\ -1 & t \leq 0 \\ 1 & t \leq 4 \\ 0 & 4 < t \end{cases}$$